THE UNIVERSITY of NORTH CAROLINA at CHAPEL HILL

Undergraduate Analysis and PDE Seminar

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1:30 - 2:30 p.m.
Zoom

On the spectrum of general-relativistic hydrogenic ions with an anomalous magnetic moment

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Abstract. The spectral lines of the hydrogen atom and hydrogenic ions have played a fundamental role in the development of quantum mechanics, first in the non-relativistic approximation (with the Schrödinger equation), and subsequently in the special-relativistic formulation (with the Dirac equation). In this talk, we report on joint work with Michael Kiessling and Shadi Tahvildar-Zadeh, in which we investigate the effects general relativity can have on hydrogenic spectral lines. We do this by studying the Dirac equation for a single electron on a curved spacetime background that models the vacuum outside the nucleus. In general relativity, a point mass with charge can be modeled by the family of Reissner-Weyl-Nordström (RWN) spacetimes. (Given the mass and charge of an atomic nucleus, this puts us well within the naked singularity sector of the RWN spacetimes). In the 1980s, Cohen and Powers showed that Einstein's theory of general relativity has dramatic effects on the quantum-mechanical theory of atomic spectra. Specifically, they showed that the corresponding Dirac Hamiltonian is not essentially self-adjoint (leading to a loss of predictability). In the early 2000s, Belgiorno et al. showed that essential self-adjointness is restored if the electron's anomalous magnetic moment is not too small. (The empirical value of the electron's anomalous magnetic moment overwhelmingly satisfies this condition.) They also showed that the discrete spectrum consists of an infinite number of eigenvalues, but they did not provide any other information about it. In this talk, we show how one can completely characterize the discrete spectrum in terms of winding numbers of heteroclinic orbits of a dynamical system on a finite cylinder and compare it with the well-known Bohr-Sommerfeld spectrum in the special-relativistic formulation.