International Conference on Recent Progress in Many-Body Theories XXI

Dynamic vortex and topological phase transition in a quantum-critical superconductor

Shang-Shun Zhang

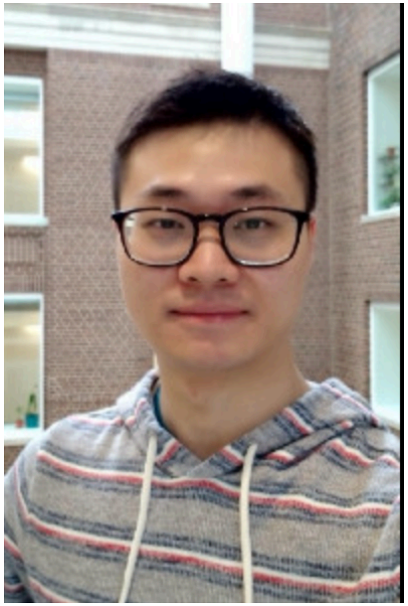
Sep. 16, 2022, Chapel Hill, NC

UNIVERSITY OF MINNESOTA
Driven to Discover®

Collaborators

Andrey Chubukov
UMN

Artem Abanov
TAMU



Yiming Wu
Stanford

Outline

I. Introduction: superconductivity from quantum critical metal

Interplay of non-Fermi liquid and superconductivity

II. Topological aspects of quantum critical superconductor

These appear in the gap function away from Matsubara axis.

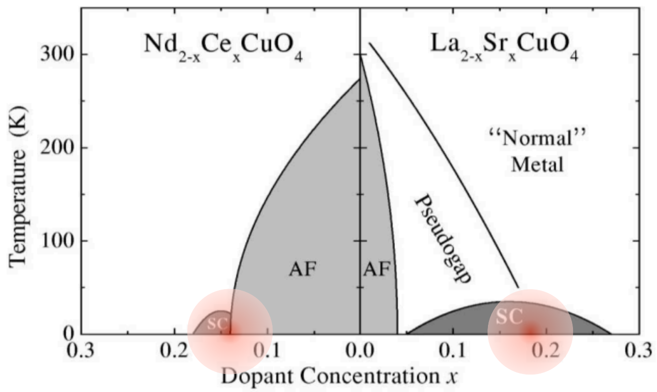
Dynamic vortices emerge in the gap function

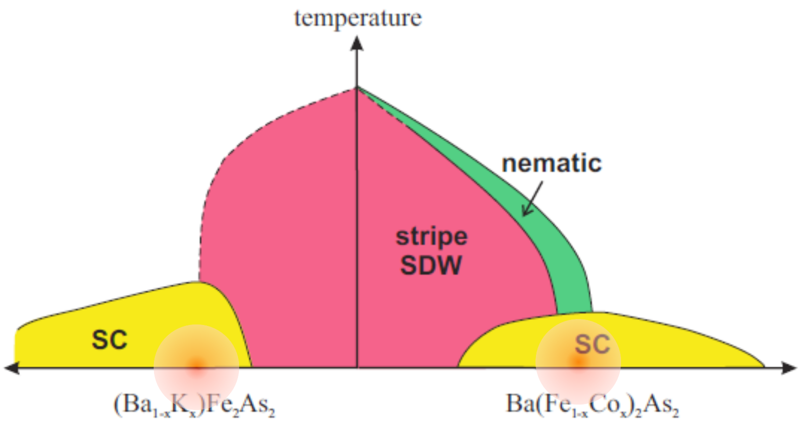
Topological phase transition in the ground state

III. Summary

Metal near a quantum critical point (QCP)

In many strongly correlated materials, superconductivity emerges in the vicinity of a QCP!





Measurements show anomalous behaviors that contradict with Fermi liquid

Normal state: linear-in-T resistivity, breakdown of Landau quasi-particle, etc.

Superconducting state: the underlying mechanism goes beyond BCS theory!

Driven force: Enhanced quantum fluctuation in proximity to a QCP

Fermion-boson model

In the low-energy spectrum, there are two basic interredients

- ✱ Fermions with a Fermi surface (FS)
- ✱ softened bosons (i.e., collective modes of electrons near a QCP)

Incomplete list of authors in this front

Abanov, Chubukov, Schmalian, 2003; Metlitski, Sachdev, 2010; Patel, Strack, Sachdev, 2015; Schatter, Lederer, Kivelson, Berg, 2016; Xu, Sun, Schattner, Berg, Meng, 2017, …

Strong-coupling problem ➔ require advanced many-body techniques

We focus on a subset of this model, where the *typical energy scale of bosons* is much smaller than the *Fermi energy*

- ✱ Momentum integration is **factorized** along and transverse to Fermi surface (FS)
- ✱ Average over FS leads to a singular frequency-dependent interaction

$$V(\Omega_m) = (\bar{g}/\Omega_m)^\gamma$$ (dubbed as the "γ model")

In BCS superconductor, V(Ω)≈const. at small frequency. It corresponds to γ=0.

