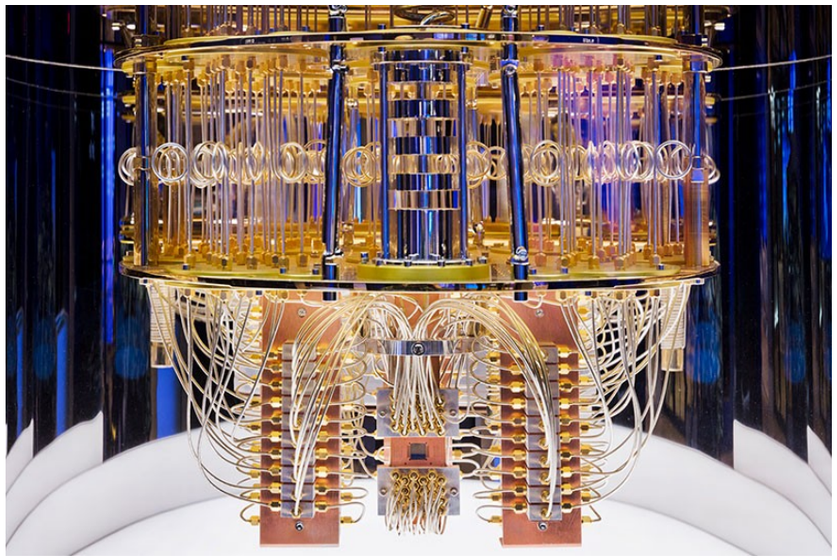
Probing Geometric Excitations of Fractional Quantum Hall States on Quantum Computers

Armin Rahmani



Ammar Kirmani, Kieran Bull, Chang-Yu Hou, Vedika Saravanan, Samah Mohamed Saeed, Zlatko Papić, Armin Rahmani, and Pouyan Ghaemi, Phys. Rev. Lett. **129**, 056801 (2022).

Armin Rahmani, Kevin J. Sung, Harald Putterman, Pedram Roushan, Pouyan Ghaemi, and Zhang Jiang, PRX Quantum **1**, 020309 (2020).

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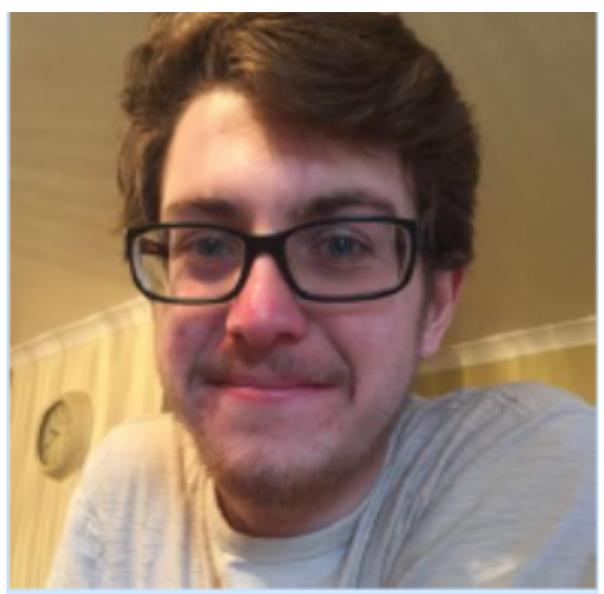
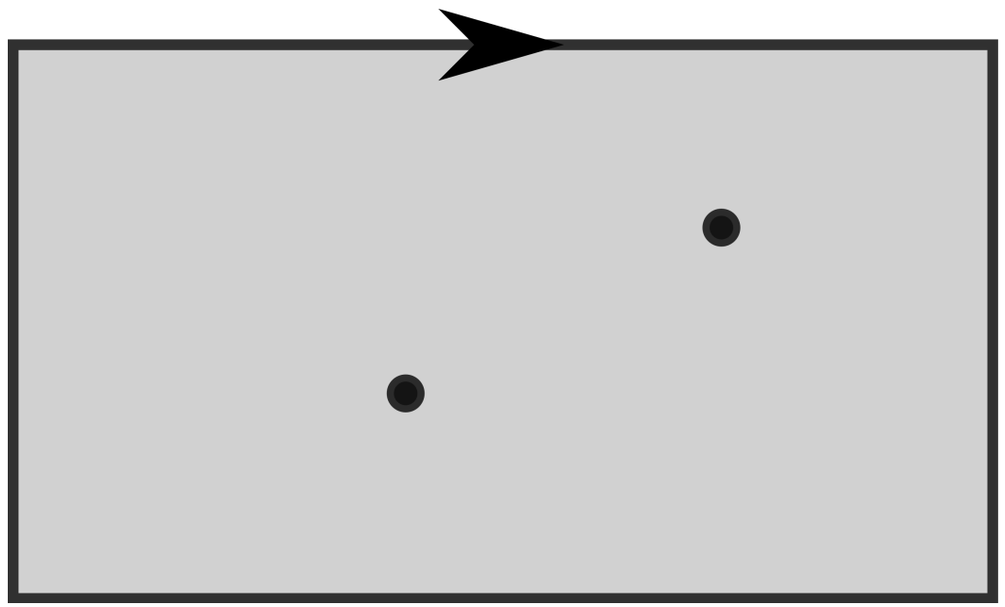


Vedika Saravanan (CUNY)

FQH topology

Fractional quantum Hall states are the canonical example of topological order in strongly interacting quantum matter.



2d electron gas in out-of-plane magnetic field (clean samples low T)
fractional Hall conductance
incompressible liquid with no local order parameter
robust chiral edge states
anyonic quasiparticle excitations

Laughlin's 1/3 state is the simplest FQH state, well described by the Laughlin's continuous 2d wave function.

FQH geometry

PRL **107,** 116801 (2011) PHYSICAL REVIEW LETTERS week ending 9 SEPTEMBER 2011

Geometrical Description of the Fractional Quantum Hall Effect

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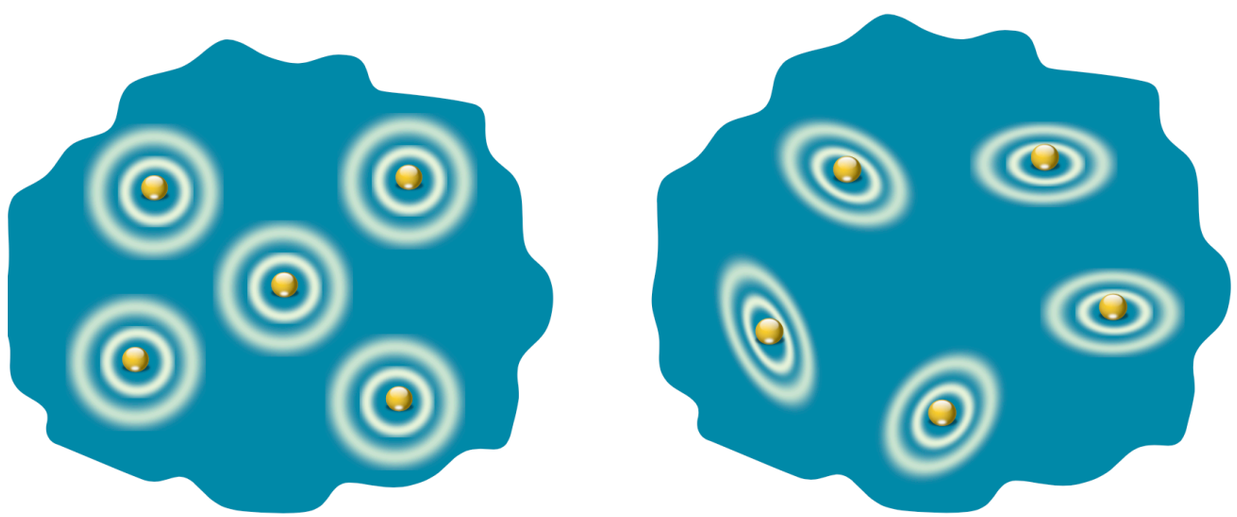
(Received 15 June 2011; published 6 September 2011)

The fundamental collective degree of freedom of fractional quantum Hall states is identified as a unimodular two-dimensional spatial metric that characterizes the local shape of the correlations of the incompressible fluid. Its quantum fluctuations are controlled by a topologically quantized "guiding-center spin." Charge fluctuations are proportional to its Gaussian curvature.

