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FFLO correlations in polarized ultracold Fermi gases

Pierbiagio Pieri

Dept. of Physics and Astronomy, University of Bologna & INFN, Sezione di Bologna, Italy

Michele Pini[1,2], G. Calvanese Strinati[1]

[1]CNR-INO, Istituto Nazionale di Ottica, Sede di Sesto Fiorentino, (FI), Italy
[2]Max Planck Institute for the physics of complex systems, Dresden, Germany

ALMA MATER STUDIORUM UNIVERSITÀ DI BOLOGNA

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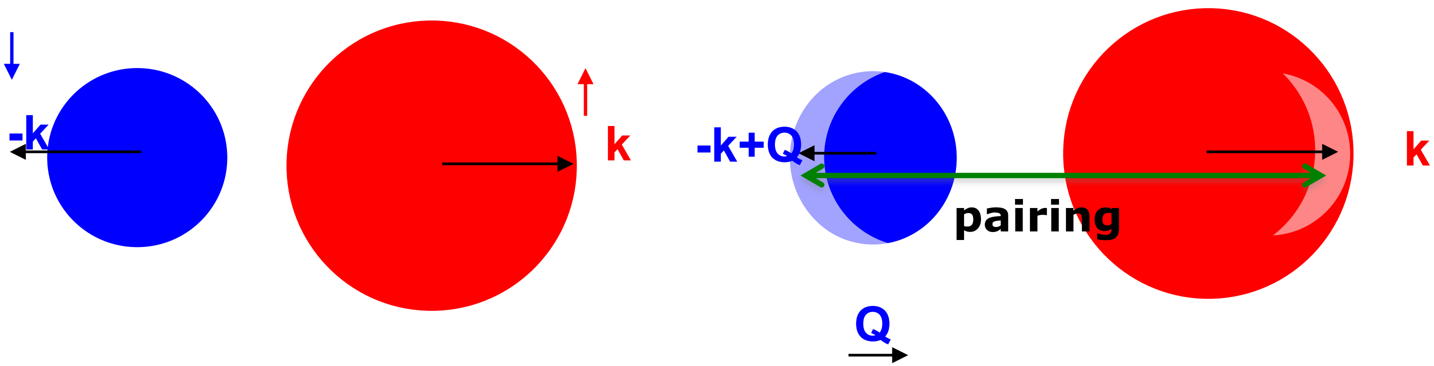
Polarized Fermi gases: search for FFLO phase

- Search for FFLO phase was one of the main motivations driving experiments with polarized Fermi gases
- Phase first proposed theoretically by Fulde & Ferrell and independently by Larkin & Ovchinnikov (1964)
- Pairing between **k** and **−k+Q** to compensate mismatch of Fermi surfaces: pairs acquire a finite center of mass momentum **Q**



$$|\Psi_{FF}\rangle = \prod_{\mathbf{k}\in\mathcal{R}} c^{\dagger}_{\mathbf{k}+\frac{\mathbf{Q}}{2}\uparrow} \prod_{\mathbf{k}\notin\mathcal{R}} (u_{\mathbf{k}} + v_{\mathbf{k}} c^{\dagger}_{\mathbf{k}+\frac{\mathbf{Q}}{2}\uparrow} c^{\dagger}_{-\mathbf{k}+\frac{\mathbf{Q}}{2}\downarrow})|0\rangle$$

Search for FFLO fluctuating pairs in the normal phase

However:

- In fully isotropic systems, **at finite T,** thermal fluctuations destroy long-range FFLO ordering [Shimahara (1998), Ohashi (2002)] possibly turning it into algebraic order [Radzihovski (2011), Jakubczyk (2017,2021)]

- Phase diagram of polarized Fermi gas (mean field at T=0 & experiments at finite temperature in harmonic trap) is dominated by **phase separation** (even though a density functional calculation with input from quantum Monte Carlo data predicted a large FFLO region at unitarity [Bulgac et al., (2008)])

Our idea:

- Search for FFLO fluctuating pairs in the normal phase

- Even if the system is the normal phase, non-condensed FFLO pairs might be present close to the FFLO/phase separation region

- If this is the case, FFLO short-range correlations and fluctuations should reveal in corresponding pairing susceptibility

Finite temperature

M. Pini, P. Pieri, G. C. Strinati, Phys. Rev. Res. **3**, 043068 (2021)

Method used in our calculations: self-consistent T-matrix

Modification of the original Nozieres-Schmitt-Rink approach for the BCS-BEC crossover in that: (i) Dyson's equation solved exactly; (ii) self-consistent G used everywhere instead of bare G_0; (iii) generalized to polarized systems with $\mu_\uparrow \neq \mu_\downarrow$

