THE UNIVERSITY
of NORTH CAROLINA
at CHAPEL HILL

Analysis and PDE Seminar

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3:00 - 4:00 p.m.
PH 328

How smooth is a C^2 surface?

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Abstract. Let $M \subset \mathbb{R}^n$ be a k-dimensional manifold, smooth of class C^2, i.e., locally the graph of C^2 maps $V \to V^\perp$, where $V \subset \mathbb{R}^n$ is a linear space of dimension k. We then ask the question:

How smooth is M?

This question might appear to be trivial, but actually it's not. In our talk we analyze the question, bringing in the notions of harmonic coordinates, the Riemann tensor, casting the Ricci curvature equation in harmonic coordinates, elliptic regularity, L^p Sobolev spaces, and BMO.

We draw conclusions about the geodesic flow on M, and, going further, about microlocal propagation of singularities for the wave equation $\partial_t^2 u - \Delta u = f$ on $\mathbb{R}_t \times M$, and on the decay of solutions to damped wave equations

$$\partial_t^2 u + a(x)\partial_t u - \Delta u = 0,$$

under control conditions (seen to be relevant due to results on the geodesic flow hinted above). The result here would be inaccessible from a naive answer to the initial question posed above.