Relevance to microscopic models

Ising-nematic QCP in 2D (collective modes with Landau damping): **γ=1/3**

P-A Lee, Bonesteel, MacDonald, Nayak, Millis, Altshuler, Ioffe, Metlitski, Mross, Sachdev, Senthil, Berg, Kivelson, Fradkin, Oagnesyan, Lederer, Trebst, Metzner, Pepin, Efetov, Maslov, Klein, Raghu, …

Spin density wave QCP in 2D (over-damped paramagnon): **γ=1/2**

Millis, Sachdev, Varma, Finkelstein, Schmalen, Metlitski, Y. Wang, Efetov, Pepin, Zaanen, Tremblay, Berg, Fernandes, Tsvelik, S-S Lee, Di Castro, Castellani, Grilli, Gaprara, …

SYK models: **0<γ<1** (depends on ratio between number of fermions & bosons)

Esterlis, Schmalian, Y. Wang, Classen, …

Iron based SC: **γ~1.2**

Kotliar, Miao, Lee, …

Phonon mediated SC (strong coupling regime): **γ=2**

Carbotte, Marsiglio, Combescot, Scalapino, Ranninger, Maksimov, Dolgov, Kivelson, Esterlis, Mazin, Yuzbashyan, Altshuler, …

In this talk, we take γ as a tunable parameter

Universal theory (all microscopic details are encoded into a single parameter)

Competition between non-Fermi liquid and superconductivity (SC)

Two effects of the interaction

1. Singular self-energy in the normal state $\Sigma(\omega_m) = \omega_0^\gamma \omega_m^{1-\gamma}$

   (No Landau quasi-particle; non-Fermi liquid)

2. It provides attraction in certain pairing channel

The two effects compete with each other!

Absence of Landau quasi-particle ➔ Cooper logarithm doesn't exist

Pairing of electrons ➔ Gaps out the spectrum and restores FL

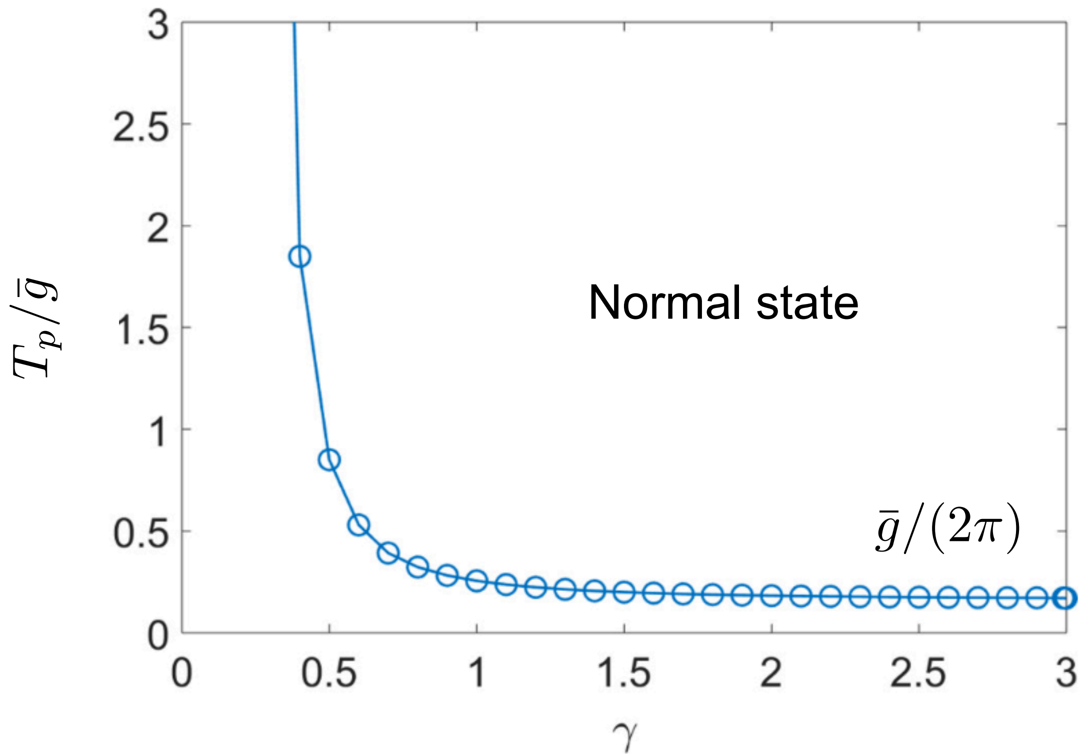
The competition is captured by the Eliashberg-like equation

$$\boxed{\begin{aligned}\Phi(\omega_m) &= \pi T \sum_{\omega'_m} \frac{\Phi(\omega'_m)}{\sqrt{\tilde{\Sigma}^2(\omega'_m) + \Phi^2(\omega'_m)}} V(\omega'_m - \omega_m) \\ \tilde{\Sigma}(\omega_m) &= \omega_m + \pi T \sum_{\omega'_m} \frac{\tilde{\Sigma}(\omega'_m)}{\sqrt{\tilde{\Sigma}^2(\omega'_m) + \Phi^2(\omega'_m)}} V(\omega'_m - \omega_m)\end{aligned}}$$

