KADANOFF CENTER *for* THEORETICAL PHYSICS

Simons Collaboration on Ultra-Quantum Matter

Bulk-Boundary Correspondence for Interacting Floquet Topological Phases in 2D

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CZ & ML, arXiv:2209.03975

CZ, in prep

Setup: interacting Floquet systems

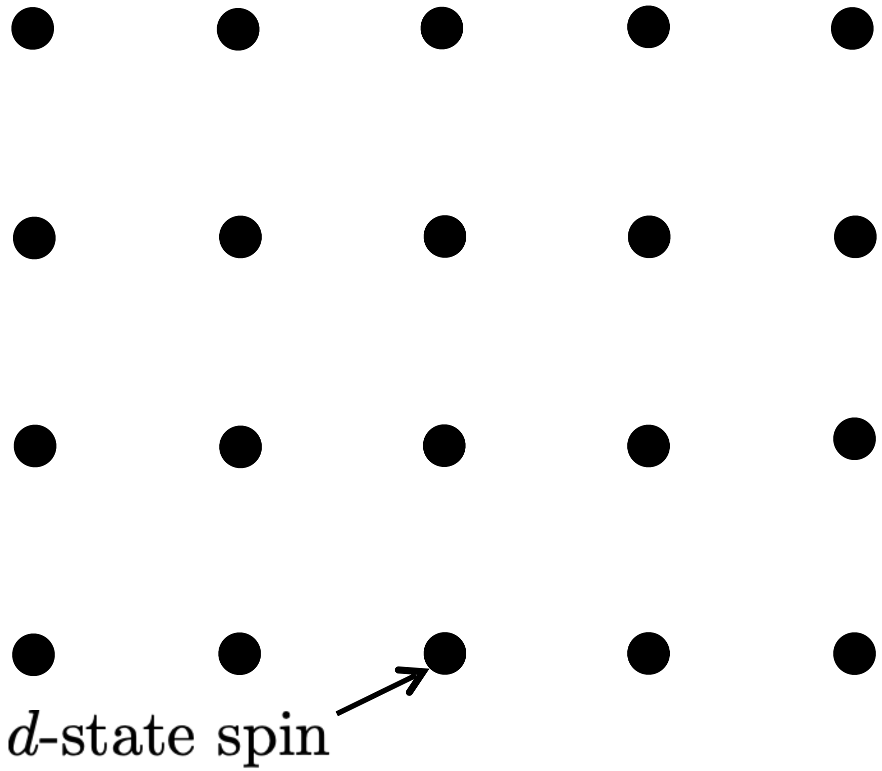
Consider a 2D system of d-state spins on a lattice, evolving under

$$H(t) = \sum_r H_r(t)$$

$$H(t) = H(t+T)$$

The system is described by
a circuit $\{U(t)\}$, where

$$U(t) = \mathcal{T} e^{-i \int_0^t dt' H(t')} \qquad t \in [0, T)$$



Why are Floquet phases interesting?

- We can use some familiar tools from stationary topological phases.
- There are new phases that cannot be realized in stationary systems.
- We can study them in the lab.

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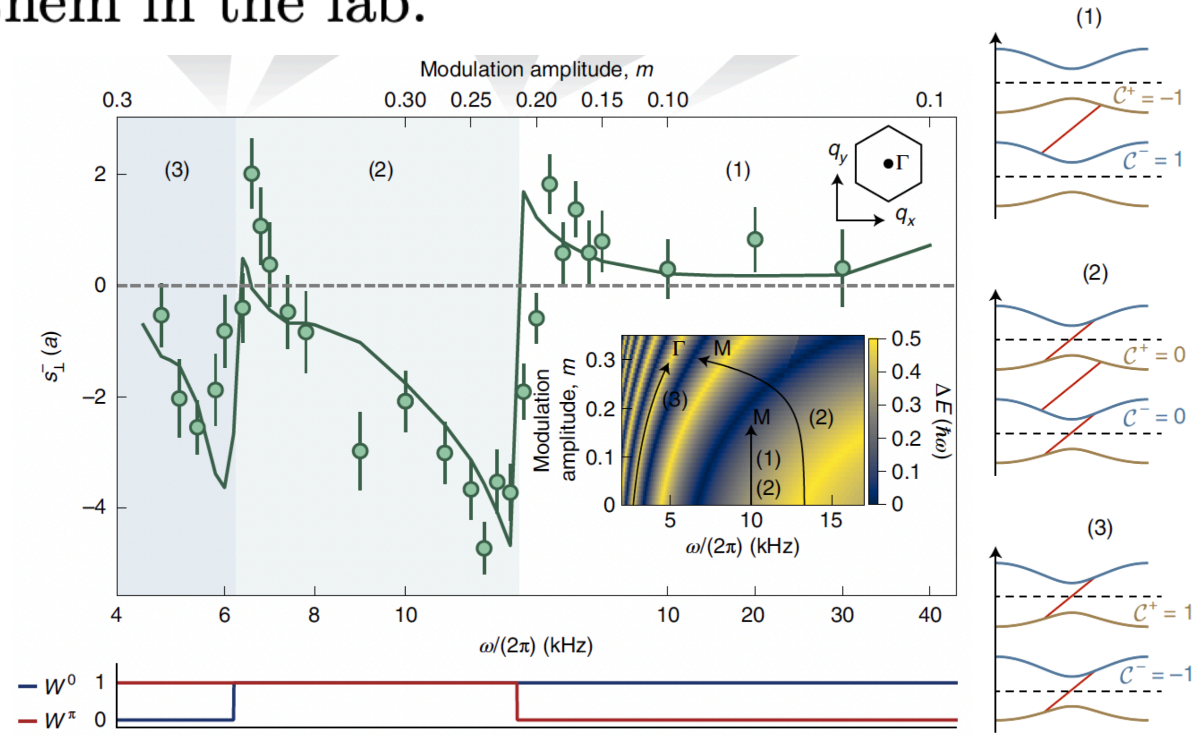


Wintersperger et al, 2020

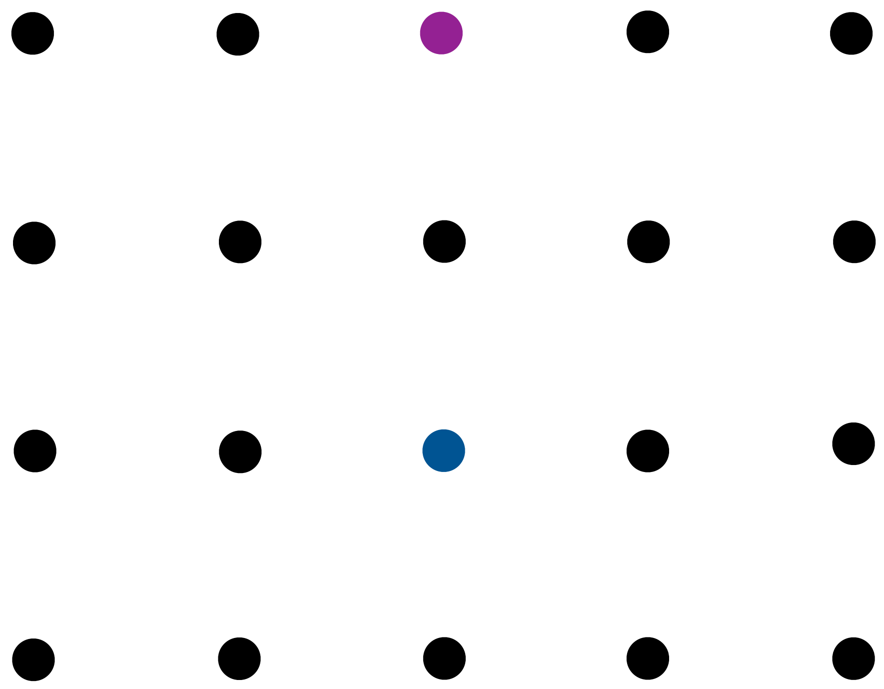
Preliminaries

- Example of a Floquet topological phase in non-interacting systems
  $\rightarrow$Example of bulk-boundary correspondence
- Definition of interacting Floquet phase
- Review of classification of interacting Floquet phases
- Main question: general interacting bulk-boundary correspondence

Non-interacting Floquet systems

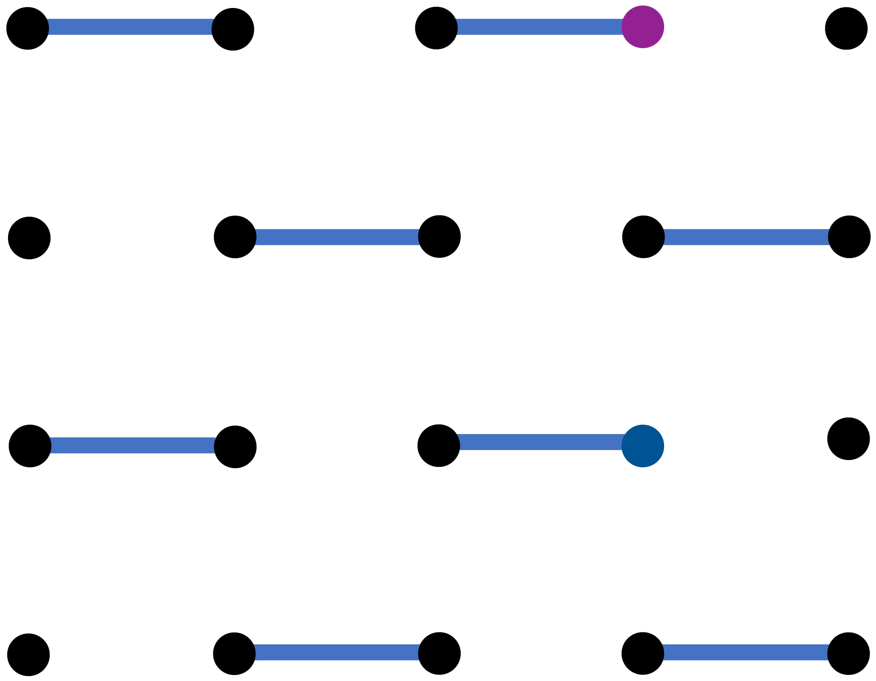
Example of nontrivial circuit:
4-step hopping circuit