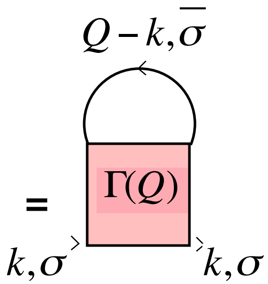
$$\Sigma_\sigma(k) = -\int \frac{d\mathbf{Q}}{(2\pi)^3} T \sum_\nu \Gamma(Q) G_{\bar{\sigma}}(k-Q)$$

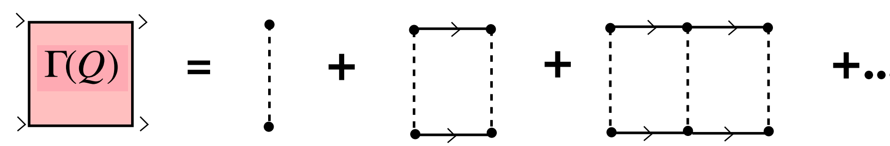
$$\Gamma(Q) = -\left[\frac{m}{4\pi a} + \int \frac{d\mathbf{k}}{(2\pi)^3}\left(T\sum_\nu G_\uparrow(k) G_\downarrow(Q-k) - \frac{m}{k^2}\right)\right]^{-1}$$

$k = (\mathbf{k}, \omega_n)$

$\omega_n = 2\pi T(n+1/2)$

$\Sigma_\sigma(k) =$





$Q = (\mathbf{Q}, \Omega_\nu)$ $\quad$ $\Omega_\nu = 2\pi T \nu$

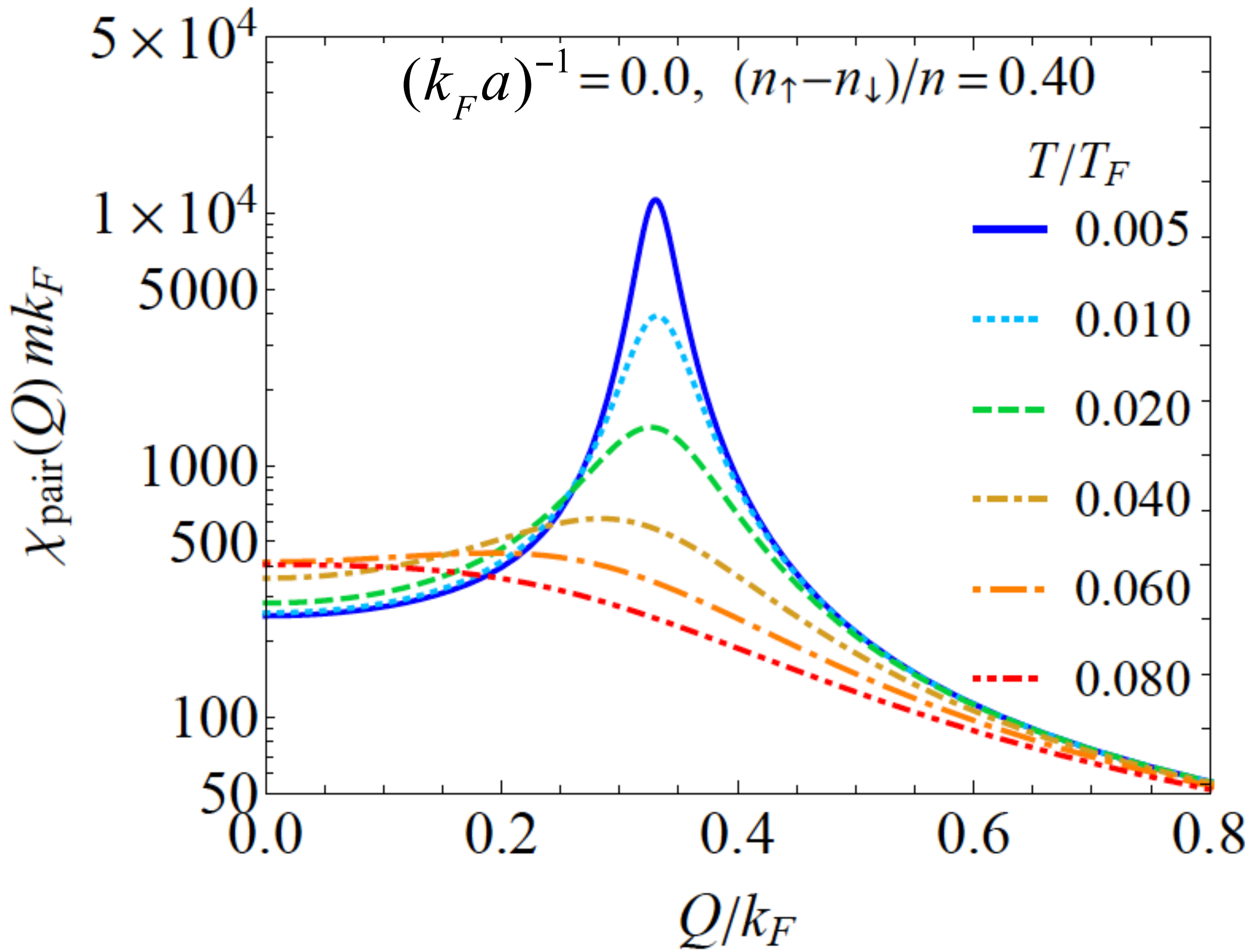
$$G_\sigma(\mathbf{k}, \omega_n) = [G_{0\sigma}(\mathbf{k}, \omega_n)^{-1} - \Sigma_\sigma(\mathbf{k}, \omega_n)]^{-1}$$