Rotational symmetry is not necessary for FQH
A metric characterizes the shape of the electron-flux composite droplets



Galilean mass tensor (band structure property of 2d quantum well)
Coulomb tensor (dielectric tensor of 3d background)

Important consequences like Hall viscosity

FQH graviton

The geometric description also sheds light on collective neutral modes

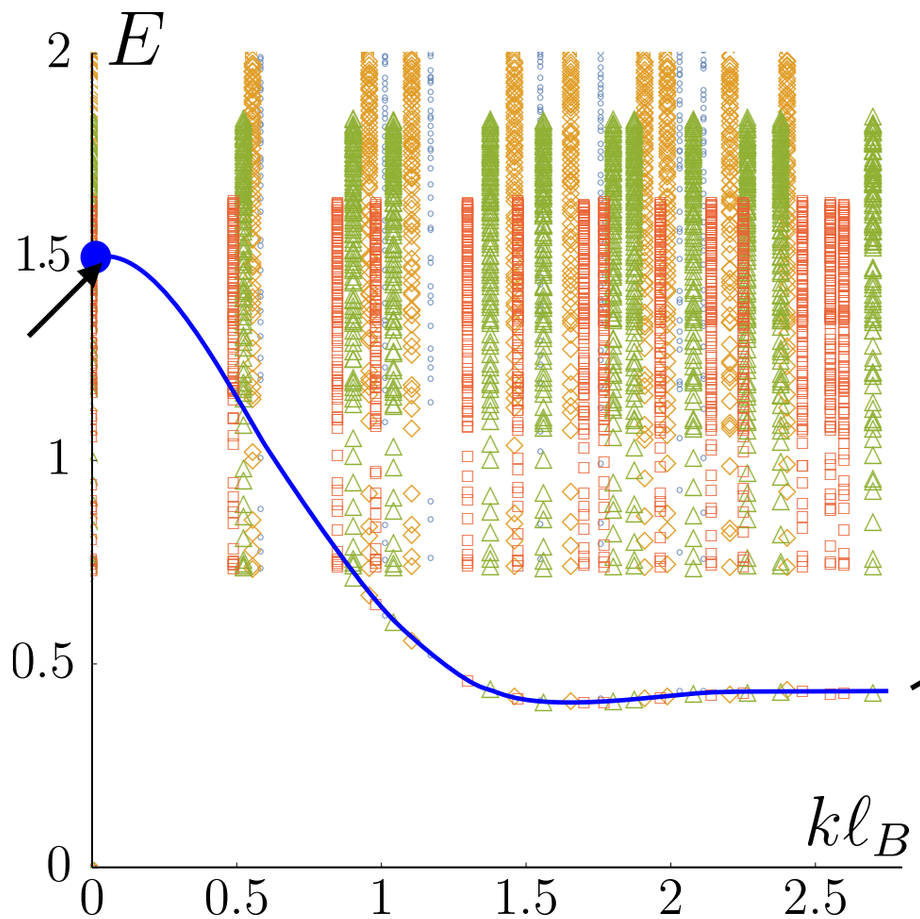
Quadrupole
(2 qh+2 qp)

L=2 Angular momentum

Metric fluctuations

Similar to *graviton*

Haldane 2011



Girvin-MacDonald-Platzmann mode (1985)

Collective density mode (magnetoroton)

Similar to Feynman's theory of superfluid He

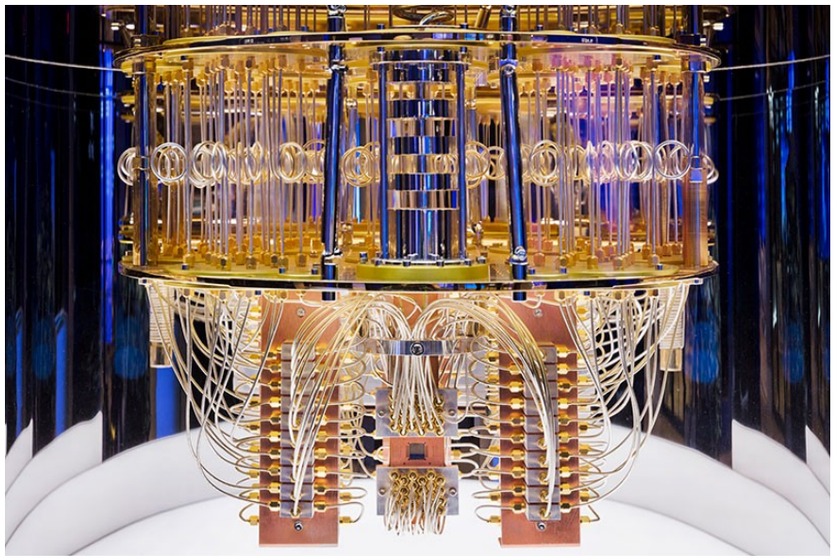
At long-wavelength described by SMA

Z. Liu, A. Gromov, and Z. Papic, Phys. Rev. B **98**, 155140 (2018).

Experimental detection of magnetoroton mode are through inelastic light scattering [Kang et al, 2001] and surface acoustic waves [Kukushkin et al, 2009]. However *k=0* cannot be probed easily because the mode enters the continuum.

NISQ devices

In the meantime, there has been significant progress in synthetic analog and digital platforms

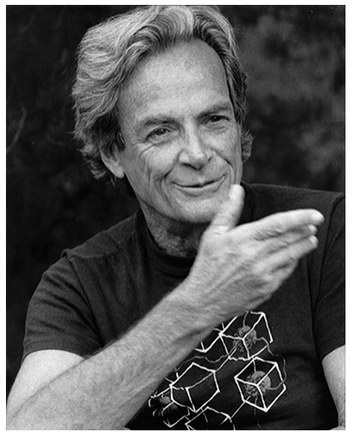


Superconducting qubits
Trapped ions
Cold atoms

Still imperfect, relatively small number of high-quality qubits, high levels of error and noise

NISQ: Noisy Intermediate-Scale Quantum devices

While we're waiting for large-scale error-corrected universal quantum computers, are there problems that benefit from the NISQ devices?



The holy grail is quantum simulation of important and classically intractable theoretical problems, using these novel experimental platforms for theoretical discovery.

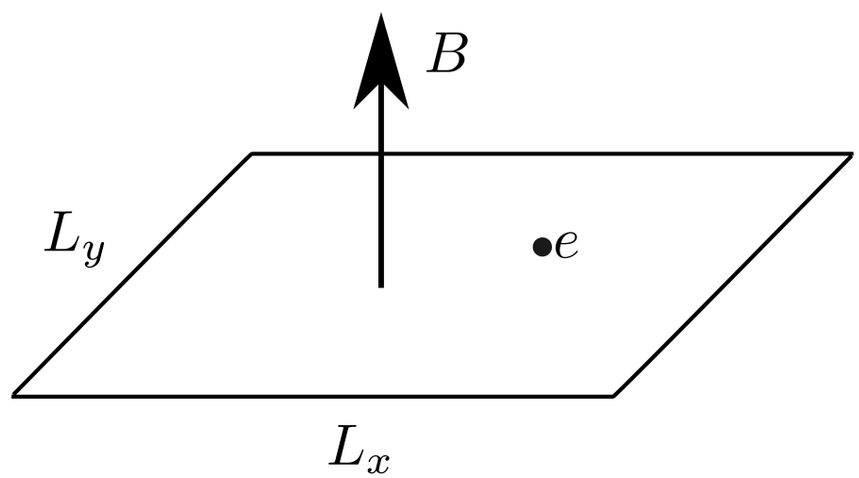
A more accessible objective is experimental observation of phenomena that are challenging to probe in nonsynthetic platforms.

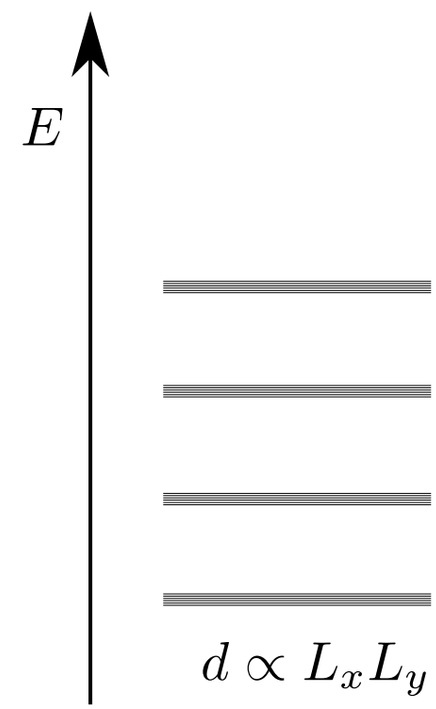
Detection of the FQH graviton

Thin-cylinder FQH

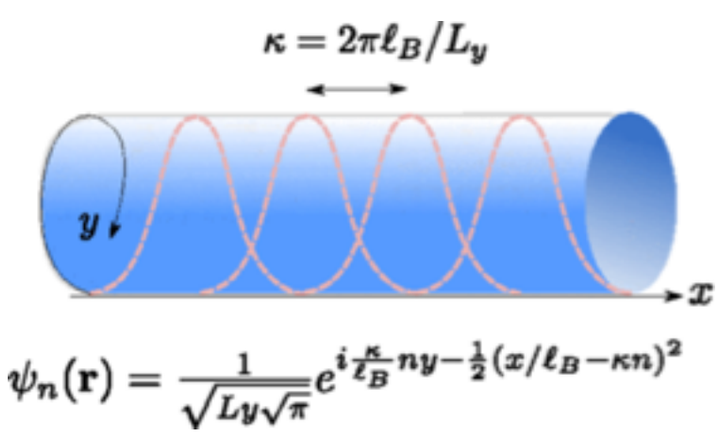
One-dimensional models of FQH are suitable for NISQ implementation

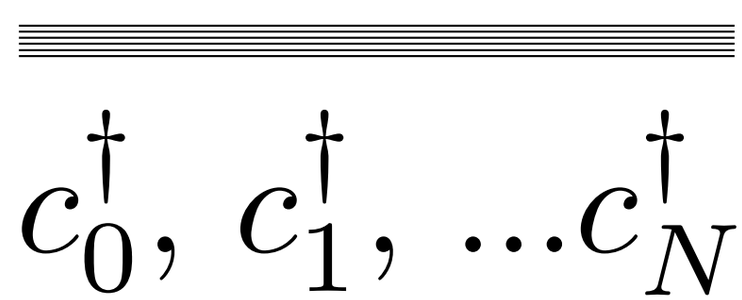
Let qubits represent orbitals in the lowest Landau levels