Rudner et al, 2013



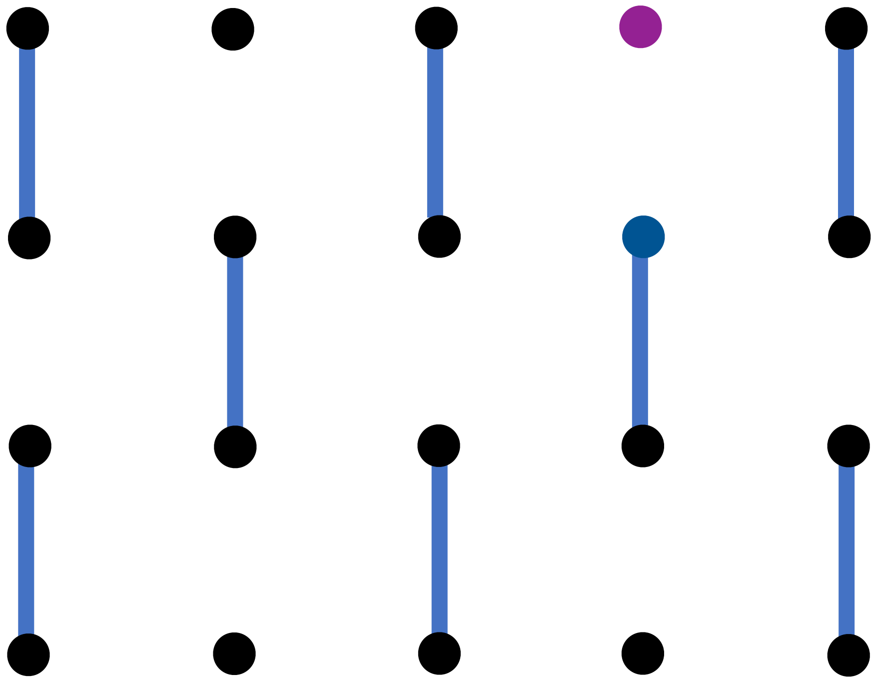
Non-interacting Floquet systems

$$0 \leq t < \frac{T}{4}$$



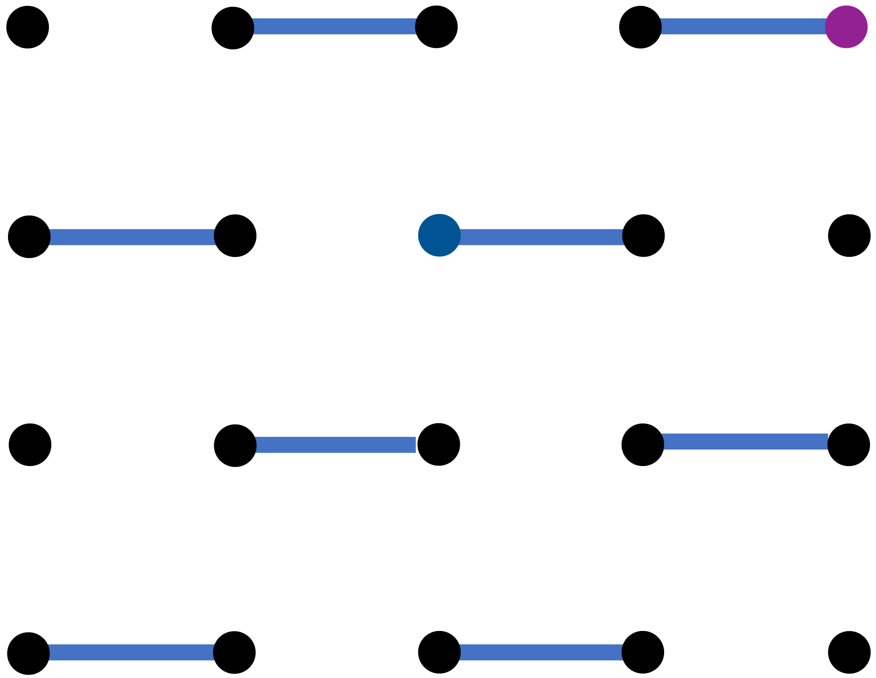
Non-interacting Floquet systems

$$\frac{T}{4} \leq t < \frac{T}{2}$$



Non-interacting Floquet systems

$$\frac{T}{2} \leq t < \frac{3T}{4}$$



Non-interacting Floquet systems

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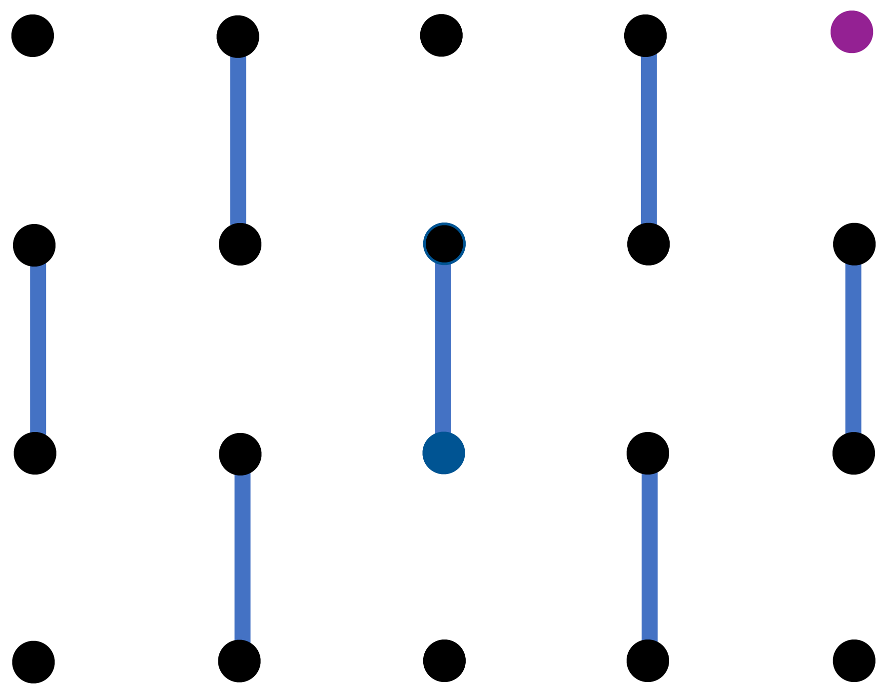
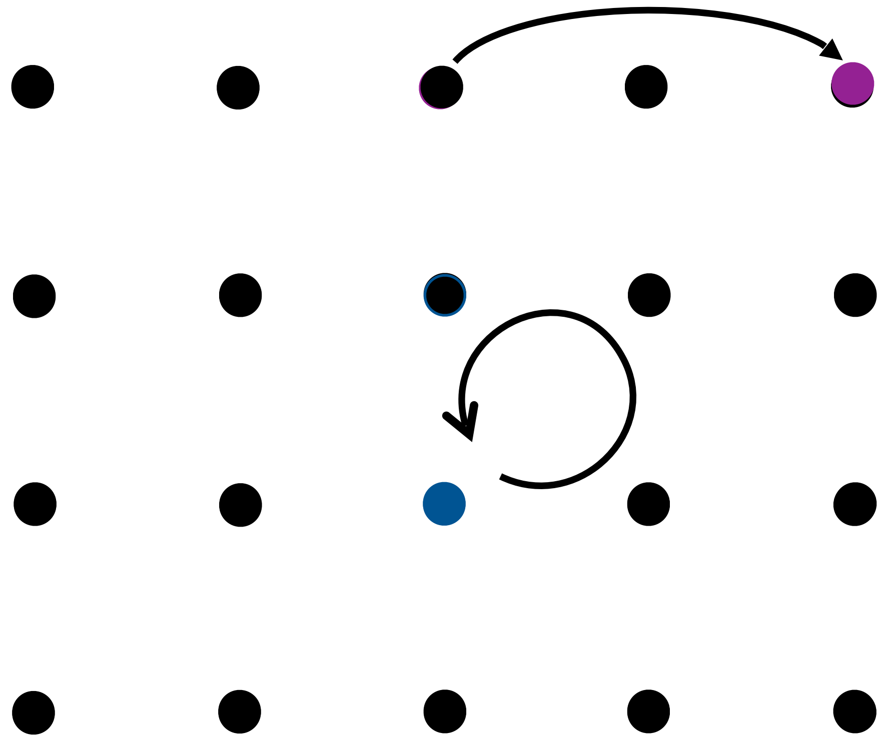


