THE UNIVERSITY
of NORTH CAROLINA
at CHAPEL HILL

Undergraduate Research Seminar

Tuesday, October 15th
3:30 - 4:30 p.m.
Zoom

Smoluchowski's coagulation equations, emergence of connectivity in network models and hierarchical clustering

Shankar Bhamidi (UNC STOR)
Akshay Sakanaveeti

Abstract. Smoluchowski's equations, originating from statistical physics and colloidal chemistry, is an (infinite) system of integrodifferential equation describing the density of particles of various sizes in populations of "particles" that coagulate into larger particles based on a kernel that incorporates the sizes of the particles. In the specific case of a multiplicative kernel, these equations predict the emergence of a macroscopic gel within finite time starting from "microscopic dust".

Two at first unrelated questions are the following:

Connectivity of networks: motivated by the importance of networks, a plethora of math models have been proposed to understand the structure of real world systems. Most of these models have a model dependent "critical time" where the connectivity of the network transitions from minute disconnected islands of small sizes to the sudden emergence of a large connected cluster of the same order as the entire system. Hierarchical clustering trees (in computer science e.g. minimal spanning trees on weight graphs) play a fundamental role in many unsupervised learning problems. Simulations predict that these objects, in the large scale limit, should exhibit interesting "fractal" like behavior and in fact it is predicted that properly rescaled, these objects converge, in the space of metric spaces, to random compact fractals.

In the first 30 mins Shankar will give an overview connecting the above three topics. In the last 30 mins Akshay will describe how one can use this intuition to get "fine scale" asymptotics for various functionals of interest and the kinds of math one requires to make progress on such questions.