pairing vertex

self-energy

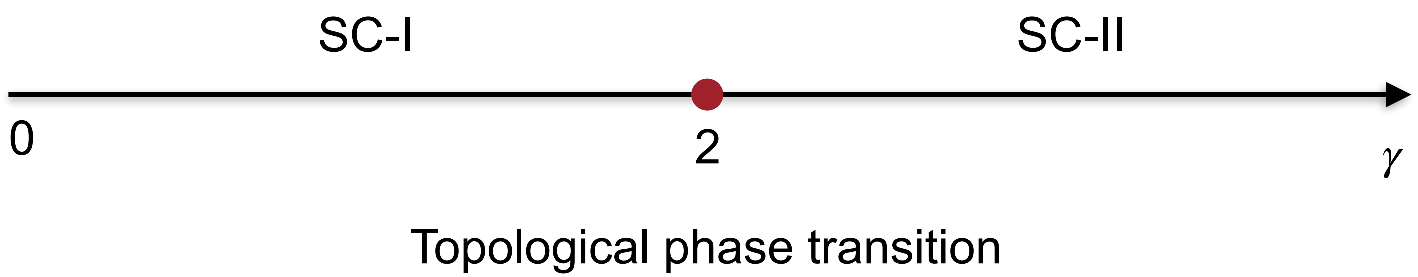
Onset temperature of pairing



Pairing occurs even if the normal state is a non-Fermi liquid!

Ground state develops superconductivity.

Ground state phase diagram



It is characterized by dynamical quantities instead of band topology!

Outline

I. Introduction: superconductivity from quantum critical metal

Interplay of non-Fermi liquid and superconductivity

II. Topological aspects of quantum critical superconductor

These show up in the gap function away from Matsubara axis.

Dynamic vortices emerge in the gap function

Topological phase transition in the ground state

III. Summary

Why γ=2 is special?

Let's look at the interaction vertex

Along Matsubara-frequency axis

$$V(\Omega_m) = (\bar{g}/\Omega_m)^\gamma$$

✱ Real and attractive for all γ. Nothing is special at γ**=2**

Along real-frequency axis

$$V'(\Omega) = \left(\frac{\bar{g}}{|\Omega|}\right)^\gamma \cos\frac{\pi\gamma}{2} \qquad V''(\Omega) = \left(\frac{\bar{g}}{|\Omega|}\right)^\gamma \sin\frac{\pi\gamma}{2}\mathrm{sgn}(\Omega)$$

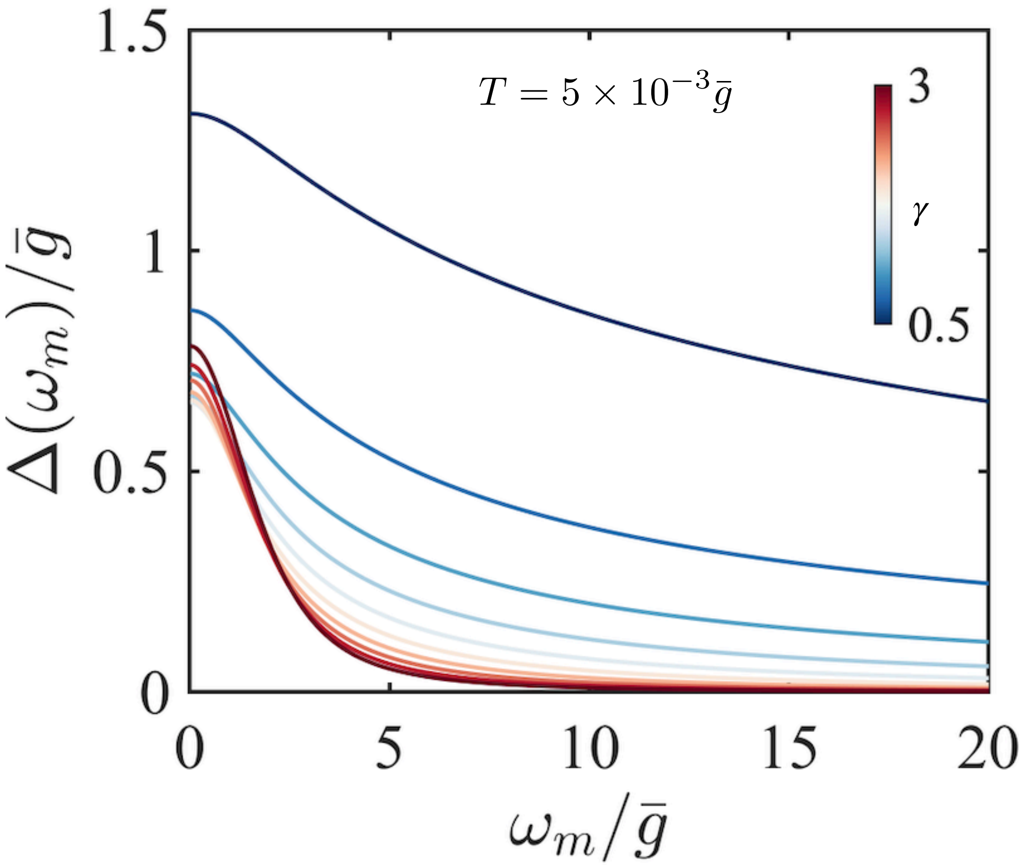
Real part: attractive ($0<\gamma<1$); repulsive ($1<\gamma<3$)