$$G_{0\sigma}(\mathbf{k}, \omega_n) = [i\omega_n - k^2/2m + \mu_\sigma]^{-1}$$

Within fully self-consistent t-matrix approach the pair susceptibility χ_{pair} is given by:

$$\chi_{\text{pair}}(\mathbf{Q}) = \Gamma(\mathbf{Q}, \Omega_\nu = 0)$$

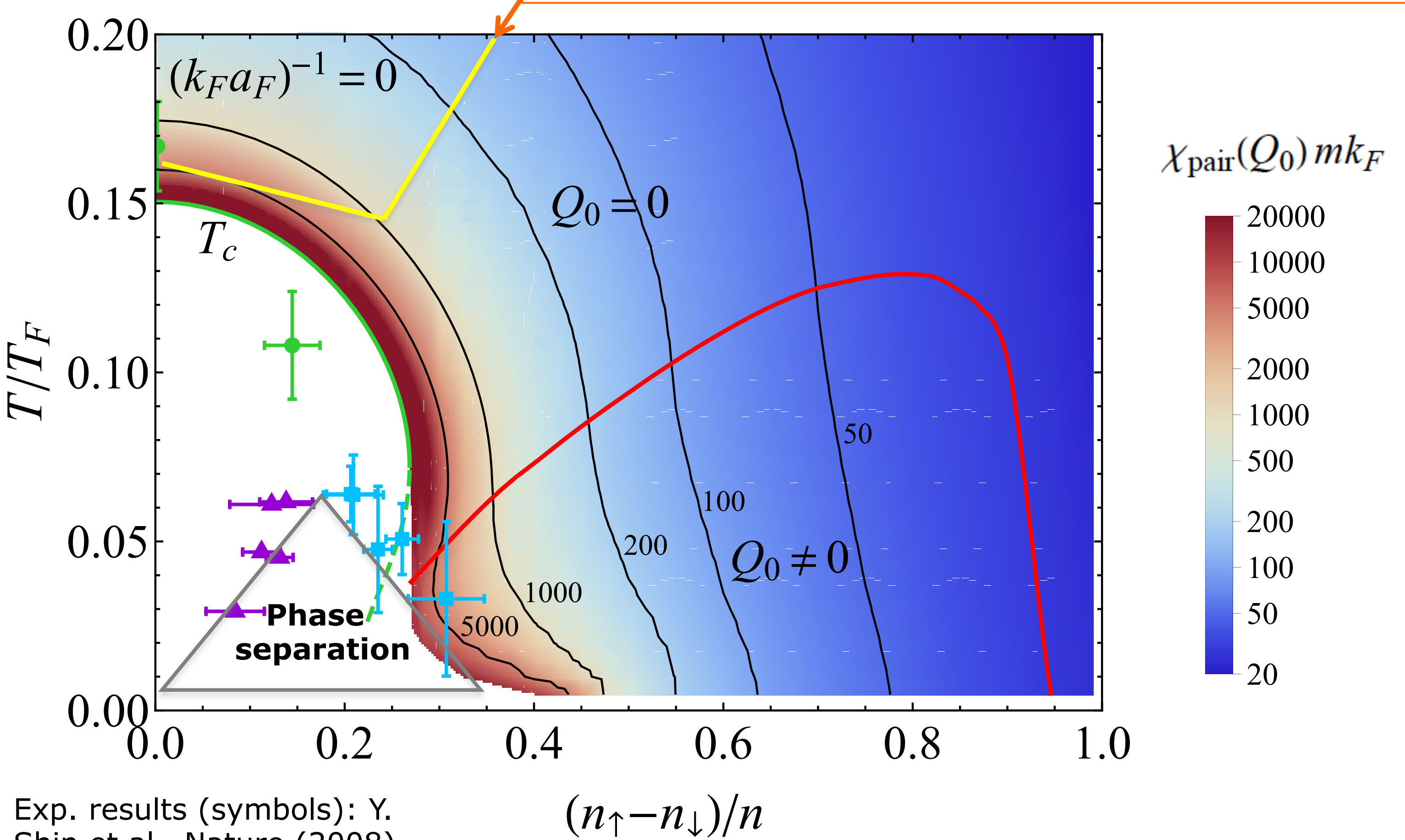
Polarized unitary Fermi gas: results for the pair susceptibility



Pronounced peak in the pair susceptibility at finite Q: **FFLO pairing fluctuations**.

