THE UNIVERSITY *of* NORTH CAROLINA *at* CHAPEL HILL

Undergraduate Analysis and PDE Seminar

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Zoom

An NLS case study: well-posedness beyond standard models

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Abstract. We consider a nonlinear Schrödinger (NLS) equation

$$iu_t + \Delta u + \lambda \mathcal{N}(u)\, u = 0,$$

where the potential $\mathcal{N}$ can be thought of as a combination of different nonlinearities, for example, $\mathcal{N}(u) = \sum a_k |u|^{\gamma_k}$, $a_k \in \mathbb{C}$ and $\gamma_k > 0$. Typically, to show the existence and uniqueness of solutions, a contraction mapping argument with Strichartz estimates is applied on the Duhamel (integral) equation

$$u(t) = e^{it\Delta} u_0 + i\lambda \int_0^t e^{i(t-s)\Delta} \big(\mathcal{N}(u)\, u\big)(s)\, ds$$

on some space, for instance, a subset of a Sobolev space H^s with certain conditions on a_k, γ_k, initial data u_0 (and time). This approach works for a power-type nonlinearity $|u|^\gamma u$ with powers $\gamma \geq 1$.

In this talk, we will discuss another approach to obtain well-posedness in the NLS equation for nonlinearities that go beyond standard models, such as an infinite series of power nonlinearities. This would include an exponential, $e^{|u|^r}$, or sinusoidal, $\sin(|u|)$, potentials among various others nonlinearities. First, we discuss the NLS on the whole space and then shift the gears to consider the periodic setting. On T^N we show the local well-posedness of solutions to the NLS equation with nonlinearities that are not possible to consider on the whole space, for example, nonlinearities with negative powers: $\frac{u}{|u|}$ and $\frac{u}{e^{|u|}}$.

We also construct a class of initial data u_0 such that there exists a unique, local solution of the inhomogeneous NLS equation $iu_t + \Delta u + \lambda V(x)|u|^\alpha u = 0$ on $\mathbb{R}^N$, where $V(x)$ has polynomial-like behavior. In addition, we consider the periodic setting of inhomogeneous NLS, in which we find local well-posedness for every $\alpha \in \mathbb{R}$. Finally, we show that this approach applies to the NLS with higher order of dispersion such as the bi-harmonic NLS.

This talk is based on research done by Gia Azcoitia, Hannah Wubben, Beckett Sanchez, Troy Roberts, Sam Kilgore, Alex D. Rodriguez, Iryna Petrenko, Oscar Riaño, and Svetlana Roudenko.