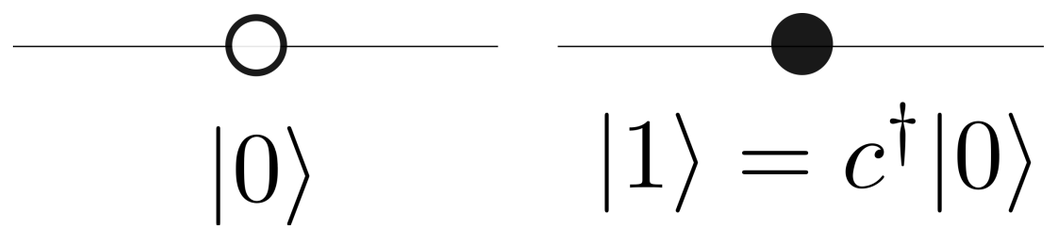


Focus on lowest Landau level, assign a fermonic creation operator to each of the degenerate orbitals in the level





Thin-cylinder FQH



General form of the electron-electron interaction Hamiltonian in this basis

$$H = \sum_{i=1}^{N_s} \sum_{k>|m|} V_{km} c^\dagger_{i+m} c^\dagger_{i+k} c_{i+m+k} c_i$$

$$N_s = L_x L_y / 2\pi$$

E. J. Bergholtz and A. Karlhede, Phys. Rev. Lett. **94**, 026802 (2005).
A. Seidel, H. Fu, D.-H. Lee, M. Leinaas, J. Moore, Phys. Rev. Lett. **95**, 266405 (2005).

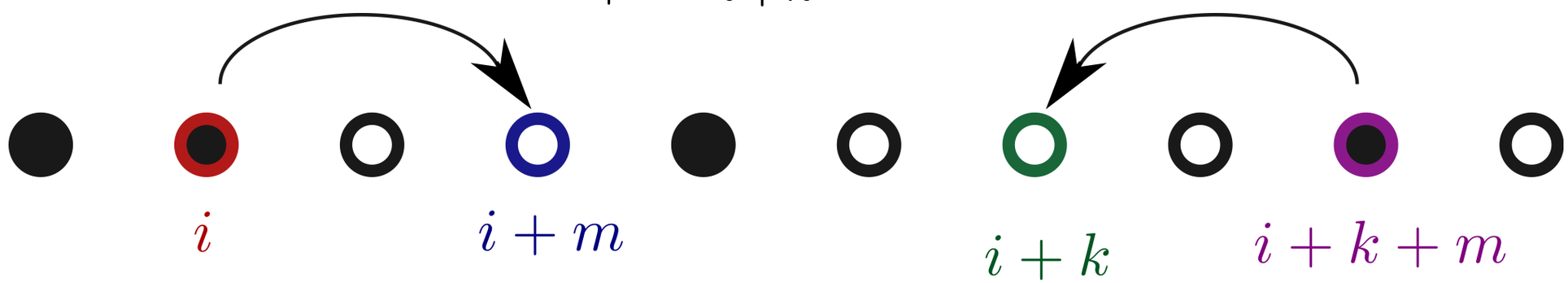
The structure of the potentials determines the state.
For the potential below, the Laughlin state is the solution

$$V_{km} \propto (k^2 - m^2) e^{-2\pi^2 (m^2 + k^2)/L_y}$$

Thin-cylinder FQH

Squeezing operator

$$c^\dagger_{i+m} c^\dagger_{i+k} c_{i+m+k} c_i$$



Potentials decay as Gaussians

We can truncate the interactions, the approximation improves for thinner cylinders.

Thin-cylinder FQH

$$H = \sum_j V_{10} n_{j+1} n_{j+2} + V_{20} n_j n_{j+2} + V_{30} n_j n_{j+3} + \sqrt{V_{10} V_{30}} \left(c_j^\dagger c_{j+3}^\dagger c_{j+2} c_{j+1} + \text{H.c.} \right)$$

Local squeezing

$$1001 \leftrightarrow 0110$$

The ground state of the above Hamiltonian is given by the expression

$$|\psi\rangle \propto \prod_i (1 - t S_i) |100100100...\rangle \qquad S_i = c_{i+1}^\dagger c_{i+2}^\dagger c_{i+3} c_i$$

$$t = \sqrt{\frac{V_{30}}{V_{10}}}$$ One parameter in 1D model, encoding the aspect ratio of 2D electron gas

M. Nakamura, Z.-Y. Wang, and E. J. Bergholtz, Phys. Rev. Lett. **109**, 016401 (2012).

Quantum algorithm for ground state

Fibonacci constraint from truncation

100 100 **100** 100 100
100 100 **011** 000 100

We cannot squeeze two neighboring blocks

Let *F(M)* be the number of states with *M* blocks

F(M)						

=

F(M-1)	100					

+

F(M-2)	011	000				

Quantum algorithm for ground state

The operator acting on the direct product CDW is not unitary. How can we construct an algorithm for making the state with standard gates?

$$1 - tS_j$$

Apply the following unitary sequentially from the end of the chain

$$U_j = e^{\phi_j (S_j - S_j^\dagger)}$$

We have showed that the above unitary operators provide the same ground state if the angles follow from the recursion relation

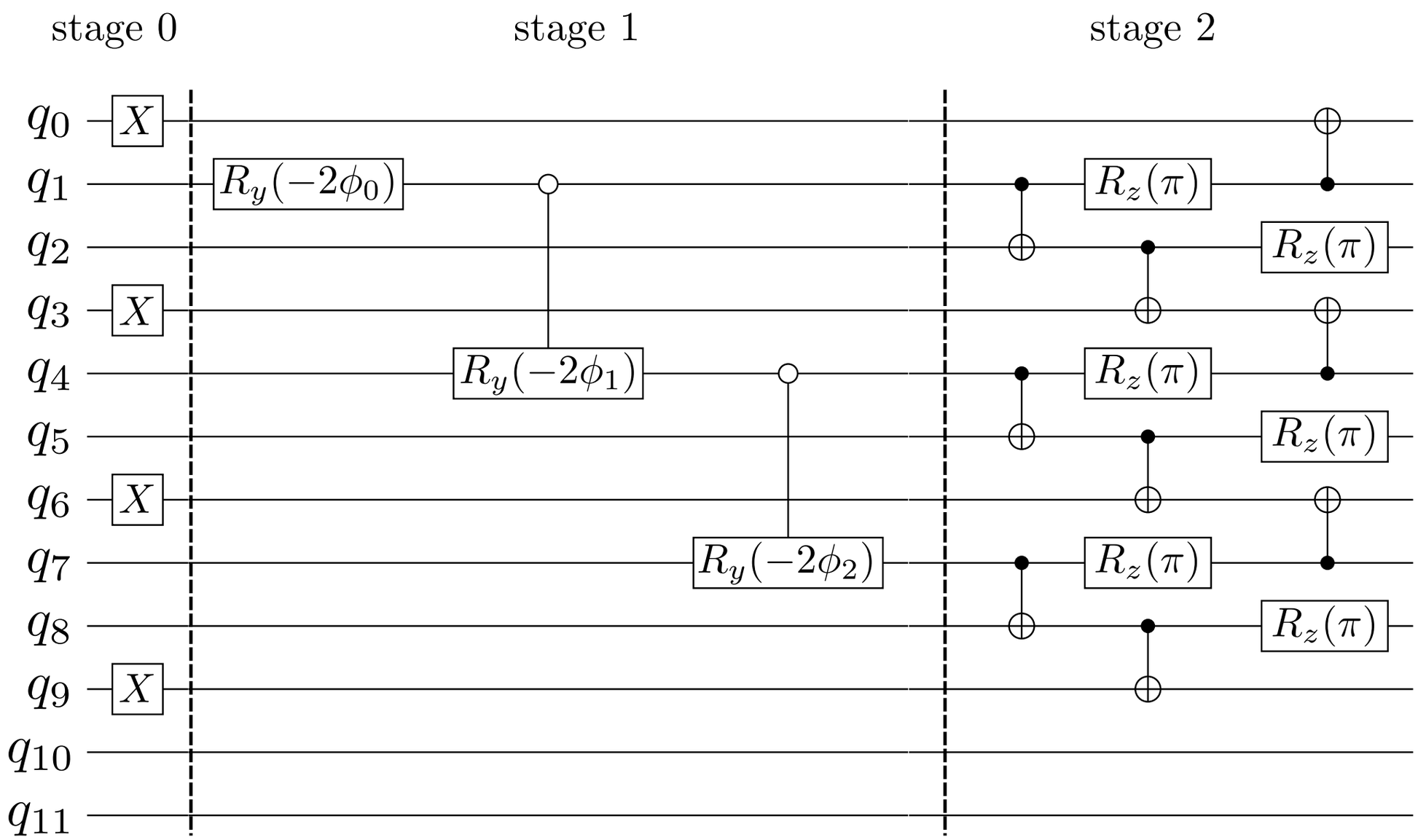
$$\boxed{\tan(\phi_{k-3}) = t\cos(\phi_k)}$$