Imaginary part changes sign at γ=2

✱ BCS limit (γ=0): purely real function and attractive

✱ γ**=2 is special**: purely real function and repulsive

Gap function along Matsubara axis



$\Delta(\omega_m \gg \bar{g}) \propto 1/\omega_m^{\gamma}$

Gap function evolves continuously as a function of γ

Gap function along real-frequency axis

Before solving the gap equation, we notice that

- ✱ Gap function is generally complex $\Delta(\omega)=|\Delta(\omega)|\exp[i\eta(\omega)]$
- ✱ Boundary behavior $\Delta(\omega=0)$ is real, $\Delta(\omega\to\infty) \sim \exp(i\pi\gamma/2)/\omega^{\gamma}$

$$\boxed{\eta(0) = 0 \qquad \eta(\infty) = \pi\gamma/2 \mod 2\pi}$$

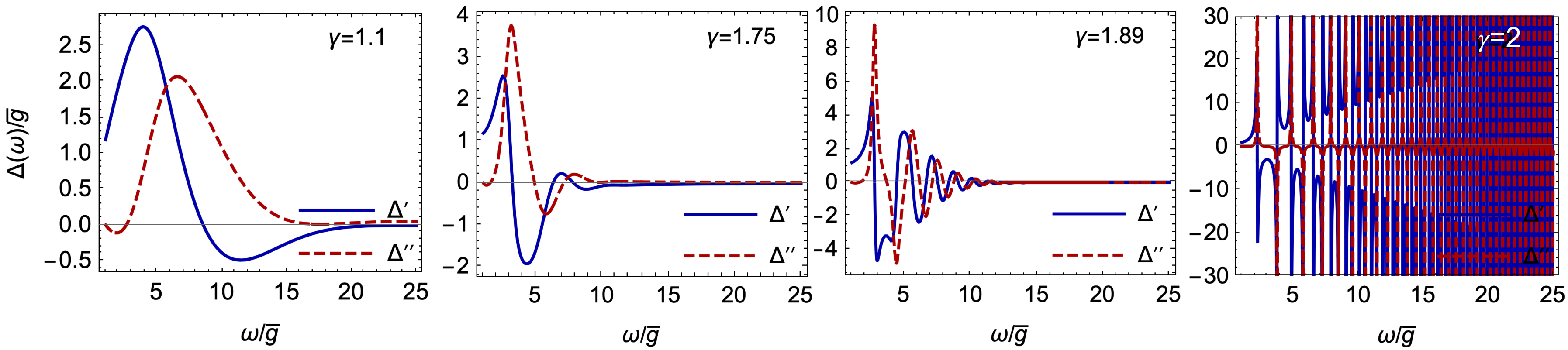
Between the two limits, phase η may wind up by integer times (W).

To determine W, we need to solve the gap equation.

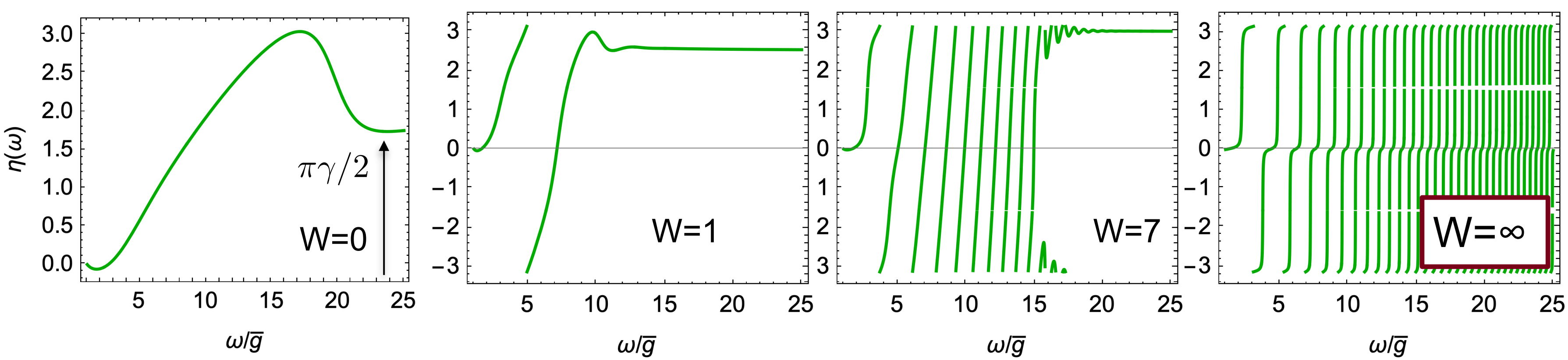
Indeed, it is non-zero for some parameter regime of γ