Non-interacting Floquet systems

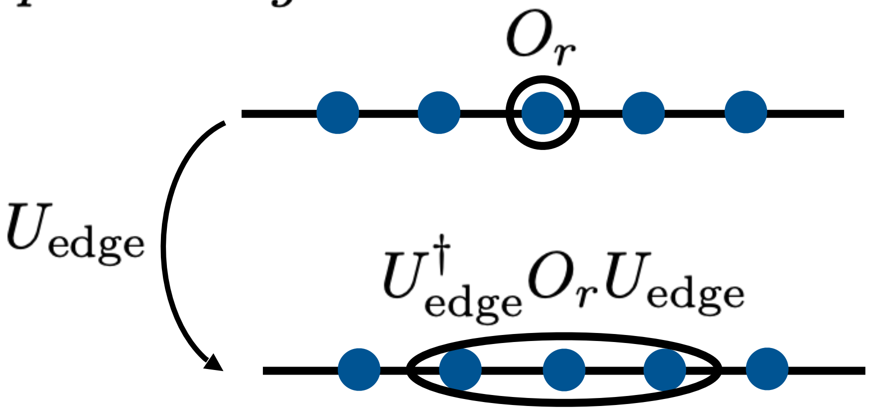
Bulk states have *trivial* stroboscopic dynamics.
Edge states are *translated*.

$$U(T) = \overbrace{U_{\text{bulk}}}^{\text{trivial}} \cdot \underbrace{U_{\text{edge}}}_{\text{anomalous}}$$



Anomalous edge unitaries

U_{edge} must be *locality preserving*:



However, U_{edge} is not *locally generated*:

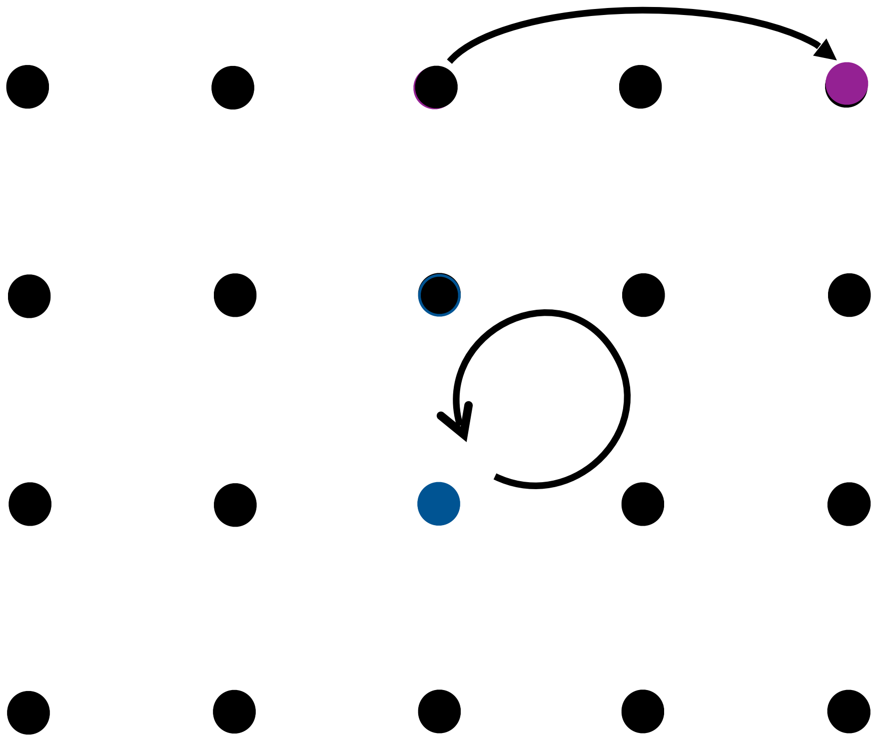
$$\text{Translation} \neq \text{1D finite-depth local unitary (FDLU)}$$ Gross et al, 2012

$$= \mathcal{T} e^{-i \int_0^T dt H_{1\text{D}}(t)}$$

$\rightarrow U_{\text{edge}}$ can only occur at the boundary of a 2D system

How do we detect the anomaly in the bulk?

The full path of unitaries $\{U(t)\}$ describes bulk *micromotion.*



Bulk Boundary Correspondence

Edge invariant: $n(U_{\text{edge}})$

Bulk invariant: $\mathcal{W}(\{U(t)\})$

$$\boxed{n(U_{\text{edge}}) = \mathcal{W}(\{U(t)\})}$$

Rudner et al, 2013

stroboscopic
edge dynamics

continuous
bulk micromotion

Many-body localized (MBL) Floquet circuits

Generic $\{U(t)\} \rightarrow$ thermalization.

To avoid this, we specialize to *many-body localized* Floquet systems

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On a *closed* geometry:

$$\boxed{U(T) = \prod_r U_r \qquad [U_r, U_{r'}] = 0}$$

U_r are
(quasi)-local unitaries

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For classification purposes, we can restrict to $\{U(t)\}$ satisfying

$$\boxed{U(T) = \mathbf{1}}$$

MBL Floquet phase

$\{U(t)\} \sim \{U'(t)\}$: there exists $\{U_s(t) : s \in [0,1]\}$ such that

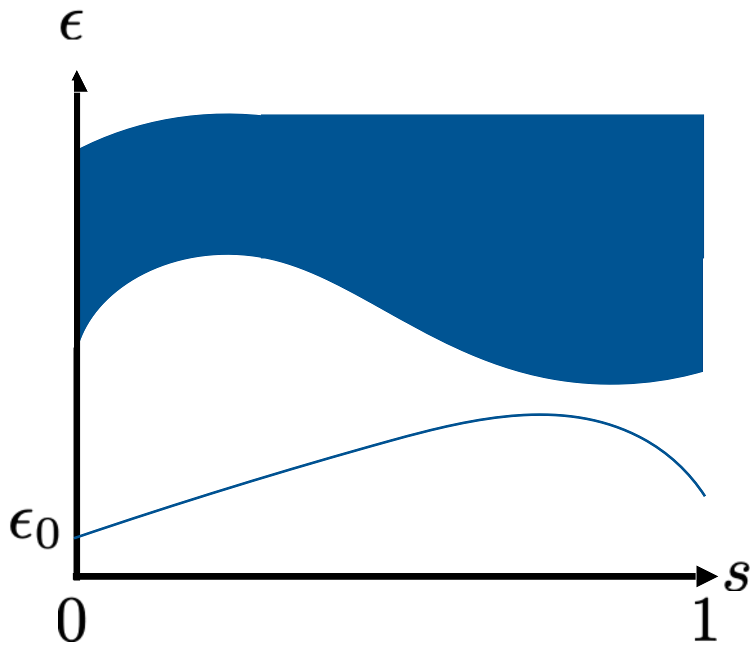
$$U_0(t) = U(t) \qquad U_1(t) = U'(t)$$

While maintaining the MBL condition for all s:

$$U_s(T) = \mathbf{1}$$

Similar to stationary definition of equivalence:

$H \sim H'$ if there exists an interpolation that does not close the gap

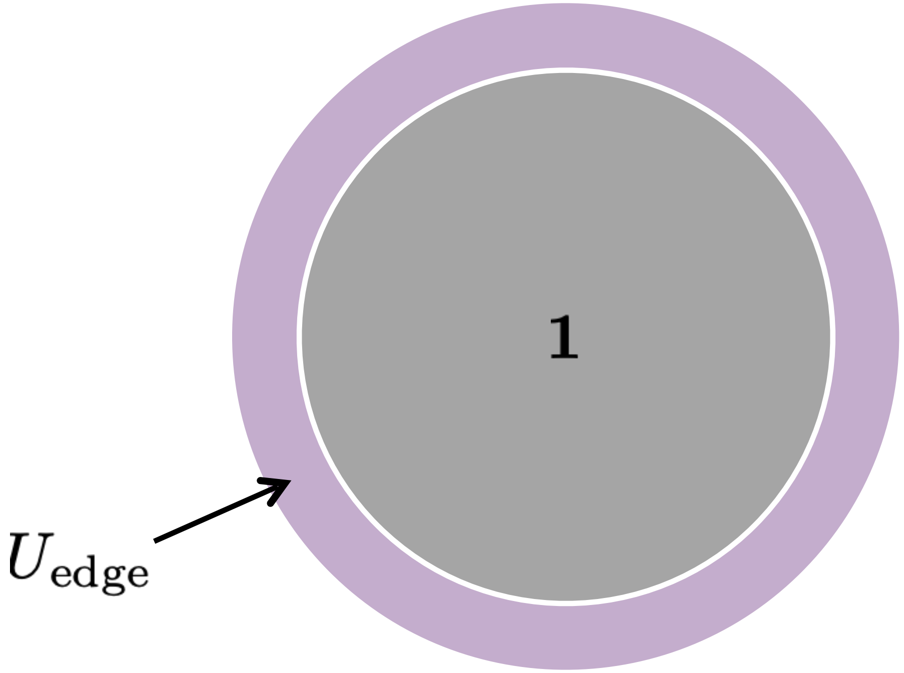


Floquet edge unitaries

If $U(T) = \mathbf{1}$ for a closed geometry, then in a geometry with an edge,

$$\boxed{U(T) = \mathcal{T} e^{-i \int_0^T dt H_R(t)} = U_{\text{edge}}}$$

U_{edge} is a 1D locality preserving unitary localized near the edge of R.