with boundary condition

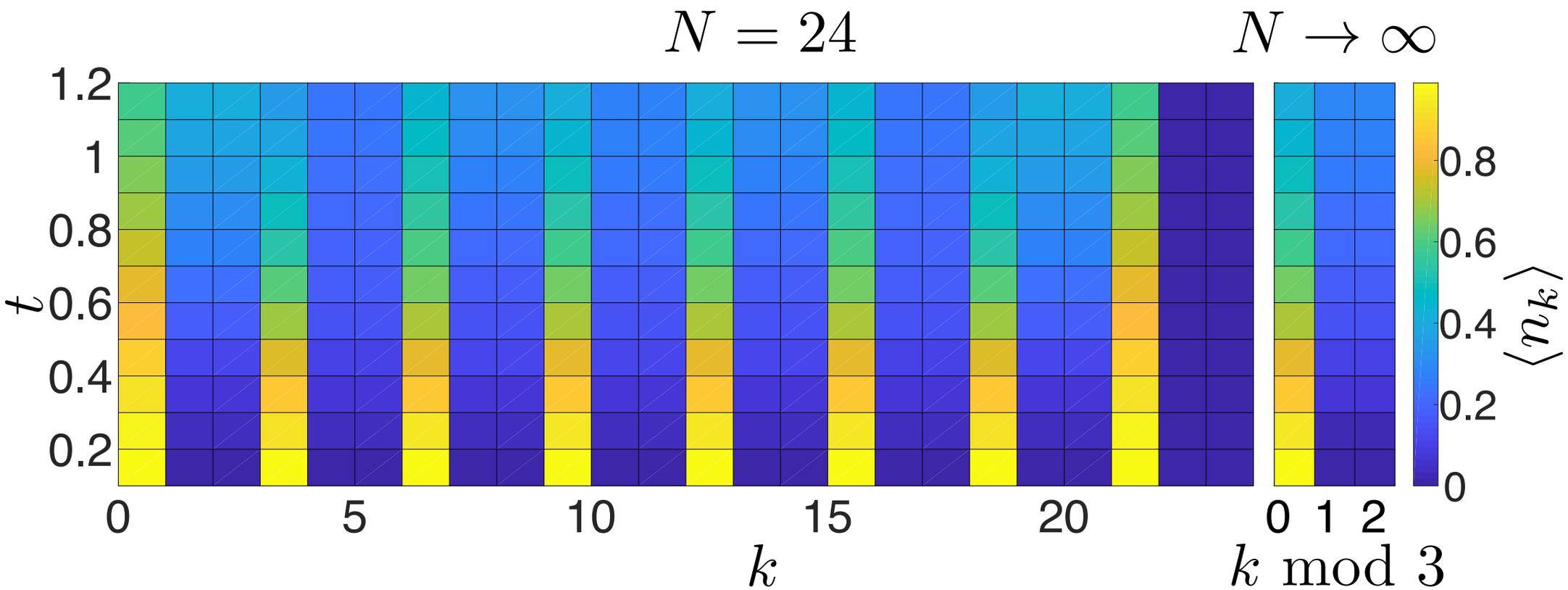
$$\phi_{N,N-1,N-2} = \arctan(t)$$

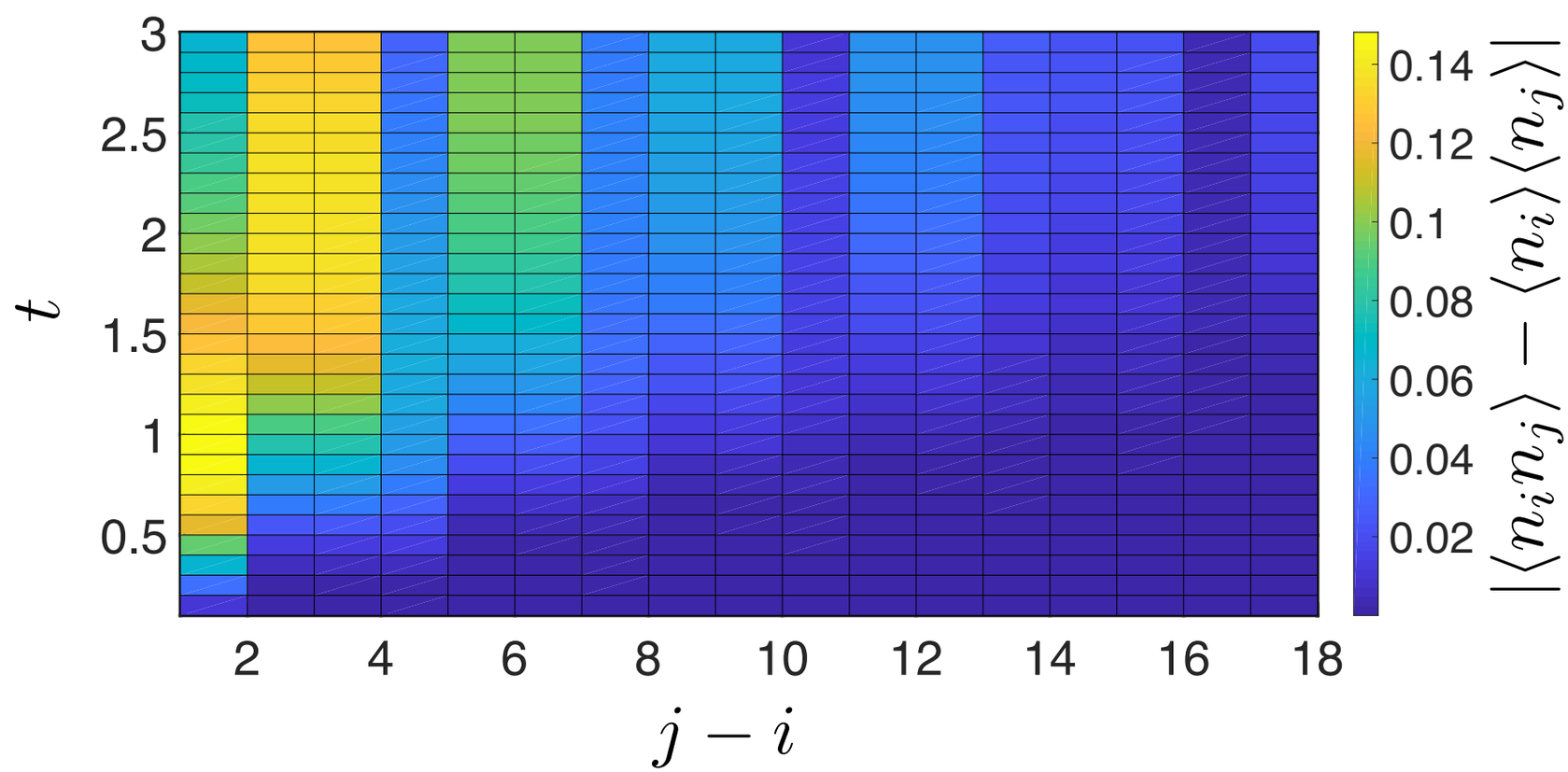
Quantum algorithm for ground state

Squeezing is a controlled rotation for reduced registers!



Verification of state preparation



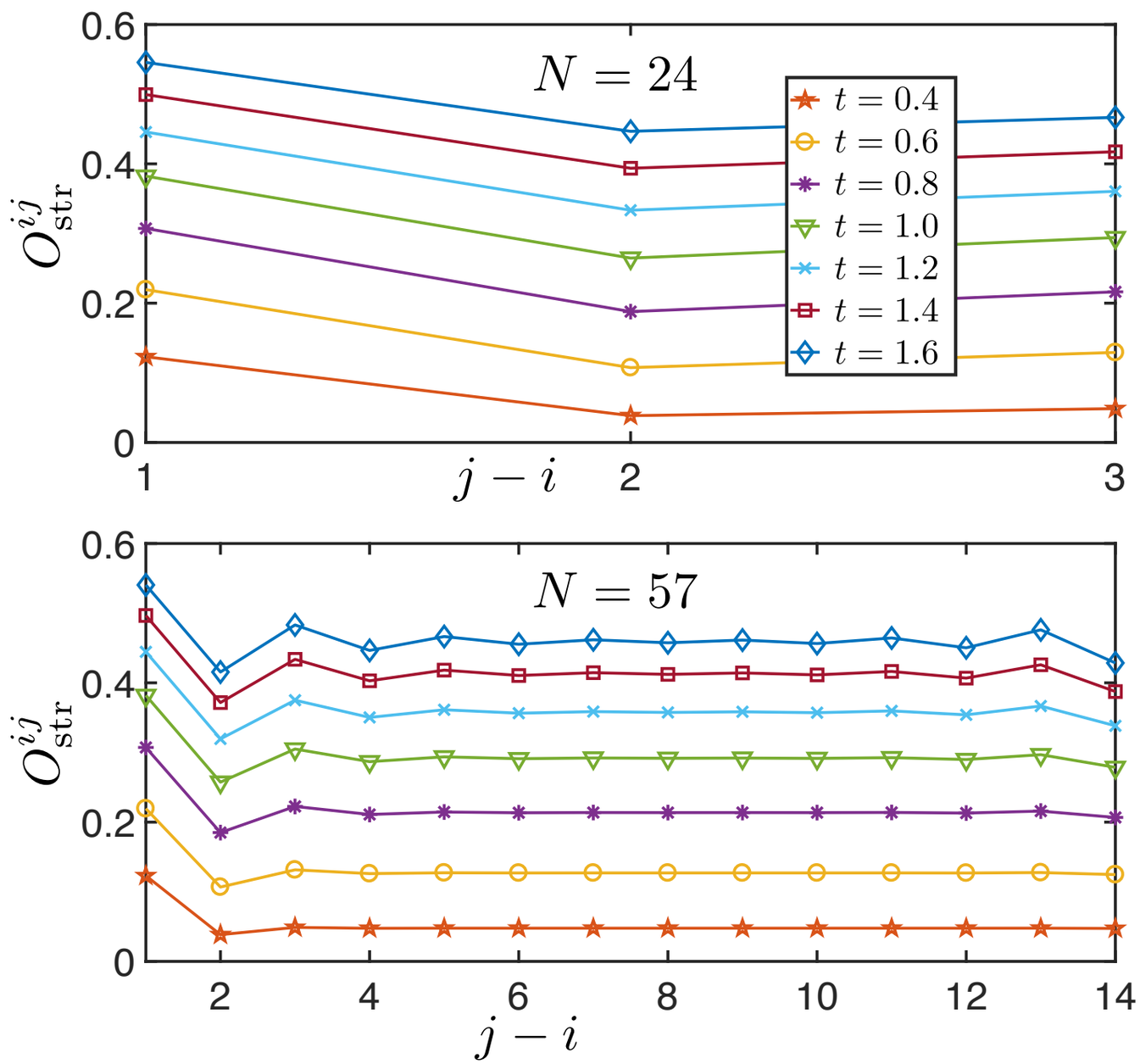


Verification of state preparation

A string operator capturing topological order (off-diagonal long-range order after a singular gauge transformation)