Gap function along real-frequency axis

We solved the gap eqn along real axis



Phase winding

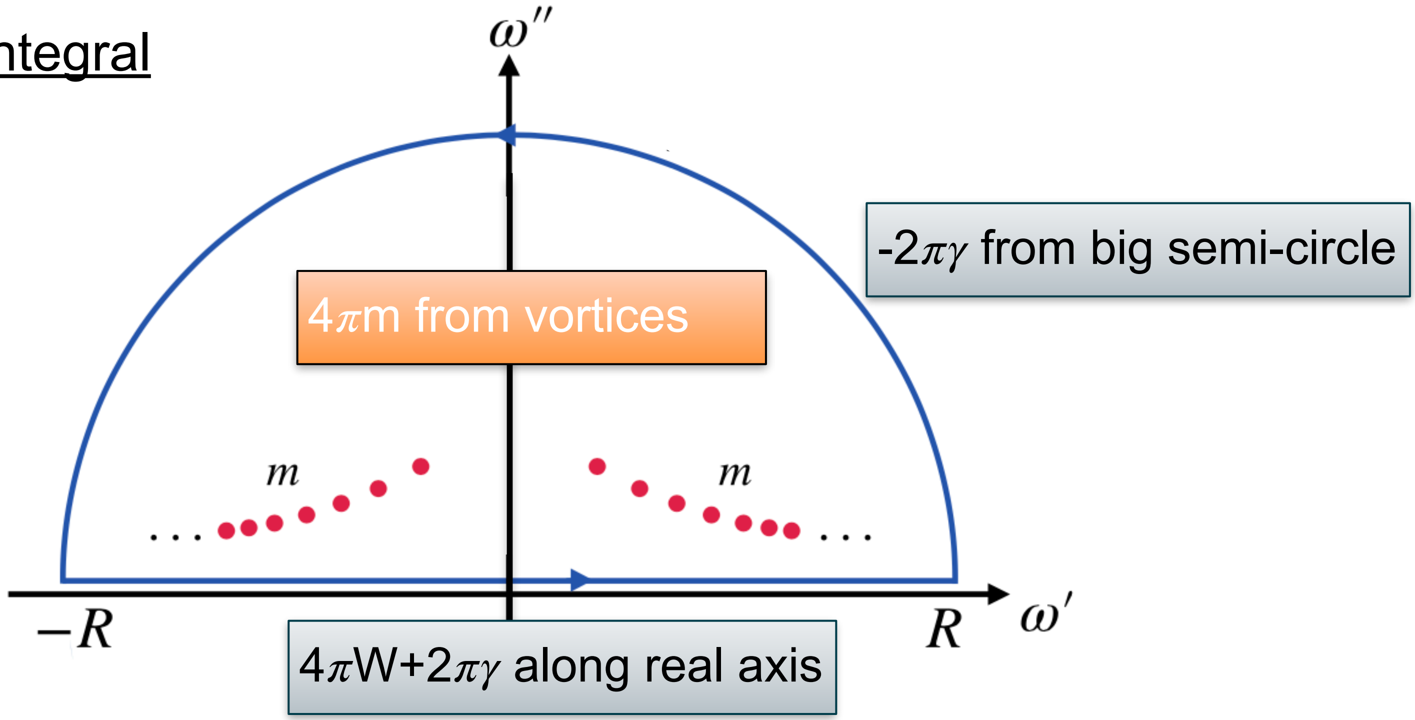


Dynamical vortices on upper frequency plane

Upper plane: $z=\omega' + i\,\omega''$, $\Delta(z) = |\Delta(z)| \exp(i\eta(z))$

Dynamical vortex: around which, phase η changes by 2π

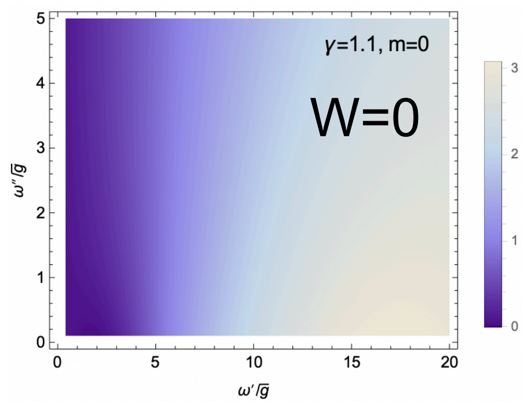
<u>Cauchy integral</u>



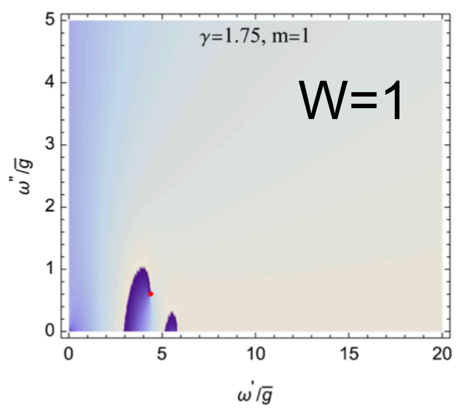
Phase change along boundary = 2π * number of vortices (2m)

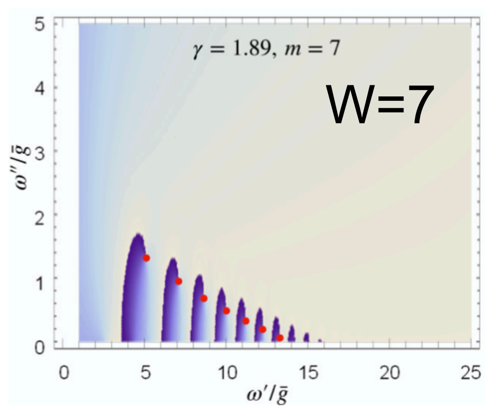
➔ m=W

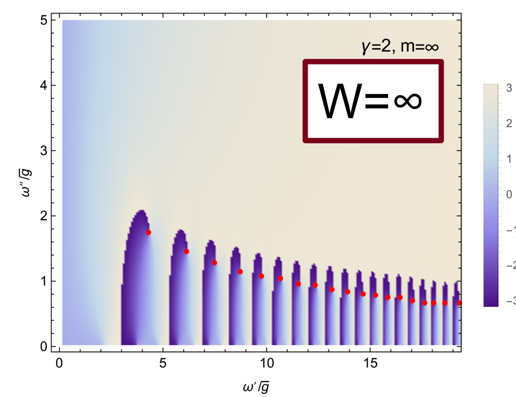
Dynamic vortices



Wu, SSZ, Abanov, Chubukov, 2021







Dynamic vortices

The dynamic vortex is a topological defect and appears in the frequency dependence in $\Delta(\omega)$