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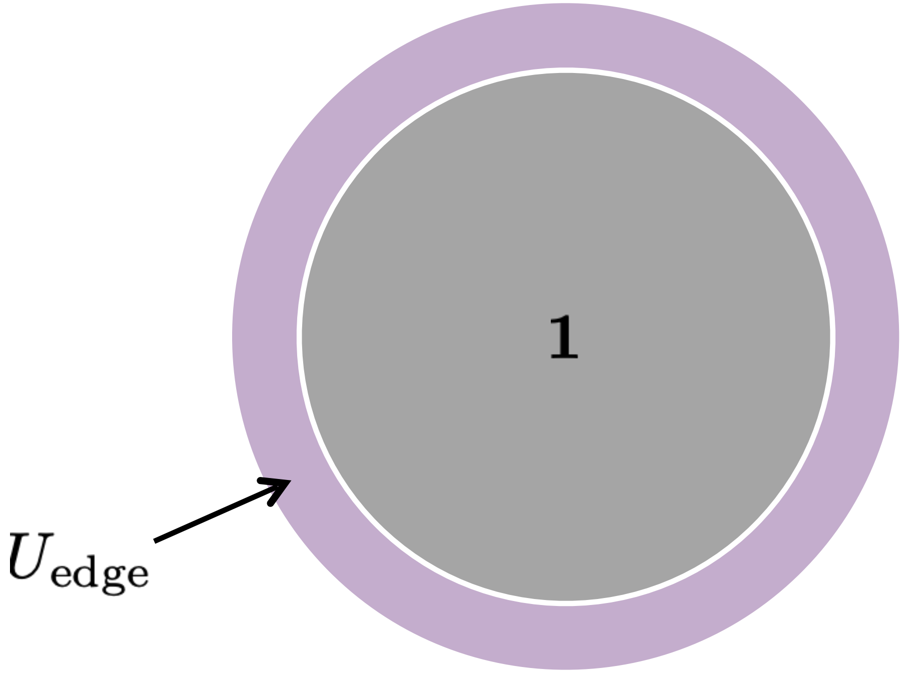


$$U_{\text{edge}} \sim U'_{\text{edge}} : U_{\text{edge}} = W \cdot U'_{\text{edge}}$$

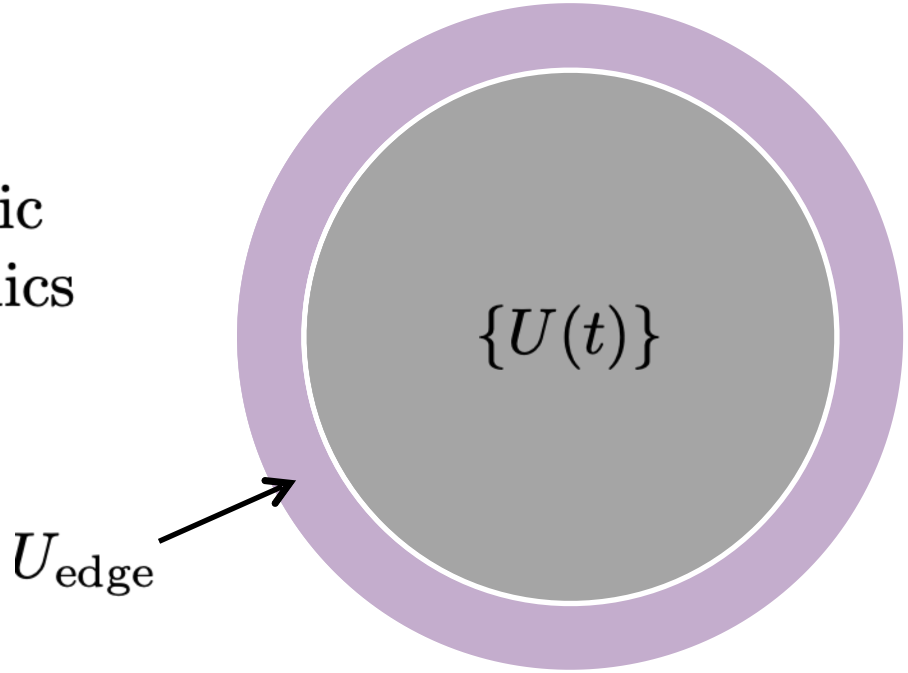
$$W = \text{1D FDLU}$$

Interacting bulk-boundary correspondence

$$\boxed{\{\text{2D Floquet phases}\} = \{U_{\text{edge}}\}}$$

continuous
bulk micromotion

stroboscopic
edge dynamics



What phases can be realized in these systems?

Floquet phases with no symmetry

$\rightarrow$ transports quantum information along the edge

Gross et al, 2012
Po et al, 2016

Floquet phases with discrete unitary symmetries

$\rightarrow$ toggles the edge between different 1D SPTs

Else & Nayak, 2016
Gong et al, 2020

Floquet phases with $U(1)$ symmetry

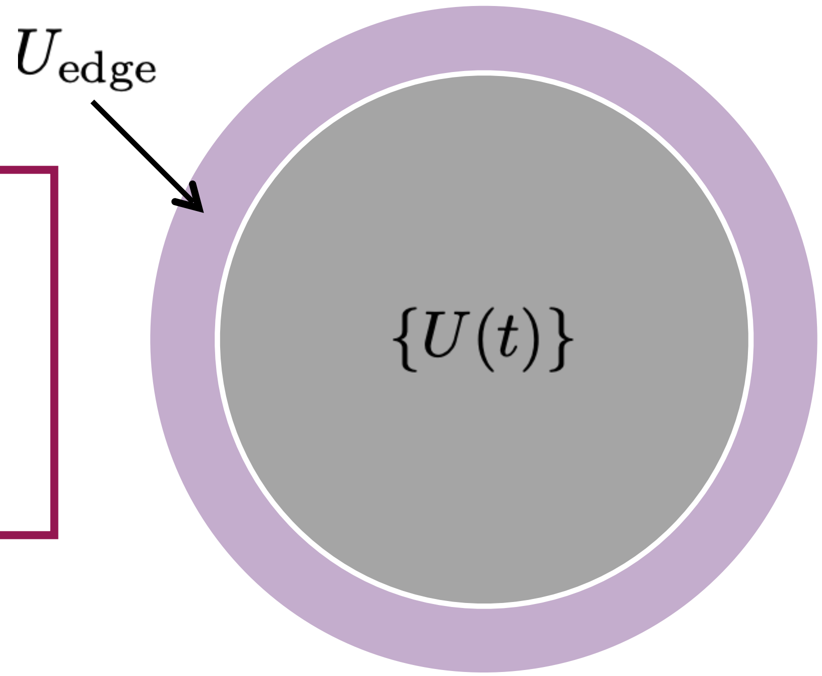
$\rightarrow$ transports charge along the edge

CZ & ML, 2020

Main question

How do we diagnose these phases using:

- Edge topological invariants $F(U_{\text{edge}})$
- Bulk topological invariants $M(\{U(t)\})$



Also, is there a general relation between edge and bulk invariants?

Answer

We can obtain edge and bulk invariants from *flows* $\Omega_{A,B}(U)$.

$$
\begin{array}{ccc}
 & \Omega_{A,B}(U) & \\
\swarrow & & \searrow \\
F(U_{\mathrm{edge}}) & & M(\{U(t)\})
\end{array}
$$

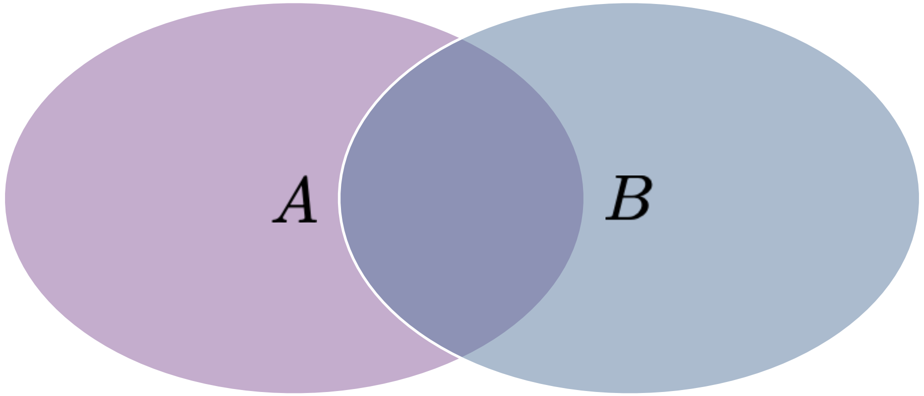
Plan for the rest of the talk

- Introduce our main tool: *flow*
- Examples of flows
- Recipe for obtaining edge and bulk invariants from flow
- Application of this recipe to various kinds of systems

Definition of flow

A *flow* $\Omega_{A,B}(U)$ is a real-valued function of:

- Two sets $A, B \subset \Lambda$, where Λ is the lattice
- A unitary U

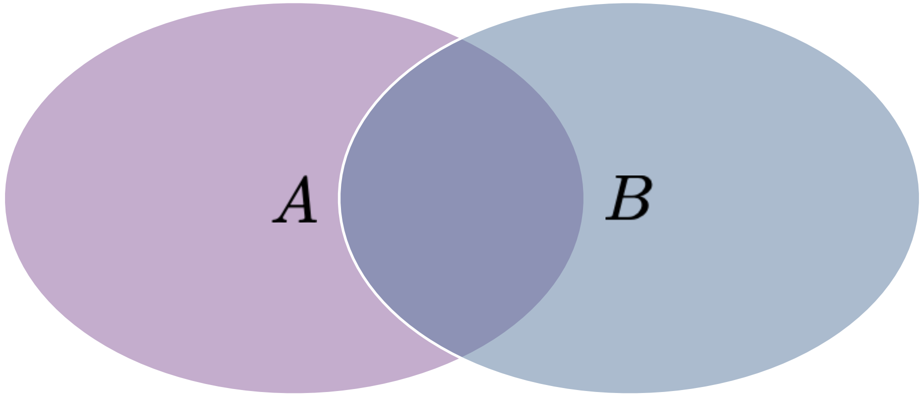


Definition of flow

* *

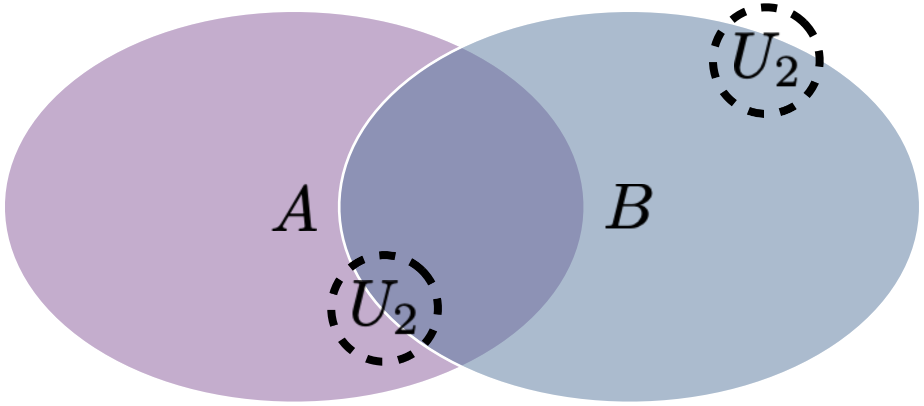
1. $\Omega_{A,B}(V_A U) = \Omega_{A,B}(U)$ if $\mathrm{supp}(V_A) \subset A$ or $\bar{A}$.
2. $\Omega_{A,B}(U V_B) = \Omega_{A,B}(U)$ if $\mathrm{supp}(V_B) \subset B$ or $\bar{B}$.
3. $\Omega_{A,B}(U)$ is additive under stacking.
4. $\Omega_{A,B}(\mathbf{1}) = 0$.

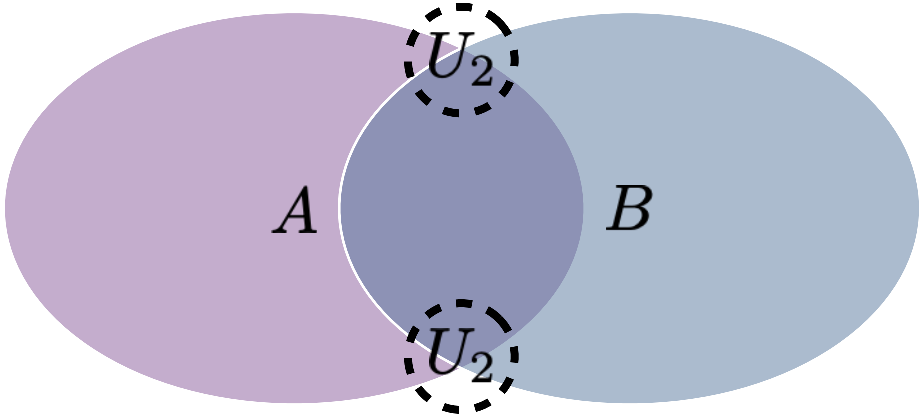
$\Omega_{A,B}(U)$ measures transport between A and B due to the action of U



Definition of flow

According to Properties 1 and 2,





$$\Omega_{A,B}(U_2U_1) = \Omega_{A,B}(U_1) \qquad \Omega_{A,B}(U_2U_1) \neq \Omega_{A,B}(U_1)$$

$\Omega_{A,B}(U)$ depends only on unitaries with support near the intersection of the boundaries of A and B

Example

Single-particle systems:

$$\Omega_{A,B}(U) = \sum_{a\in A}\sum_{b\in B} |U_{ab}|^2 - \delta_{a,b}$$

Can check that:

*

1. $\Omega_{A,B}(V_A U) = \Omega_{A,B}(U)$ if $\mathrm{supp}(V_A) \subset A$ or $\bar{A}$.
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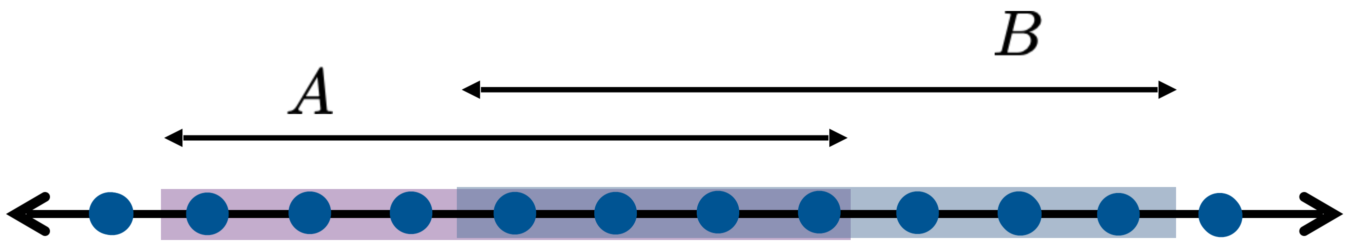
*

Edge invariant from flow

$$\boxed{F(U_{\text{edge}}) = \Omega_{A,B}(U_{\text{edge}}) \text{ in an overlapping geometry}}$$



Note that the boundaries of A and B have zero intersection in this geometry.

$\rightarrow$ *all* local gates are deep in $A, \overline{A}, B$, or $\overline{B}$.

$\rightarrow F(U_{\text{edge}}) = F(W \cdot U_{\text{edge}})$ if W is a FDLU

Bulk invariant from flow

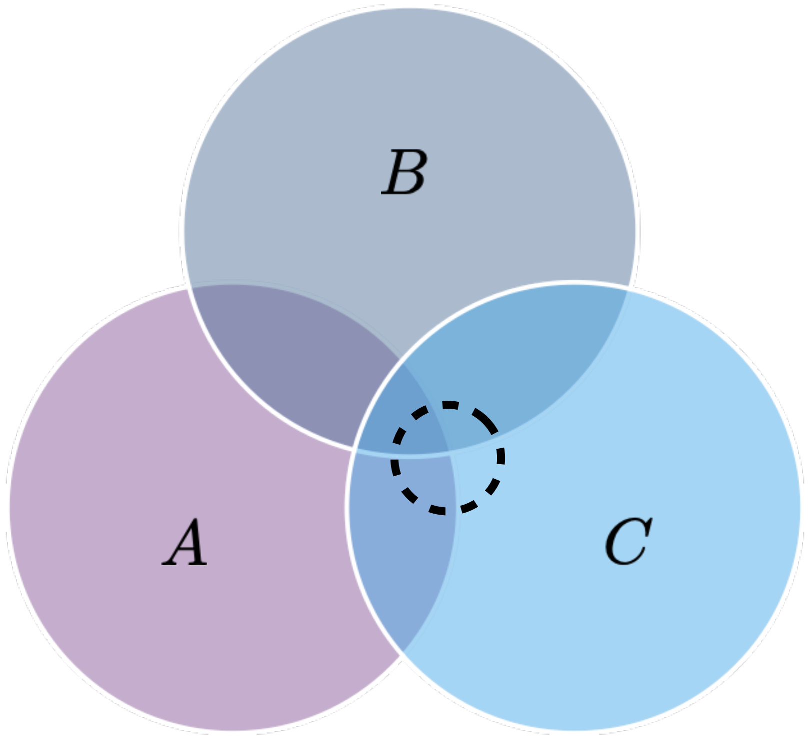
$$\boxed{M(\{U(t)\}) = \Omega^{C}_{A,B}(\{U(t)\})}$$

$A, B, C \in \Lambda$: three overlapping sets

$\{U(t)\}, t \in [0, T)$: full path of unitaries

The boundaries of A and B
intersect at two points:
one in C, one not in C.

$$\Omega^{C}_{A,B}(\{U(t)\}) = \int_0^T dt \frac{\partial}{\partial t_C} \Omega_{A,B}(U(t))$$