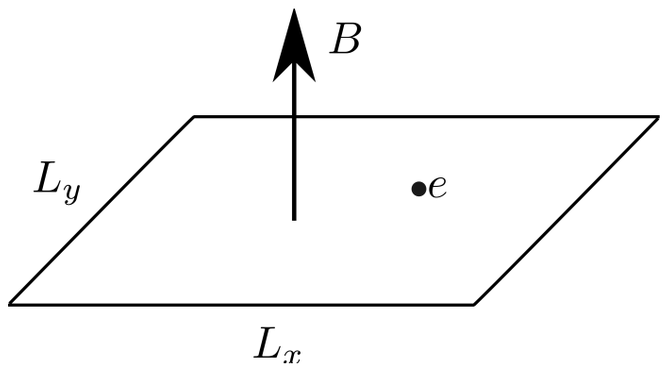
$$O_{\mathrm{str}}^{ij} = -\left\langle \left[\prod_{k=i+1}^{j-1} (-1)^{n_{3k+3}} (-1)^{n_{3k+1}} \right] (n_{3i+3} - n_{3i+1})(n_{3j+3} - n_{3j+1}) \right\rangle$$

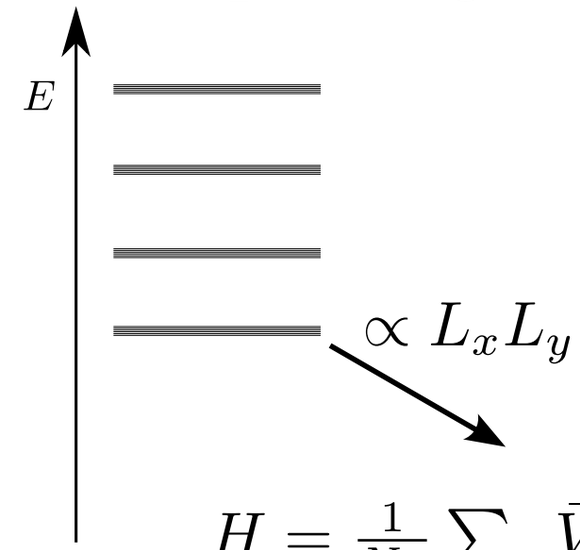
S. Girvin and D. Arovas, Physica Scripta 27, 156 (1989).



Probing the graviton

1) Endow the Hamiltonian with metric
2) Perform a geometric quench that couples to graviton





$V(|\mathbf{r}_i - \mathbf{r}_j|)$

$H = \frac{1}{N_\phi} \sum_{\mathbf{q}} \bar{V}_{\mathbf{q}} \bar{\rho}_{\mathbf{q}} \bar{\rho}_{-\mathbf{q}}$

$\bar{\rho}_{\mathbf{q}}$ is the density projected onto the Landau level

$\rho_{\mathbf{q}} = \sum_j e^{i\mathbf{q}.\mathbf{R}_j} \longrightarrow$ guiding-center coordinate

For the $\nu = 1/3$ Laughlin state, we have

$$V_{\mathbf{q}} = (1 - |\mathbf{q}|^2) e^{-|\mathbf{q}|^2/4}$$

Interaction

Landau-level form factor

$$\boxed{|\mathbf{q}|^2 = g_{\alpha\beta} q_\alpha q_\beta}$$

Probing the graviton

$$\boxed{|\mathbf{q}|^2 = g_{\alpha\beta} q_\alpha q_\beta}$$

Integrating out one momentum gives the 1D model

$$\hat{H} = \sum_{j=0}^{N_\phi - 1} \sum_{k > |m|} V_{k,m} c^\dagger_{j+m} c^\dagger_{j+k} c_{j+m+k} c_j$$

$$\boxed{V_{k,m} \propto (k^2 - m^2) \exp\left(-\kappa^2 \frac{(k^2 + m^2 - 2ikm g_{12})}{2 g_{11}} \right)}$$

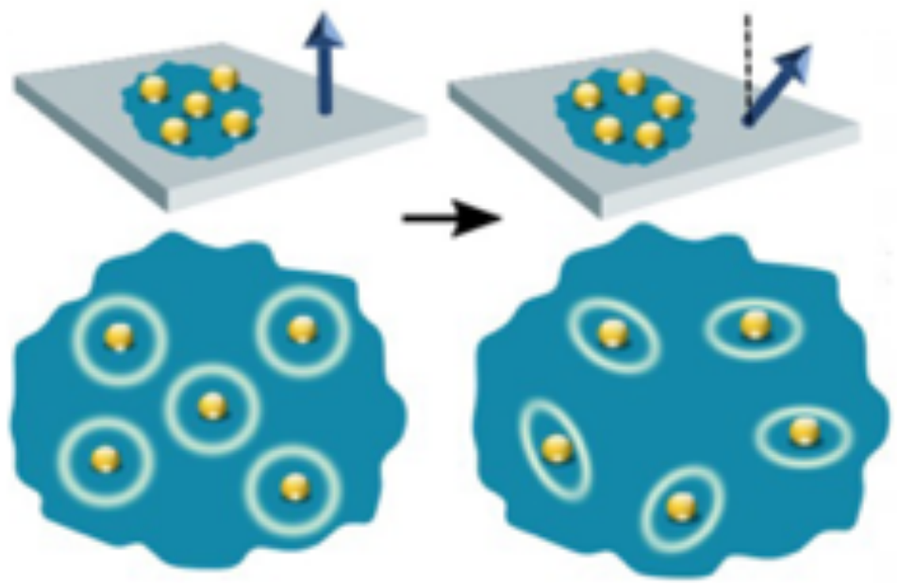
$$\kappa = 2\pi / L_2$$

Parameterization of general unimodular metric: rotation and stretch

$$g = \begin{pmatrix} \cosh Q + \cos\phi \sinh Q & \sin\phi \sinh Q \\ \sin\phi \sinh Q & \cosh Q - \cos\phi \sinh Q \end{pmatrix}$$

Probing the graviton

Geometric quantum quench



Z. Liu, A. Gromov, and Z. Papic, Phys. Rev. B **98**, 155140 (2018).

1) Create a Hamiltonian with an extrinsic metric g
2) Start in the ground state for a trivial extrinsic metric g
3) Suddenly change the extrinsic metric
4) Unitary evolution gives rise to time-dependent state similar to ground state with a time-dependent g'
5) Emergent intrinsic metric (wave function property)

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} \alpha & 0 \\ 0 & 1/\alpha \end{pmatrix}$$

Probing the graviton

PHYSICAL REVIEW X **7,** 041032 (2017)

Bimetric Theory of Fractional Quantum Hall States

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(Received 24 May 2017; published 10 November 2017; corrected 8 February 2018)

We present a bimetric low-energy effective theory of fractional quantum Hall (FQH) states that describes the topological properties and a gapped collective excitation, known as the Girvin-Macdonald-Platzman (GMP) mode. The theory consists of a topological Chern-Simons action, coupled to a symmetric rank-2 tensor, and an action *à la* bimetric gravity, describing the gapped dynamics of a spin-2 mode. The theory is formulated in curved ambient space and is spatially covariant, which allows us to restrict the form of the effective action and the values of phenomenological coefficients. Using bimetric theory, we calculate the projected static structure factor up to the k^6 order in the momentum expansion. To provide further support for the theory, we derive the long-wave limit of the GMP algebra, the dispersion relation of the GMP mode, and the Hall viscosity of FQH states. The particle-hole (PH) transformation of the theory takes a very simple form, making the duality between FQH states and their PH conjugates manifest. We also comment on the possible applications to fractional Chern insulators, where closely related structures arise. It is shown that the familiar FQH observables acquire a curious geometric interpretation within the bimetric formalism.