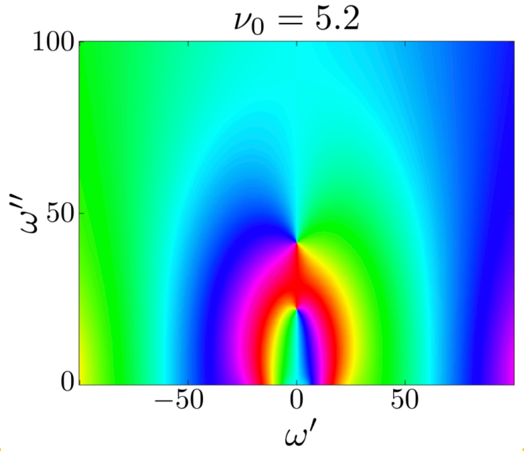
➔ It is not detectable by order parameter of pairing defined as equal-time correlator $$\Delta(\boldsymbol{r}_1, \boldsymbol{r}_2) \sim \langle c_{\boldsymbol{r}_1}(t) c_{\boldsymbol{r}_2}(t) \rangle$$

This is similar as odd-freq. pairing ($\Delta(\omega)$=-$\Delta(-\omega)$)—dynamical order

Linder, Balatsky, RMP, 2019

➔ Necessary condition: strongly retarded interaction!

Retarded effect is not specific to QCP, e.g., dynamic vortex also exists in electron-phonon superconductor.

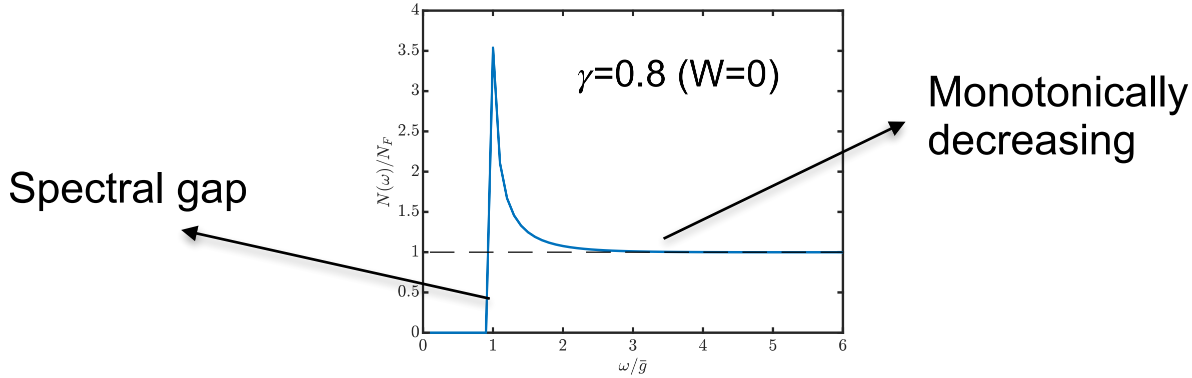


Christensen, Chubukov, 2021

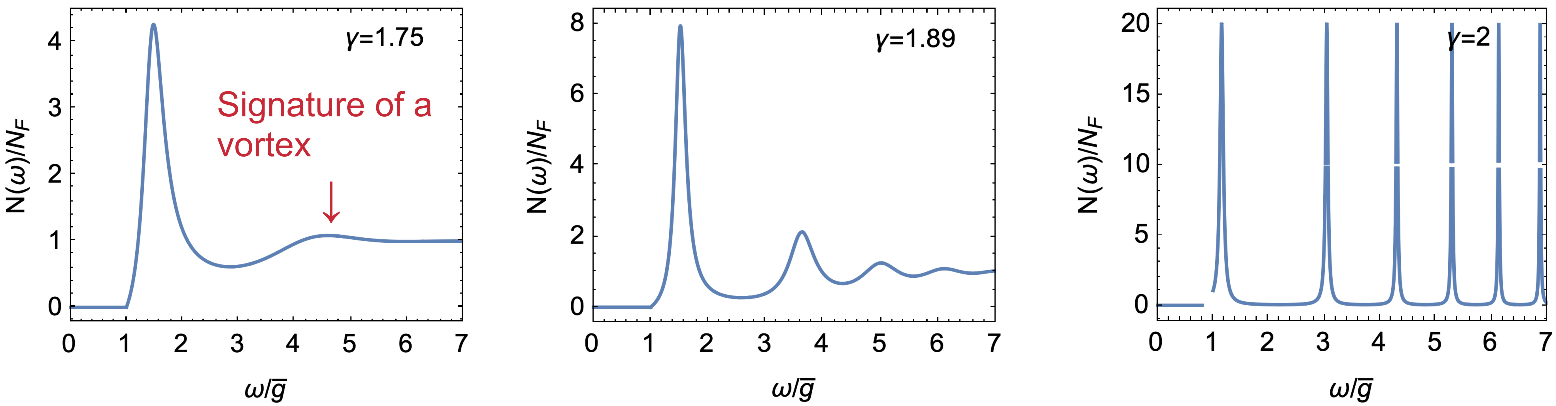
Measurable effects of dynamic vortex?

Single-electron density of states (DoS)!