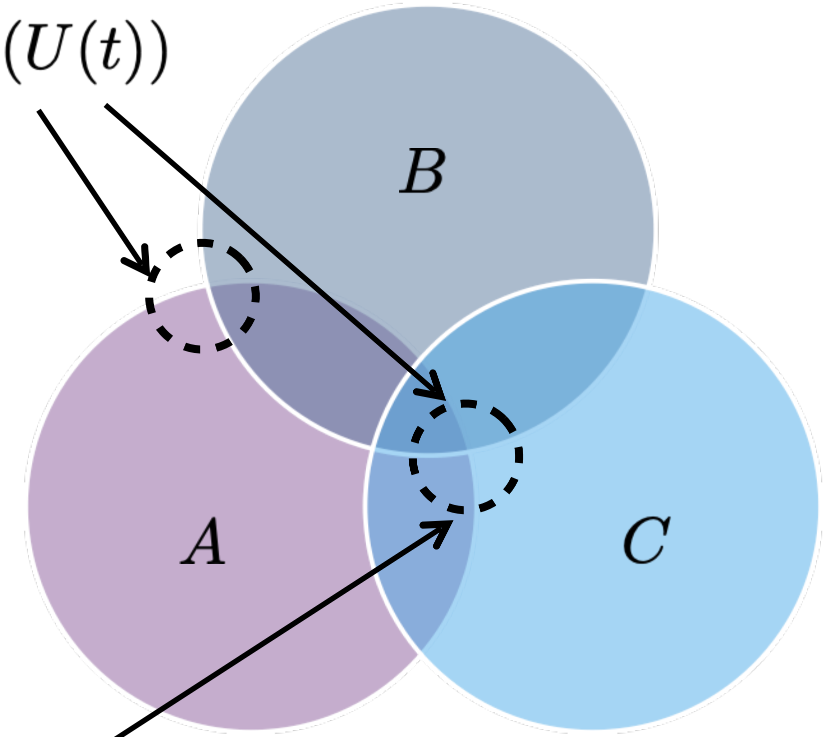


Bulk invariant from flow

$\frac{\partial}{\partial t_C}$: counts only flow from gates in C.

$$\frac{\partial}{\partial t_C}\Omega_{A,B}(U(t))$$
$$= \lim_{\epsilon\to 0}\frac{1}{\epsilon}\left[\Omega_{A,B}\left(e^{-i\epsilon H_C(t)}U(t)\right) - \Omega_{A,B}\left(U(t)\right)\right]$$

$$\frac{d}{dt}\Omega_{A,B}(U(t))$$



$M(\{U(t)\})$ only involves time-evolved operators localized in here

Example: non-interacting systems

Definition of flow:

$$\Omega_{A,B}(U) = \sum_{a\in A}\sum_{b\in B} |U_{ab}|^2 - \delta_{a,b}$$

projector into $A \subset \Lambda$

$$= \mathrm{Tr}(U^\dagger P_A U P_B) - \mathrm{Tr}(P_A P_B)$$

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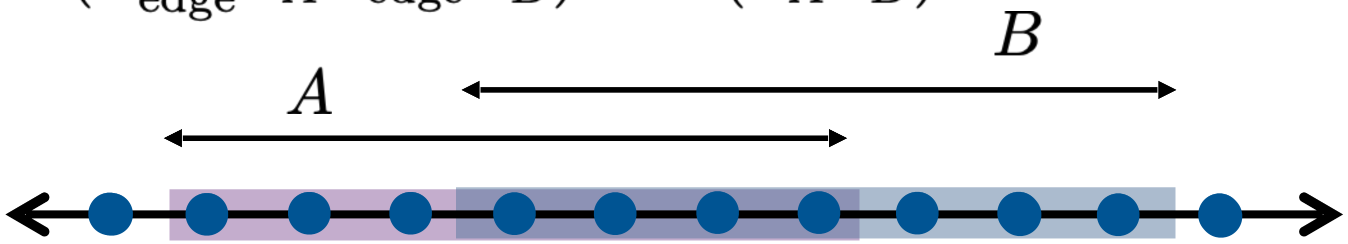
Edge invariant: $F(U_{\text{edge}}) = \mathrm{Tr}(U_{\text{edge}}^\dagger P_A U_{\text{edge}} P_B) - \mathrm{Tr}(P_A P_B)$

$\mathrm{Tr}(P_A P_B) = 4$



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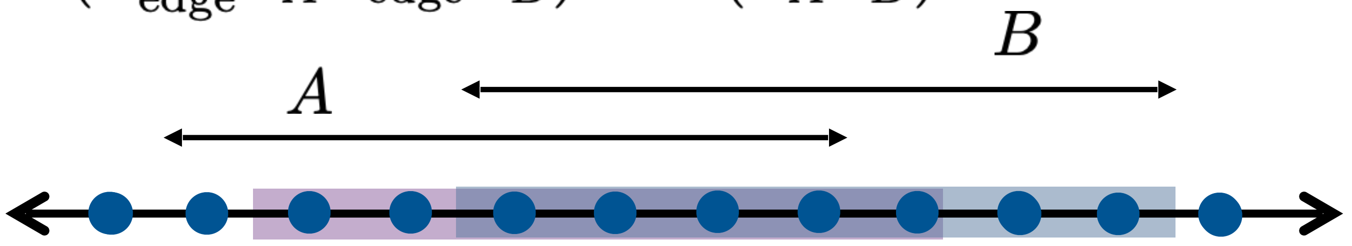
Edge invariant: $F(U_{\mathrm{edge}}) = \mathrm{Tr}(U^\dagger_{\mathrm{edge}} P_A U_{\mathrm{edge}} P_B) - \mathrm{Tr}(P_A P_B)$

$\mathrm{Tr}(P_A P_B) = 4$

$\mathrm{Tr}(U^\dagger_{\mathrm{edge}} P_A U_{\mathrm{edge}} P_B) = 5$



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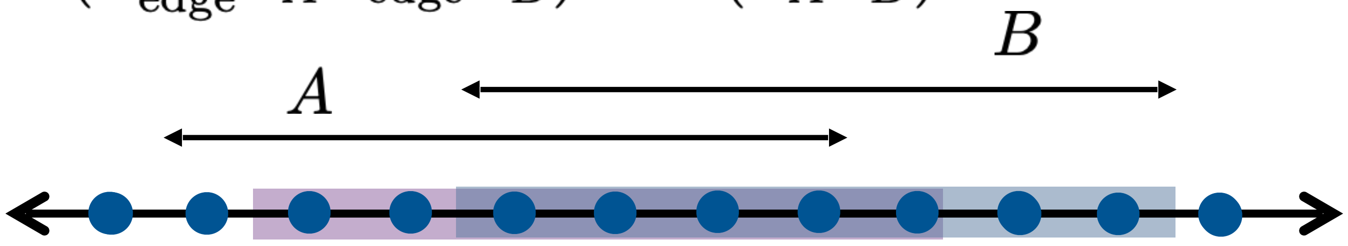
unit translation: $F(U_{\text{edge}}) = 1$



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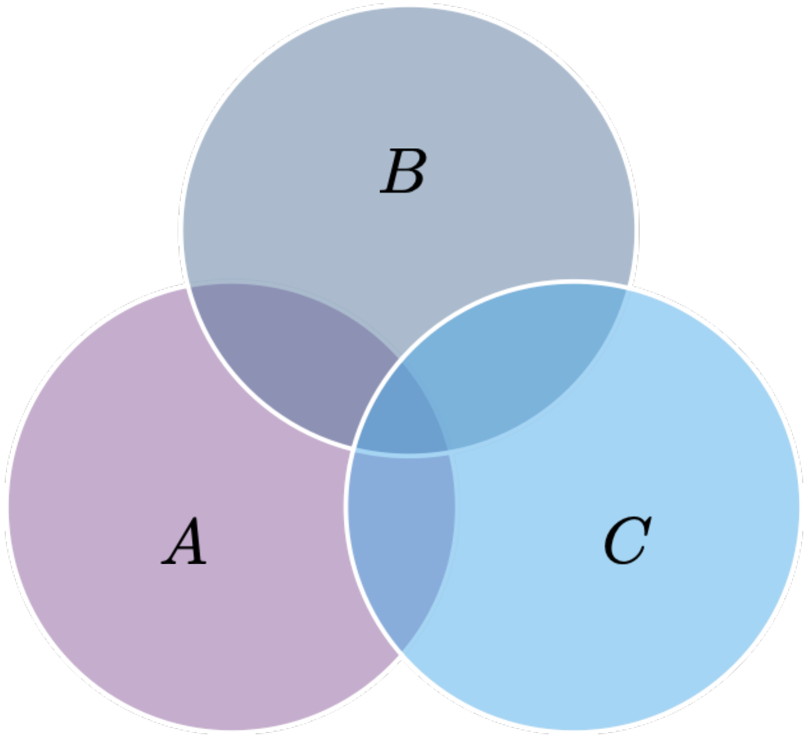
$$M(\{U(t)\}) = \int_0^T dt \frac{\partial}{\partial t_C} \Omega_{A,B}(U(t))$$



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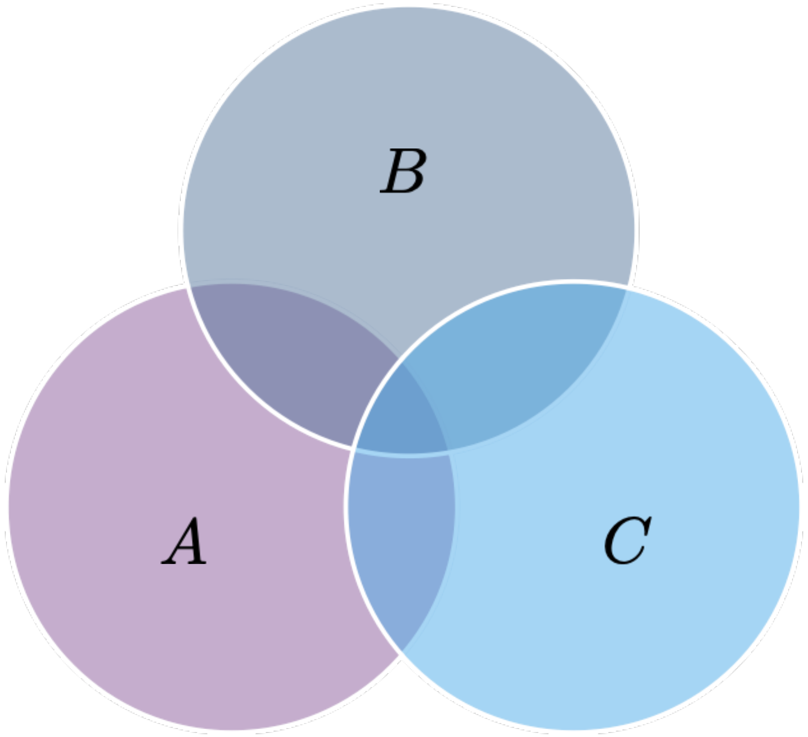
$$= i\int_0^T dt \mathrm{Tr}\left(U^\dagger(t)[H_C(t), P_A]U(t)P_B\right)$$