Simple predictions in the linear regime

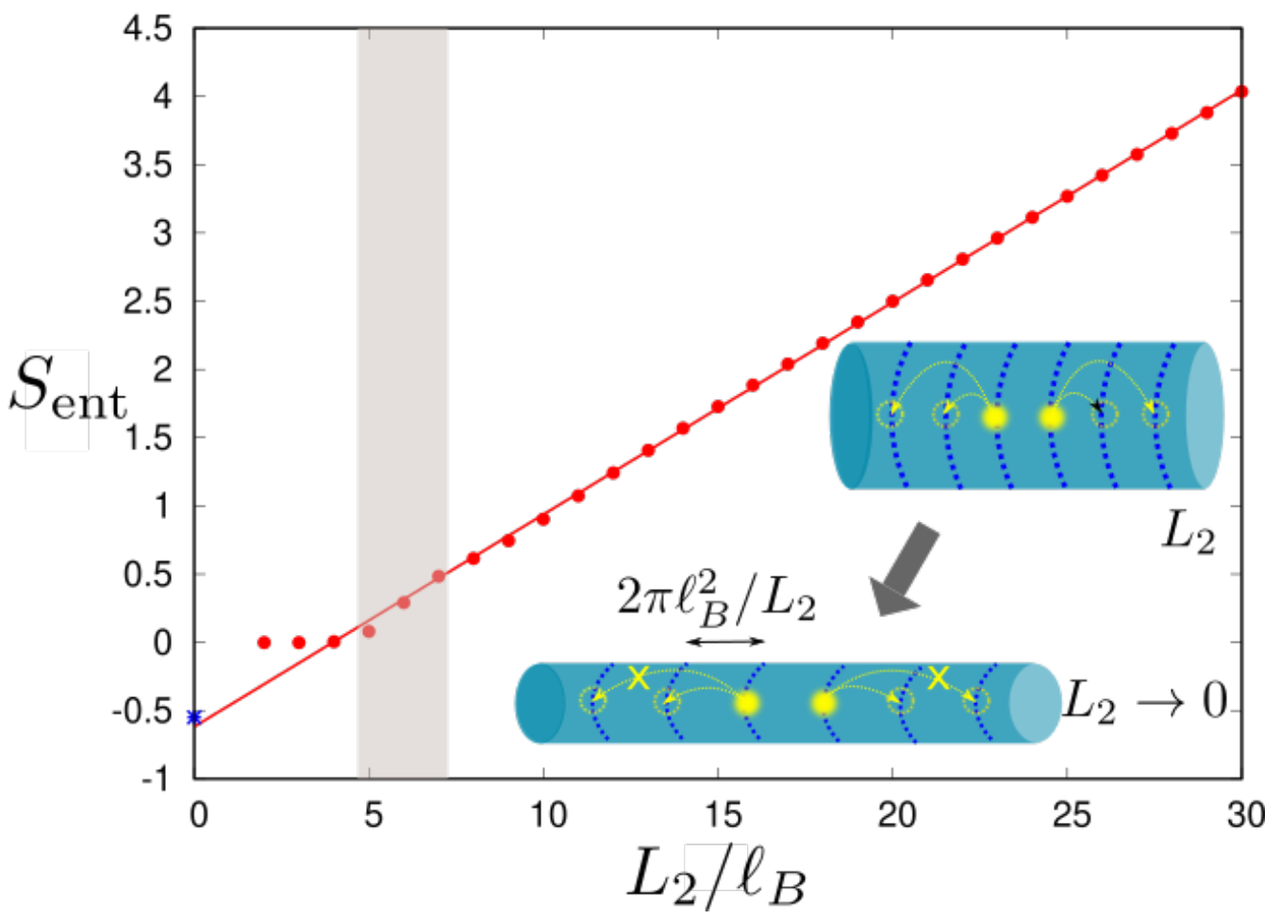
$$\widetilde{Q}(t) = \pm 2A \sin \frac{E_g t}{2}, \quad \widetilde{\phi}(t) = \pm \frac{\pi}{2} - \frac{E_g t}{2}$$

E_g is the energy of the graviton mode

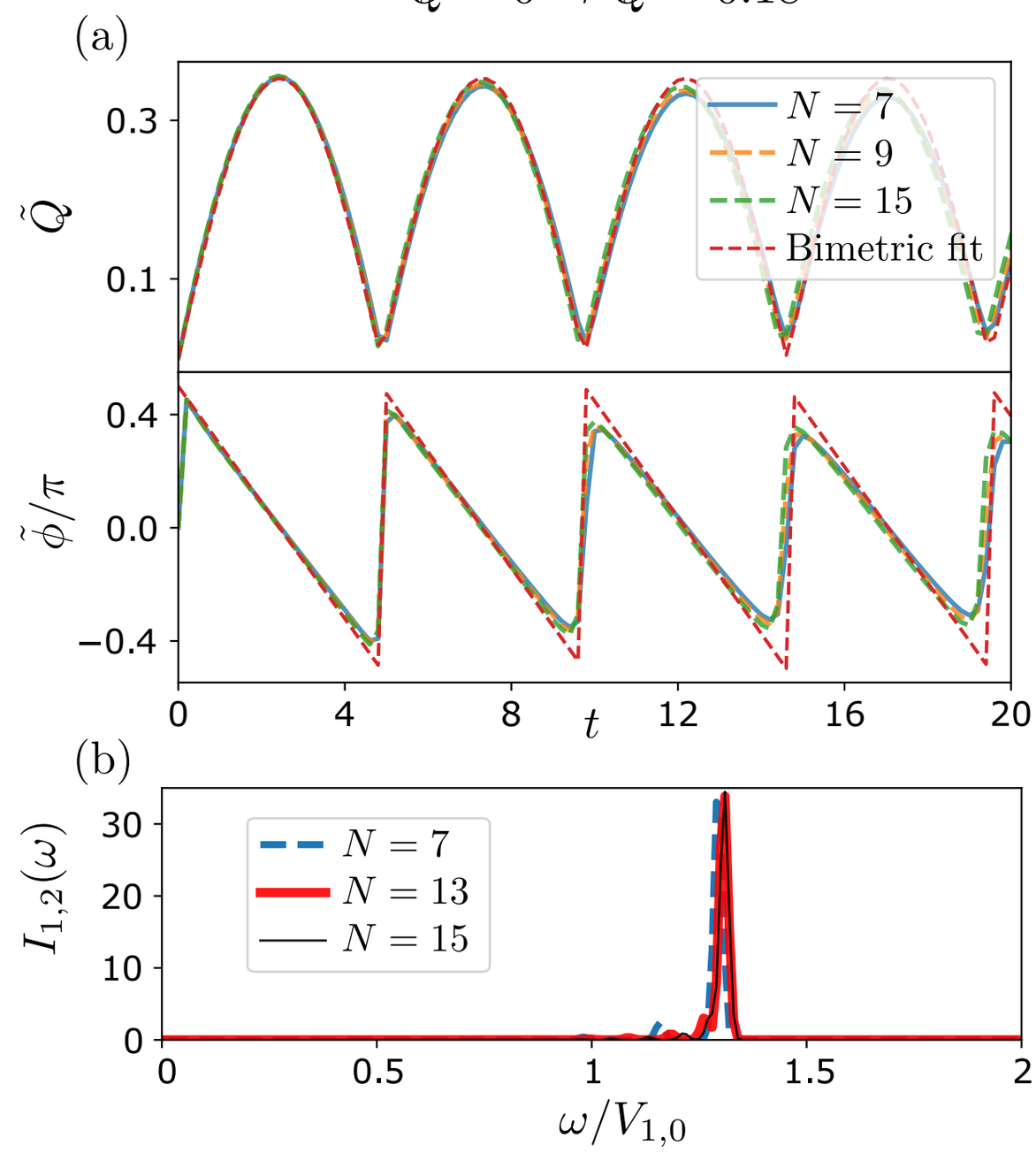
Probing the graviton

The model with metric is still solvable upon truncation!

$$|\psi_0\rangle = \prod_j \left(1 - \sqrt{\frac{V_{3,0}}{V_{1,0}}} e^{i\frac{8\pi^2\ell_B^2}{L_2^2}\frac{g_{12}}{g_{11}}} \hat{S}_j\right) |\ldots 100100\ldots\rangle$$



$Q = 0 \to Q = 0.18$



$$I_{1,2}(\omega) = \sum_j \delta(\omega - \epsilon_j)|\langle j|\hat{\mathcal{V}}_{1,2}|0\rangle|^2$$

Spectral function for quadrupolar generalized pseudopotential [Bo Yang et al, PRL **118**, 146403 (2017)]

Implementing the quench in quantum device

Mapping to reduced-register spin chain

$$|100, 100, 100, \ldots\rangle \to |\mathbb{0}, \mathbb{0}, \mathbb{0}, \ldots\rangle$$

$$|100, 011, 000, \ldots\rangle \to |\mathbb{0}, \mathbb{1}, \mathbb{0}, \ldots\rangle$$

$$\sum_{j=0}^{3N-2} \hat{n}_j \hat{n}_{j+1} \to \sum_{\ell=1}^{N-1} \mathcal{N}_\ell$$

$$\sum_{j=0}^{3N-4} \hat{n}_j \hat{n}_{j+3} \to N - 1 - 3 \sum_{\ell=1}^{N-1} \mathcal{N}_\ell + \sum_{\ell=1}^{N-3} \mathcal{N}_\ell \mathcal{N}_{\ell+2} + \mathcal{N}_1 + \mathcal{N}_{N-1}$$