In the absence of dynamic vortex:



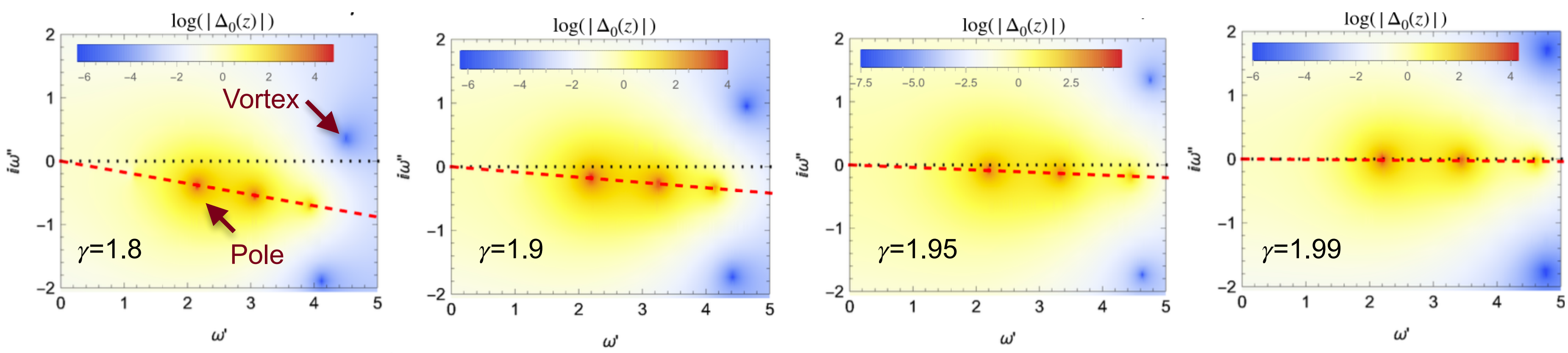
When a vortex crosses the real axis ➔ Δ=0 at vortex core, N(ω)=N_F ➔ a bump in DoS



Other possible effects: Josephson ac currents

There are also poles!

Pade approximation gives



SSZ, Wu, Abanov, Chubukov, 2021

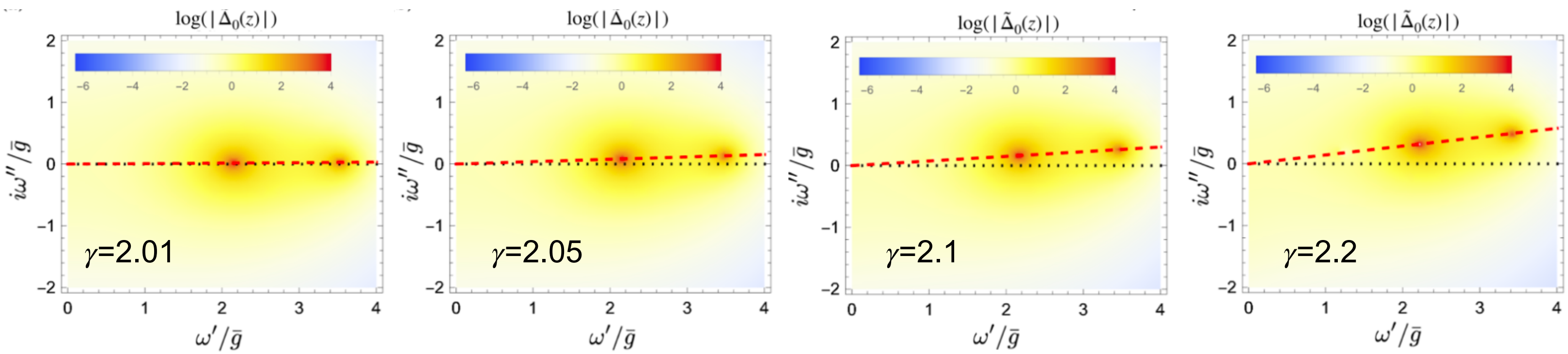
At γ</=2, gap function is analytic on **upper** plane! ➔ physically allowed!

This line of poles align with real axis at γ=2 ➔ Again, γ=2 is **special**!

What happened at γ>2?

What happened at γ>2?

Pade approximation shows, this line of poles rotate to upper half plane



This is **unphysical** solution, because it violets the causality principle!

e.g., single-particle density of states is negative

To resolve this issue:

The gap function has to switch to another Riemann surface at γ>2!

Switch to different Riemann surface (γ>2)

More precisely, look at the gap function near the spectral gap ω_0 (lower-edge of the density of states)

$$\boxed{D(\omega) - 1 \propto \delta^\nu, \delta = \omega_0 - \omega - i0^+}$$

D(ω)= $\Delta(\omega)/\omega$

Exponent ν is universally determined as a function of γ

Key point: ν=2 at γ=2 but takes fractional exponent nearby.

➔ branch-cut singularity!

Above the spectral gap, $\delta=\omega_0-\omega<0$, there are **multiple** ways to write down the gap function

$$\delta^\nu = |\delta|^\nu e^{i(2p+1)\nu\pi}$$

p: integer (indicates different Riemann surfaces)

Switch to different Riemann surface (γ>2)

Which Riemann surface does the physical gap function reside on?

Requirement: density of states must be positive!