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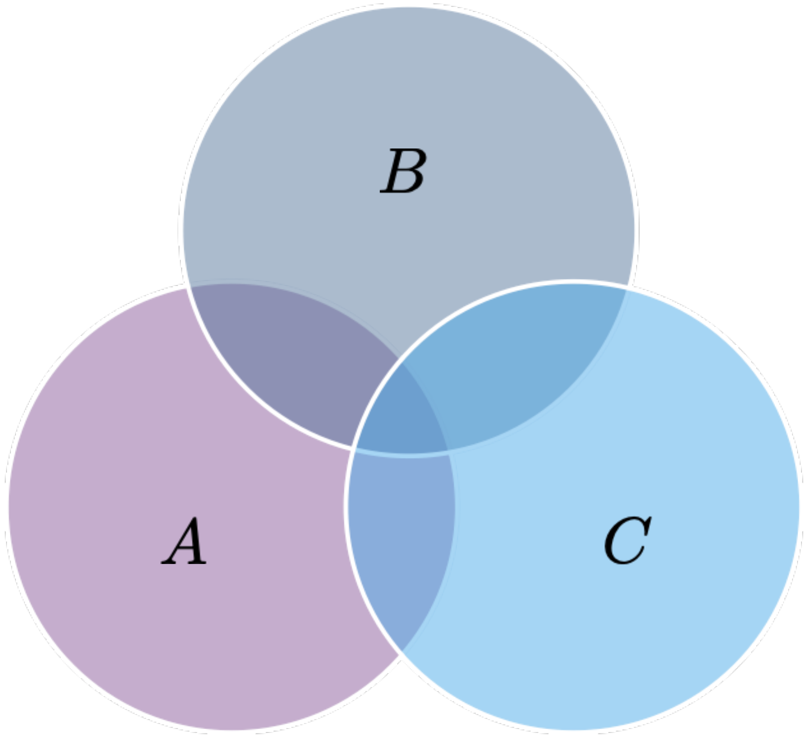
$$= i \int_0^T dt \mathrm{Tr}\left(U^\dagger(t)[H_C(t), P_A] U(t) P_B\right)$$

$$H_C(t) = \frac{1}{2}\left(P_C H(t) + H(t) P_C\right)$$

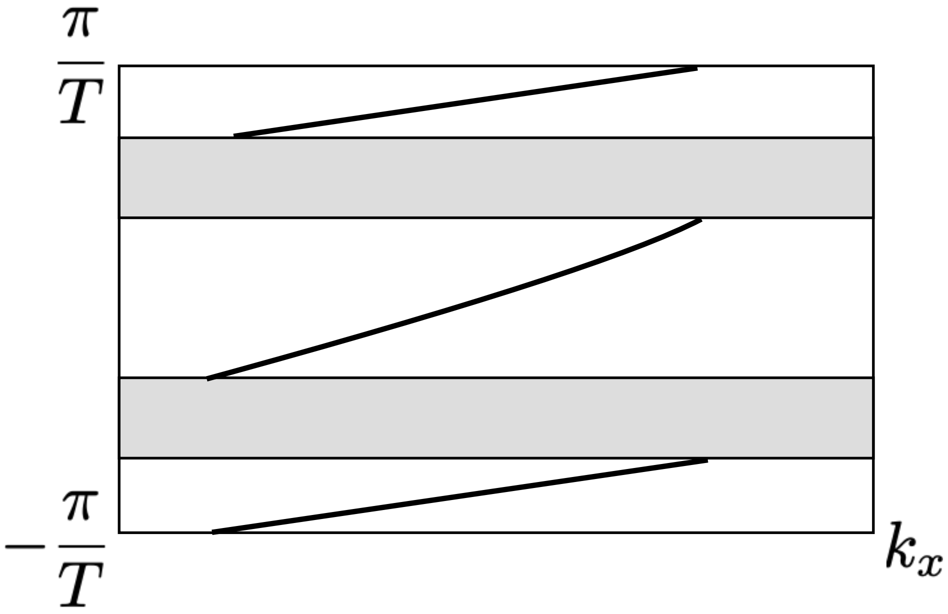


Relation to winding number

Choose A = left half plane, B = upper half plane, and C = whole plane.

$$M(\{U(t)\}) = \frac{1}{8\pi^2}\int dt dk_x dk_y \mathrm{Tr}\left(U^\dagger \frac{\partial}{\partial t}U\left[U^\dagger\frac{\partial}{\partial k_x}U, U^\dagger\frac{\partial}{\partial k_y}U\right]\right)$$

Rudner et al, 2013

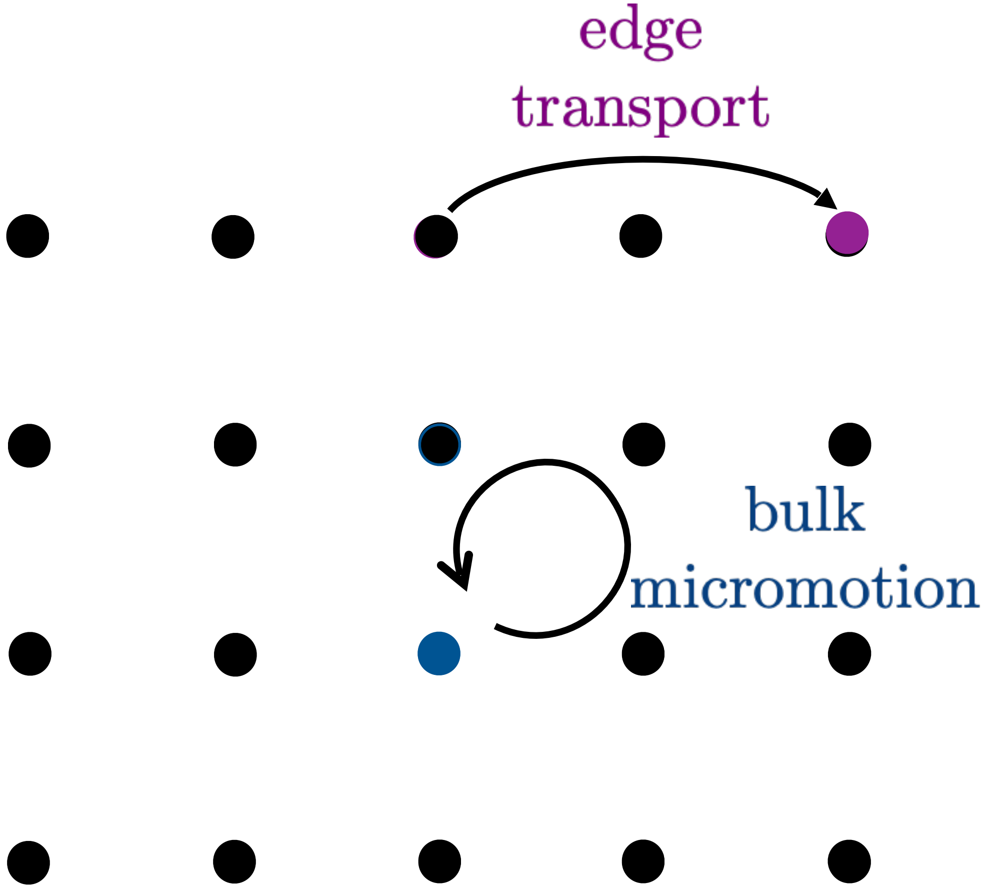


Example: non-interacting systems

Can show that:

$$F(U_{\text{edge}}) = n(U_{\text{edge}})$$

$$M(\{U(t)\}) = \mathcal{W}(\{U(t)\})$$



General recipe

Flow: find a $\Omega_{A,B}(U)$ that satisfies $*$

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$*$

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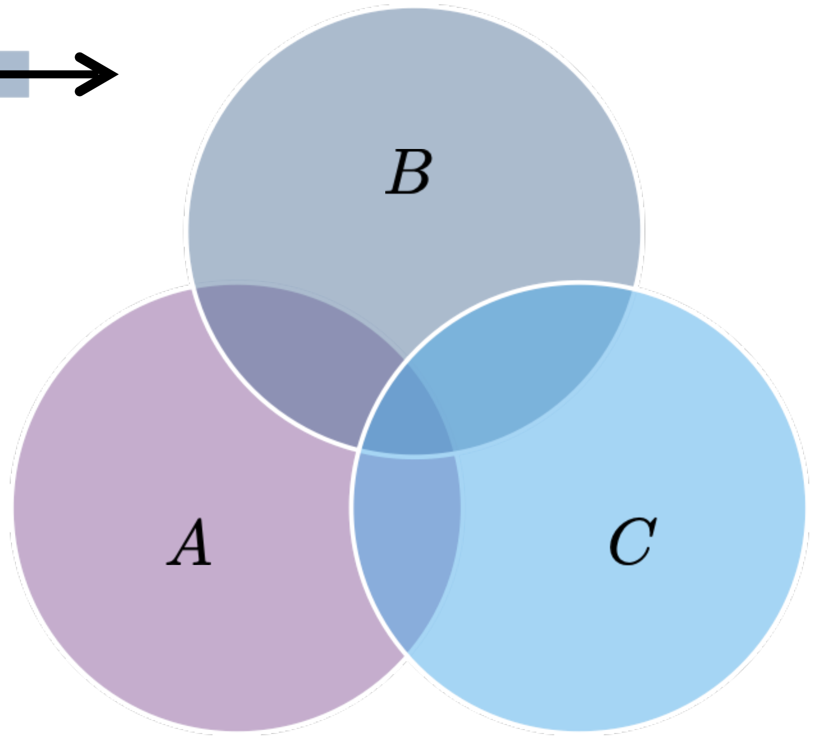