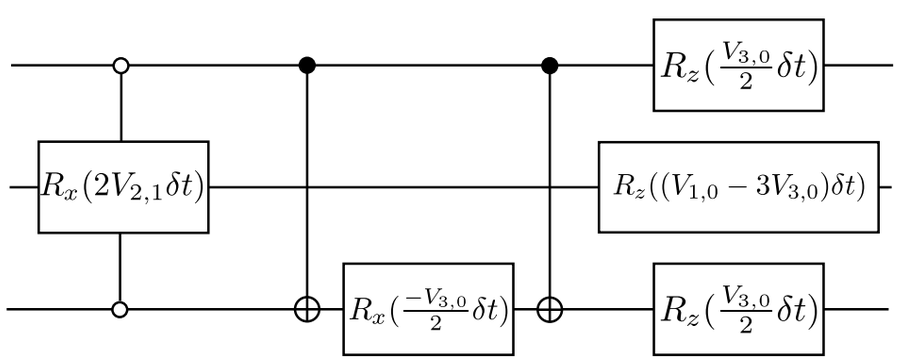
$$\sum_{j=0}^{3N-4} V_{2,1} c_{j+1}^\dagger c_{j+2}^\dagger c_{j+3} c_j + {V_{2,1}}^* c_j^\dagger c_{j+3}^\dagger c_{j+2} c_{j+1}$$
$$\to [\mathrm{Re}(V_{2,1}) X_1 - \mathrm{Im}(V_{2,1}) Y_1](1 - \mathcal{N}_2) + (1 - \mathcal{N}_{N-2})[\mathrm{Re}(V_{2,1}) X_{N-1} - \mathrm{Im}(V_{2,1}) Y_{N-1}]$$
$$+ \sum_{\ell=2}^{N-2} (1 - \mathcal{N}_{\ell-1})[\mathrm{Re}(V_{2,1}) X_\ell - \mathrm{Im}(V_{2,1}) Y_\ell](1 - \mathcal{N}_{\ell+1}).$$

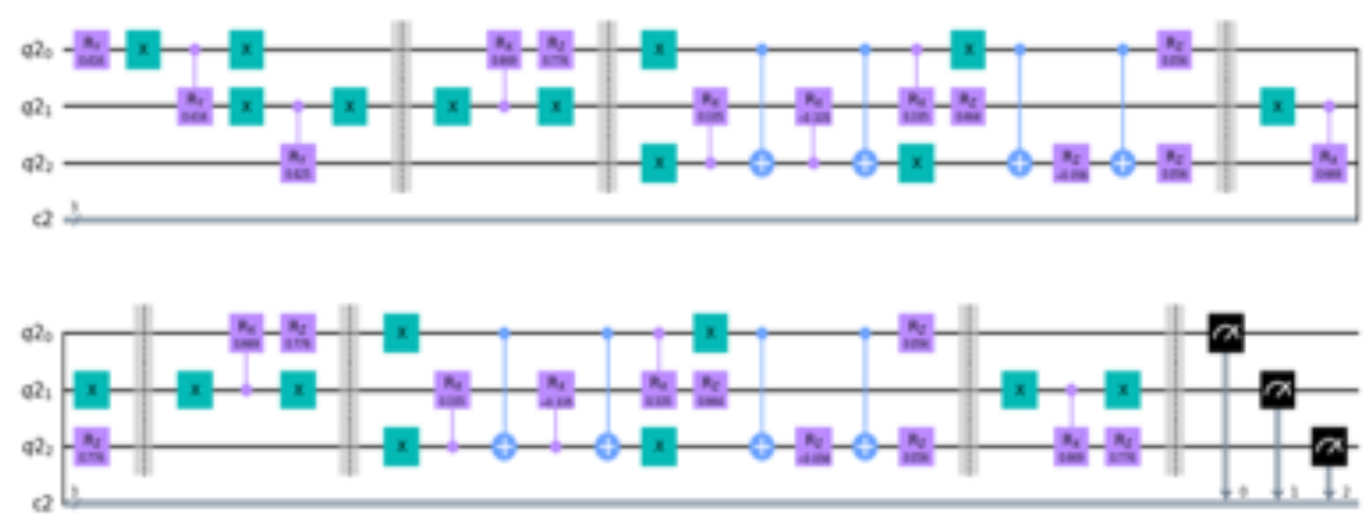
Implementing the quench in quantum device

Trotterize the evolution $$e^{-it\sum_\ell H_\ell} \approx \left[\prod_l U_\ell(\delta t)\right]^k \qquad \delta t = t/k$$

Key building block

$$U_\ell(\delta t) \approx e^{-i\frac{V_{3,0}}{4}Z_{\ell-1}\delta t} e^{-i\frac{V_{1,0}-3V_{3,0}}{2}Z_\ell \delta t} e^{-i\frac{V_{3,0}}{4}Z_{\ell+1}\delta t}$$
$$\times\, e^{-i\frac{V_{3,0}}{4}Z_{\ell-1}Z_{\ell+1}\delta t} e^{-iV_{2,1}(1-\mathcal{N}_{\ell-1})X_\ell(1-\mathcal{N}_{\ell+1})\delta t}$$



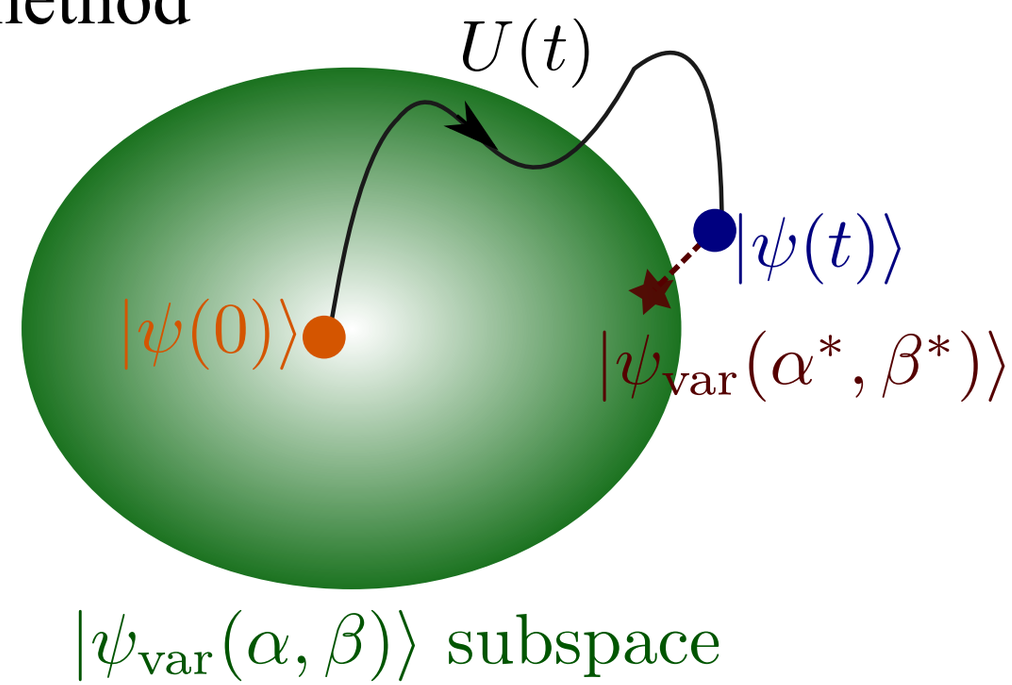


IBM Qiskit

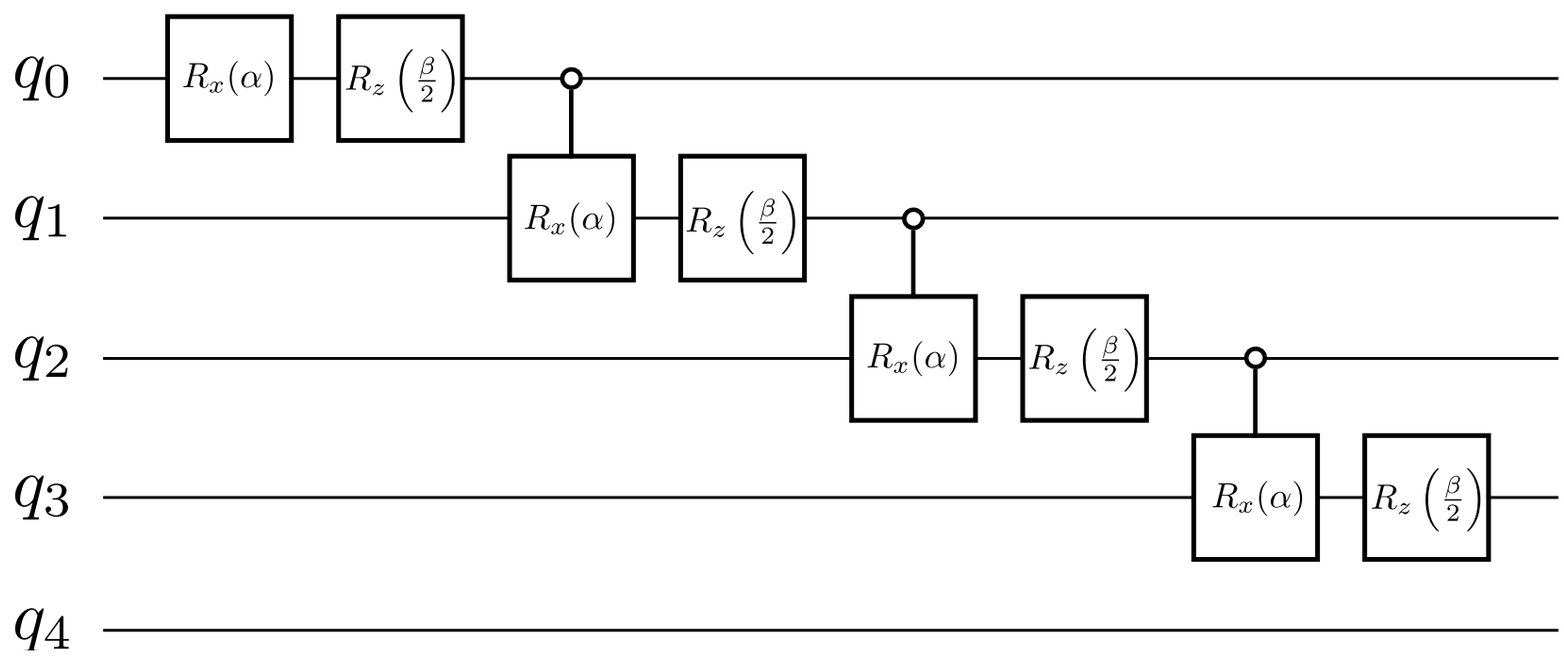
Implementing the quench in quantum device