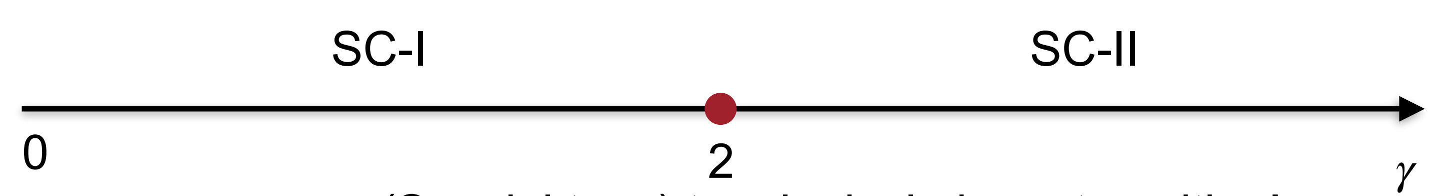
$$N(\omega) = N_0 \mathrm{Im} \sqrt{\frac{1}{D^2(\omega) - 1}}.$$

Conclusion:

p=-1 for γ<2

p=0 for γ>2

➔ Different Riemann surface



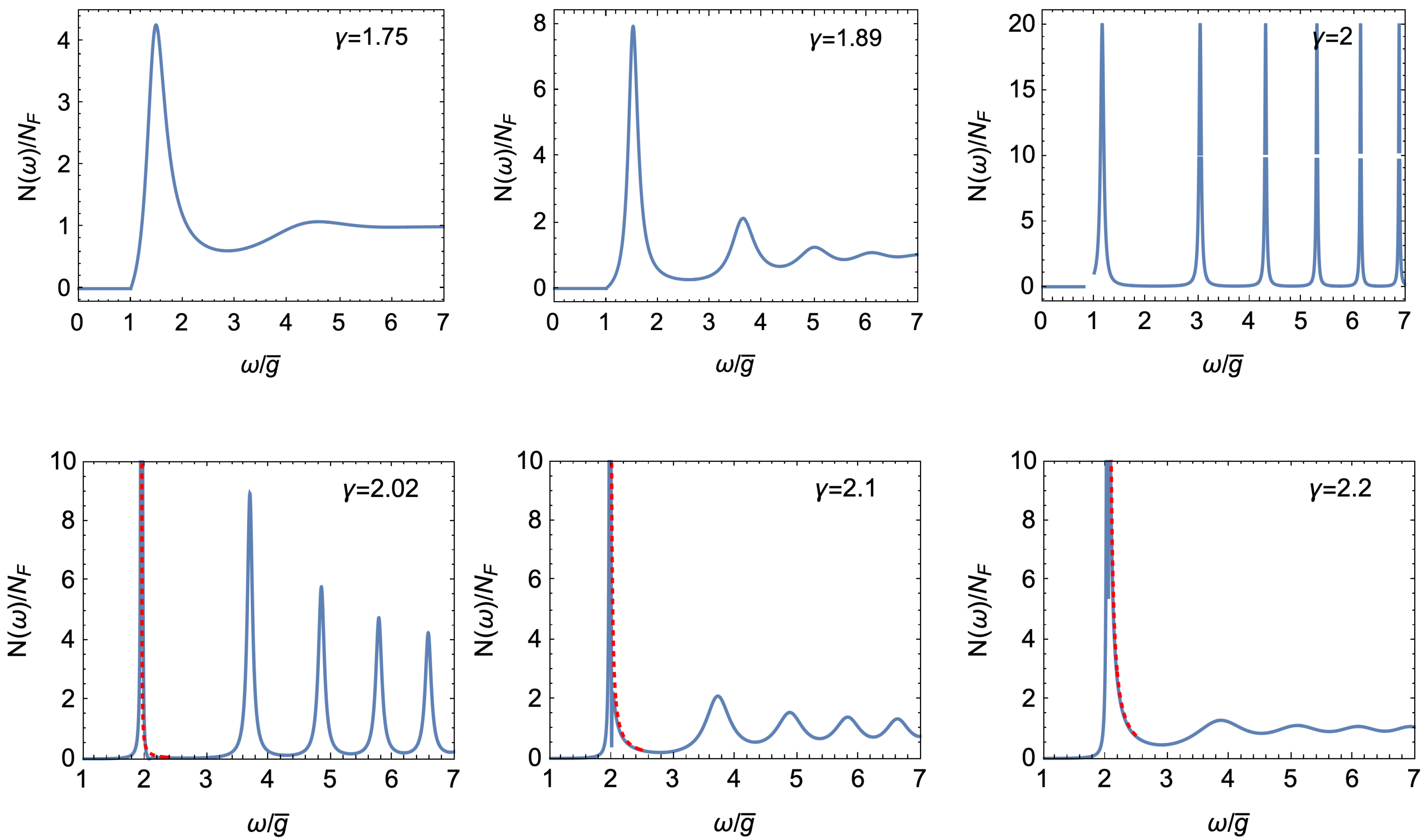
(Special type) topological phase transition!

Measurable effects of the transition?

Look at the single-electron density of state

At γ<2

At γ>2



Summary

We discussed (dynamic) topological aspect of a quantum critical superconductor

- Dynamic vortices, special topological transition at γ=2
- These features appear away from Matsubara axis
- Measurable effects of the dynamic vortex

There are additional features, e.g. linearized gap equation has solution at T=0, condensation energy spectrum, unconventional low-energy excitations, etc..

Artem, Chubukov, 2020

SSZ, Wu, Abanov, Chubukov, arXiv: 2208.13888

SSZ, Chubukov, to appear

Thanks for your attention!