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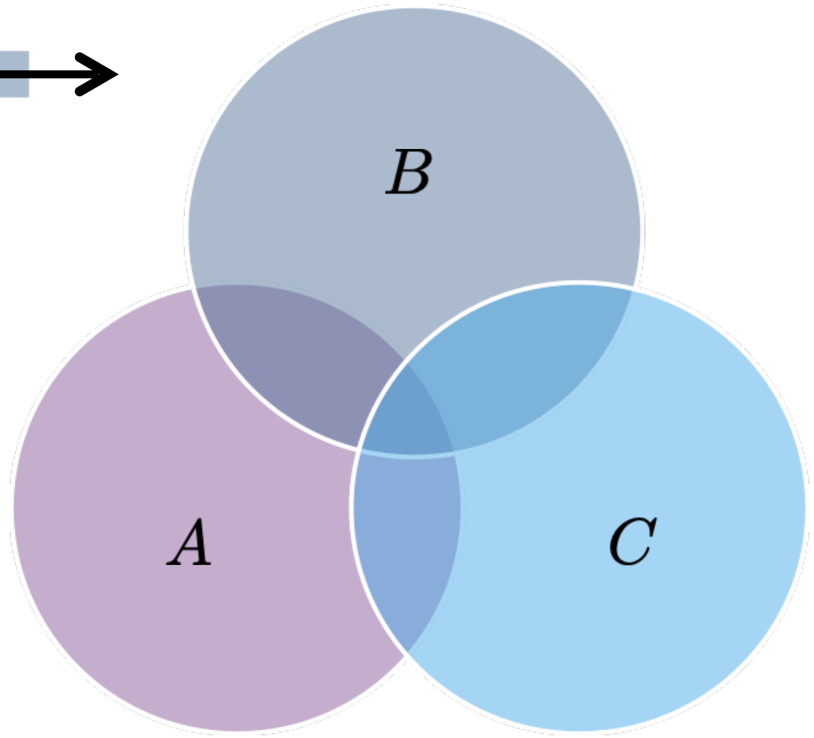
$$\boxed{\begin{array}{c} U_{\text{edge}} = U(T) \\ \to F(U_{\text{edge}}) = M(\{U(t)\}) \end{array}}$$



Flows for interacting systems

No symmetry:

$$\Omega_{A,B}(U) = \log\left[\frac{\eta(\mathcal{A}, U^\dagger \mathcal{B} U)}{\eta(\mathcal{A}, \mathcal{B})}\right] \qquad \eta(\mathcal{A}, \mathcal{B}) = \sqrt{\sum_{O_A \in \mathcal{A}, O_B \in \mathcal{B}} |\overline{\mathrm{Tr}}(O_A^\dagger O_B)|^2}$$

With $U(1)$ symmetry:

$$\Omega_{A,B}(U) = \langle Q_A U^\dagger Q_B U \rangle - \langle Q_A Q_B \rangle \qquad \rho = \frac{e^{\mu Q}}{\mathrm{Tr}(e^{\mu Q})}$$

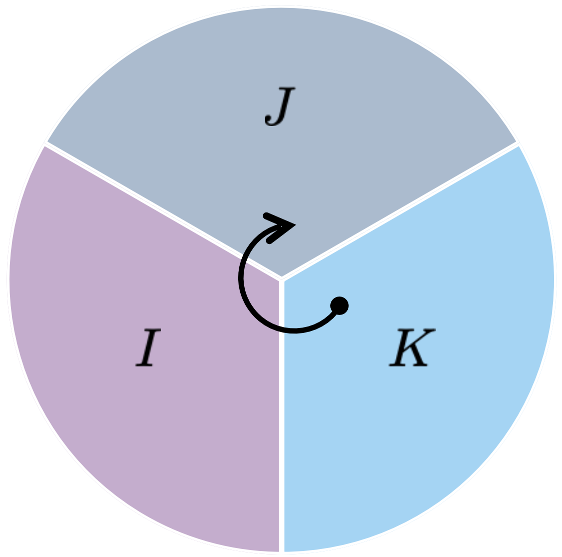
Edge and bulk invariants

	Non-interacting	Interacting, no symmetry	Interacting, $U(1)$ symmetry	Interacting, discrete symm
Edge	✓ Rudner et al, 2013	✓ Gross et al, 2012 Po et al, 2016	✓ CZ & ML, 2020	?
Bulk	✓ Nathan et al, 2017	?	?	?

Edge and bulk invariants

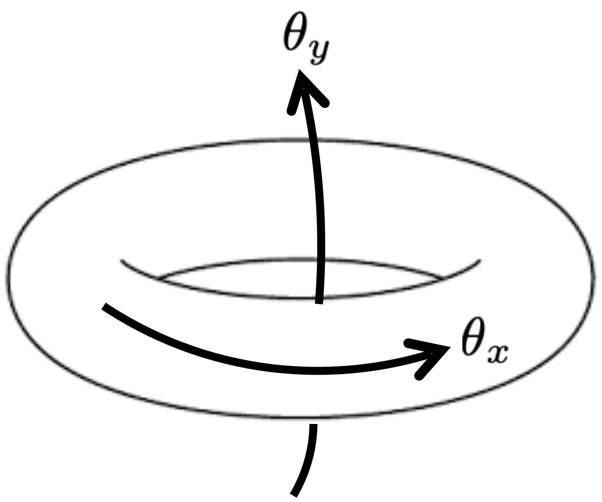
	Non-interacting	Interacting, no symmetry	Interacting, $U(1)$ symmetry	Interacting, discrete symm
Edge	✓ Rudner et al, 2013	✓ Gross et al, 2012 Po et al, 2016	✓ CZ & ML, 2020 CZ & ML, 2022	✓ CZ, in prep
Bulk	✓ Nathan et al, 2017 CZ & ML, 2022	✓ CZ & ML, 2022	✓ CZ & ML, 2022	?

Additional results

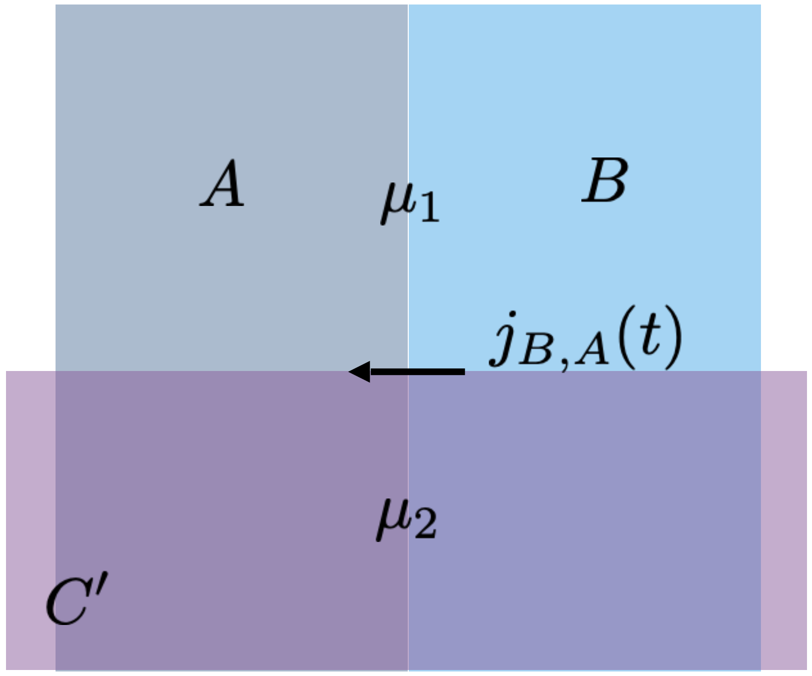


Flows for non-interacting systems and interacting with $U(1)$ symmetry using currents

$\rightarrow$ Physical measurable signatures



Bulk invariant for interacting systems with $U(1)$ symmetry using flux insertion



Physical observable: conserved current between regions of different chemical potential/filling

Outlook

- Bulk boundary correspondence for SPT pumping Floquet systems
- Topological invariants for beyond-cohomology SPT entanglers
- Application to fermionic systems
- Bulk boundary correspondence for Nth roots of MBL Floquet systems (i.e. dynamical honeycomb model)
- Bulk boundary correspondence in higher dimensions
- Stability to perturbations away from MBL Floquet

Edge and bulk invariants

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Bulk	✓ Nathan et al, 2017 CZ & ML, 2022	✓ CZ & ML, 2022	✓ CZ & ML, 2022	?