Variational method

(a)



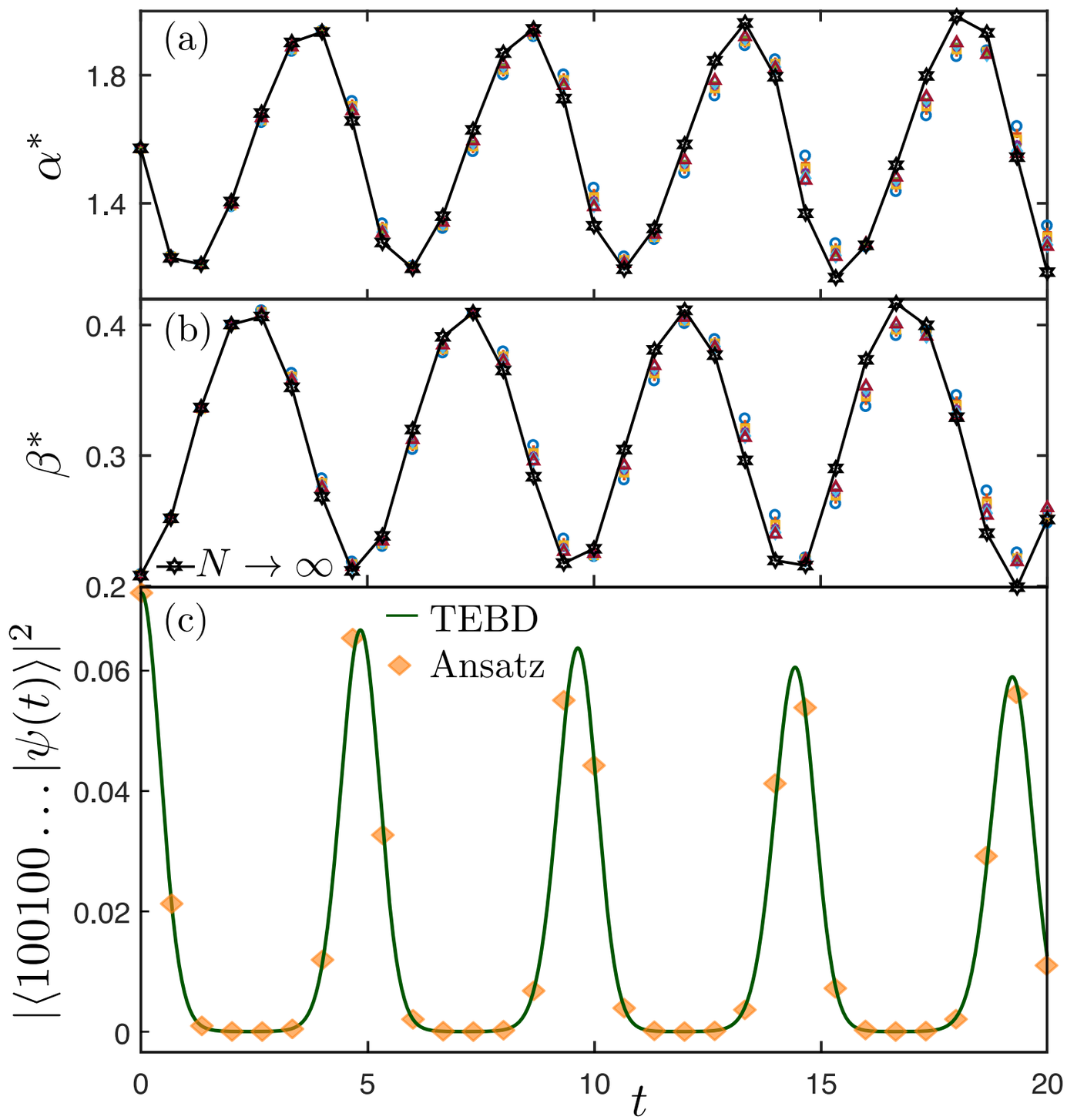
(b)



$$|\psi_{\rm var}(\alpha, \beta)\rangle = \prod_{\ell} e^{-i\alpha\mathcal{N}_\ell} e^{-i\beta(1-\mathcal{N}_{\ell-1})X_\ell} |\mathbb{0}\mathbb{0}\mathbb{0}\ldots\rangle$$

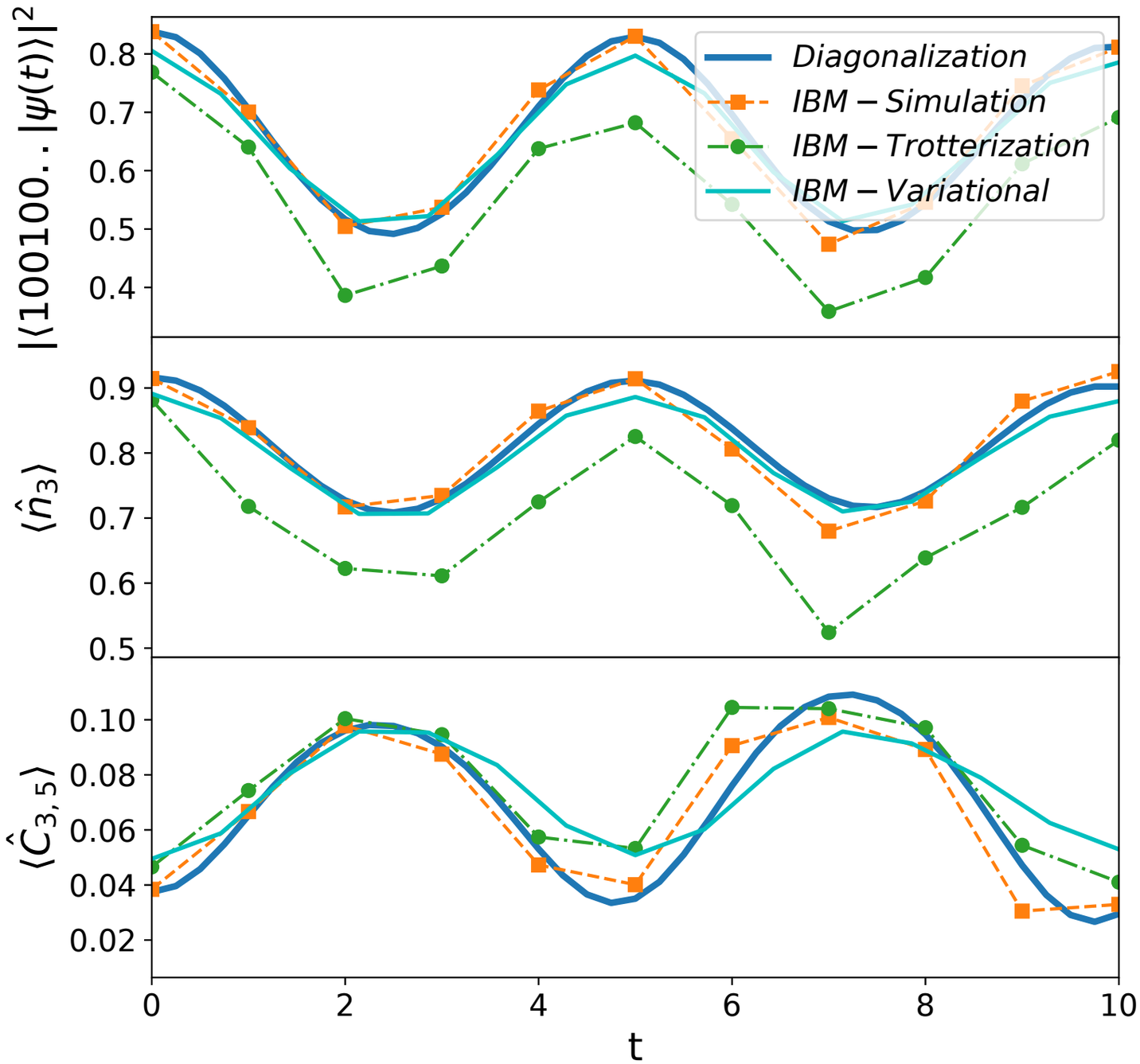
Implementing the quench in quantum device

Variational method



Implementing the quench in quantum device

Quantum hardware results



Deep trotterization circuits implemented with noise-aware error mitigation and optimized quantum compiling

Summary

- We introduced a 1D model capturing the physics of 1/3 FQH state endowed with a metric
- The geometric quench of the model is governed by the biometric theory of a spin-2 graviton
- We implemented a geometric quench on the IBM device using error-mitigated trotterization
- We observed coherent oscillations in multiple quantities with a single graviton frequency, providing a physical probe of the FQH graviton on a synthetic platform
- We found a variational algorithm that generates the post-quench state in the thermodynamic limit with only two parameters

Outlook

- Probing topological charged excitations and their braiding using the 1D construction
- Generalization to other fractional states