THE UNIVERSITY
of NORTH CAROLINA
at CHAPEL HILL

Undergraduate Analysis and PDE Seminar

February 17, 2023
1:30 - 2:30 p.m.
Zoom

Spectral and scattering theory for waveguides

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Adam Clay

Abstract. Waveguides are Euclidean domains in which waves, such as electromagnetic or acoustic waves, are transmitted or guided along a channel, such as a cable. The basic example is a strip in the plane given by the Cartesian product of a line and an interval, for example $\mathbb{R} \times (0, 1)$.

In the first part of the talk, Kiril Datchev will give an introduction to the study of waves in this setting, including the different types of spectrum that arise and the role that they play, and explain how these are related to the geometry of the waveguide. We will see how confining or trapping aspects of the geometry (the direction of the interval in the example above) can lead to discrete spectrum and stationary waves, while unbounded aspects (the direction of the line in the example above) lead to continuous spectrum and decaying waves. This discussion is based on joint work with Tanya Christiansen.

The richest behavior is seen when trapping is substantial, and in general the behavior here is poorly understood. But in the second part of the talk Adam Clay will present a simple example with heavy trapping, namely a waveguide given by a strip with a jump in the wavespeed, where a detailed analysis is possible. In this setting we will see that a significant number of eigenvalues occur, and that they obey precise Weyl asymptotics analogous to the celebrated ones for a bounded domain or manifold.