SLMath Spotlight: Lagrangian Surfaces with Arunima Bhattacharya

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Dear Friends,

The Simons Laufer Mathematical Sciences Institute (SLMath) continues our Spotlight interview series below, featuring Arunima Bhattacharya.
Dr. Bhattacharya spoke to us about Lagrangian surfaces and the importance of role models in math.

SLMath is a landmark of US and world collaborative mathematical research. Over 1,700 mathematical scientists visit our Berkeley, California headquarters each year. It is a place where breakthroughs are made, research areas are created, and brilliant careers are launched.

Your continued support allows us to welcome these gifted scientists, who will, in the span of their careers, transform mathematics. Please read on for an interview with Arunima Bhattacharya.

Best regards,

Tatiana Toro

Tatiana Toro
Director, MSRI / Simons Laufer Mathematical Sciences Institute (SLMath)
Professor of Mathematics, University of Washington

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SLMath Community Spotlight

Meet an SLMath Postdoc| Arunima Bhattacharya

Interviewed by Evelyn Lamb and Uta Lorenzen

Arunima Bhattacharya was a postdoctoral fellow in the Fall 2022 program Analytic and Geometric Aspects of Gauge Theory. She is now a tenure-track assistant professor at the University of North Carolina at Chapel Hill.

Can you tell us about how you got started in math?

I've always enjoyed doing math. Fortunately, for me, I come from a family of academics. My dad is a professor of chemistry, and my mom is a professor of English literature, so they both understand the academic career and academic life. They didn't force me to be in academia, but they were very happy about the fact that I chose to be in academia. I would say that I'm quite lucky because I knew what academic life is like before I got into it. They're not super happy that their only child lives so far away, but they are very supportive of the fact that I'm pursuing something that I think adds meaning to my life.

What are some of the motivating questions for your work?

My research is in geometric analysis with a focus on fully nonlinear second- and fourth-order elliptic partial differential equations that arise naturally in differential geometry. I primarily study geometric variational problems applying tools from minimal surface theory, Lagrangian geometry, Kähler geometry, geometric measure theory, and the theory of elliptic equations.

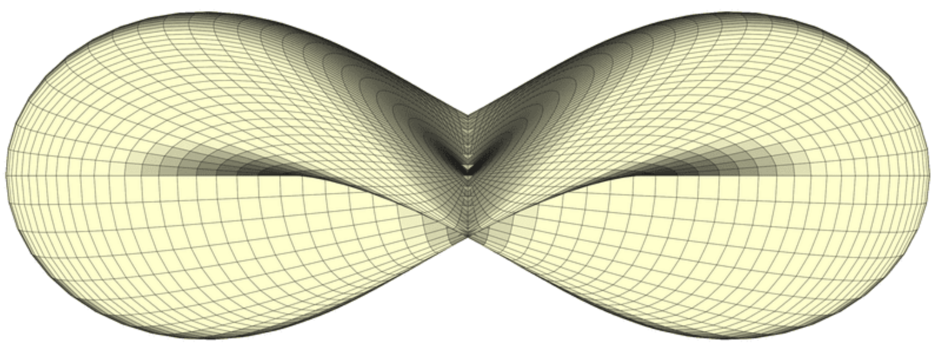
Area minimization problems amongst Lagrangian surfaces lead to the study of certain fourth order PDEs that I work on. Lagrangian surfaces are those that are half codimensional, that is, they sit inside spaces of twice their dimension. For example, a one-dimensional circle on the surface of a two-dimensional sphere is a Lagrangian. Geometrically motivated variational problems for the volume functional, in the Lagrangian setting, give rise to nonlinear fourth-order elliptic equations. A key tool for the study of second-order elliptic equations is Schauder theory, which is by now well-developed with equations in divergence form occupying an important place. In fourth order, an inherent analog is an equation in linear or nonlinear form that has a double divergence structure. Equations such as bi-harmonic functions, extremal Kähler metrics, the Willmore surface, the Hamiltonian stationary equations, which are closely linked to elastic mechanics, enjoy this structure. Fourth-order equations in double divergence form share similar features with second-order equations in divergence form; however, they remain a less explored but an important developing area of geometric analysis.

When I hear partial differential equations, I think of applied math. Does the motivation come from specific applications?

My work on some fully nonlinear second order equations, like the Lagrangian mean curvature equation, is closely related to mirror symmetry in physics. The range of the angle of a Lagrangian submanifold, also known as the Lagrangian phase, plays a dominant role in determining regularity for the potential of the Lagrangian surface. In the 1970s, Harvey and Lawson showed that the mean curvature vector of such a submanifold is determined by its angle. When the angle is constant, the Lagrangian submanifold turns out to be volume minimizing. These Lagrangian submanifolds are special and so they deserve a special name: they are called the special Lagrangian submanifolds. In the 90s, Strominger, Yau, and Zaslow conjectured that the existence of such submanifolds is the key to understanding mirror symmetry. This is a problem that requires mathematics from diverse areas ranging from symplectic geometry to derived categories to nonlinear partial differential equations. I study the nonlinear PDE part of this problem.

What does it mean for you to be a mathematician?

I think the one thing that connects all stages of your career as a mathematician is that you want to make a significant contribution to mathematics in your own way. This has always been one of my goals: I want to contribute something that is meaningful. The second thing, which is equally important to me, is to create an atmosphere where underrepresented mathematicians, especially women, feel more comfortable to pursue the beauty of this subject. I've been actively involved in promoting diversity since grad school, and I take it seriously. At UNC (and previously at UW), I am an active member of the diversity committee. At UO, I was the workshop chair for a group called the CMiS (community for minorities in STEM) where I helped organize various workshops. There are also summer workshops in India for undergraduate women in mathematics, which I plan to join next year as a mentor. For me, having role models has been very motivating. I think that if you don't see enough people like you doing math, you start to question yourself. But when you do see more people, who look like you, doing some amazing mathematics, it can be very inspiring and can make you think, "I want to be like her."



Lagrangian mean curvature flow of Whitney spheres - Scientific figure by Andreas Savas-Halilaj, available on ResearchGate.

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MSRI is becoming the Simons Laufer Mathematical Sciences Institute (SLMath) in 2022-2023.