Phase diagram of the polarized unitary Fermi gas

Recent complex Langevin QMC by F. Attanasio et al., PRA (2022) found $Q_0 = 0$ above this line



Exp. results (symbols): Y. Shin et al., Nature (2008)

Quite extended FFLO fluctuation region in the phase diagram

M. Pini, P. Pieri, G. C. Strinati, Phys. Rev. Res. **3**, 043068 (2021)

Zero temperature

M. Pini, P. Pieri, G. C. Strinati, on arxiv soon

Self-consistent T-matrix exactly at T=0

Finite T

$$\Sigma_\sigma(k) = -\int \frac{d\mathbf{Q}}{(2\pi)^3} T \sum_\nu \Gamma(Q) G_{\bar{\sigma}}(k-Q)$$

$$k = (\mathbf{k}, \omega_n) \qquad \omega_n = 2\pi T(n+1/2)$$

$$Q = (\mathbf{Q}, \Omega_\nu) \qquad \Omega_\nu = 2\pi T \nu$$

$$G_\sigma(\mathbf{k}, \omega_n) = [G_{0\sigma}(\mathbf{k}, \omega_n)^{-1} - \Sigma_\sigma(\mathbf{k}, \omega_n)]^{-1}$$

$$G_{0\sigma}(\mathbf{k}, \omega_n) = [i\omega_n - k^2/2m + \mu_\sigma]^{-1}$$

$$\Gamma(Q) = -\left[\frac{m}{4\pi a} + \int \frac{d\mathbf{k}}{(2\pi)^3}\left(T\sum_\nu G_\uparrow(k) G_\downarrow(Q-k) - \frac{m}{k^2}\right)\right]^{-1}$$

T = 0

$$\Sigma_\sigma(k) = -\int \frac{d\mathbf{Q}}{(2\pi)^3} \int \frac{d\Omega}{2\pi} \Gamma(Q) G_{\bar{\sigma}}(k-Q)$$

$$k = (\mathbf{k}, \omega)$$

with ω and Ω continuous

$$Q = (\mathbf{Q}, \Omega)$$

$$G_\sigma(\mathbf{k}, \omega) = [G_{0\sigma}(\mathbf{k}, \omega)^{-1} - \Sigma_\sigma(\mathbf{k}, \omega)]^{-1}$$

$$G_{0\sigma}(\mathbf{k}, \omega) = [i\omega - k^2/2m + \mu_\sigma]^{-1}$$

$$\Gamma(Q) = -\left[\frac{m}{4\pi a} + \int \frac{d\mathbf{k}}{(2\pi)^3}\left(\int \frac{d\omega}{2\pi} G_\uparrow(k) G_\downarrow(Q-k) - \frac{m}{k^2}\right)\right]^{-1}$$

We continue to work with imaginary frequencies: $i\omega$ and $i\Omega$, continuous at zero T.

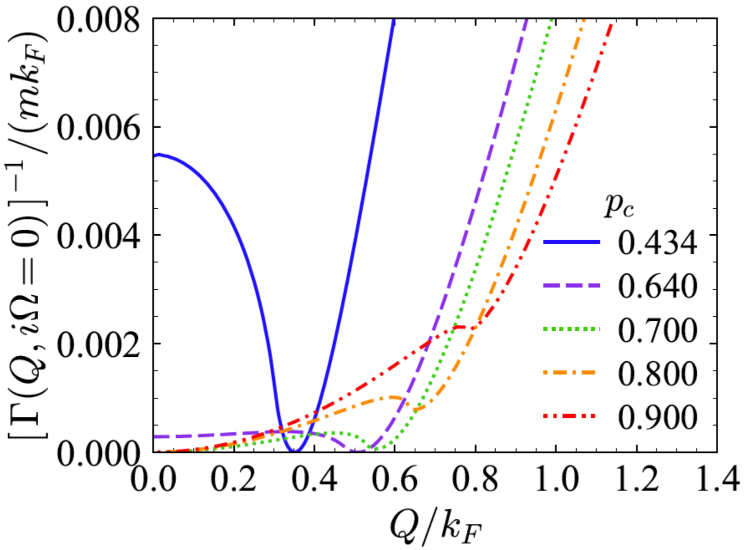
Phase diagram at T=0

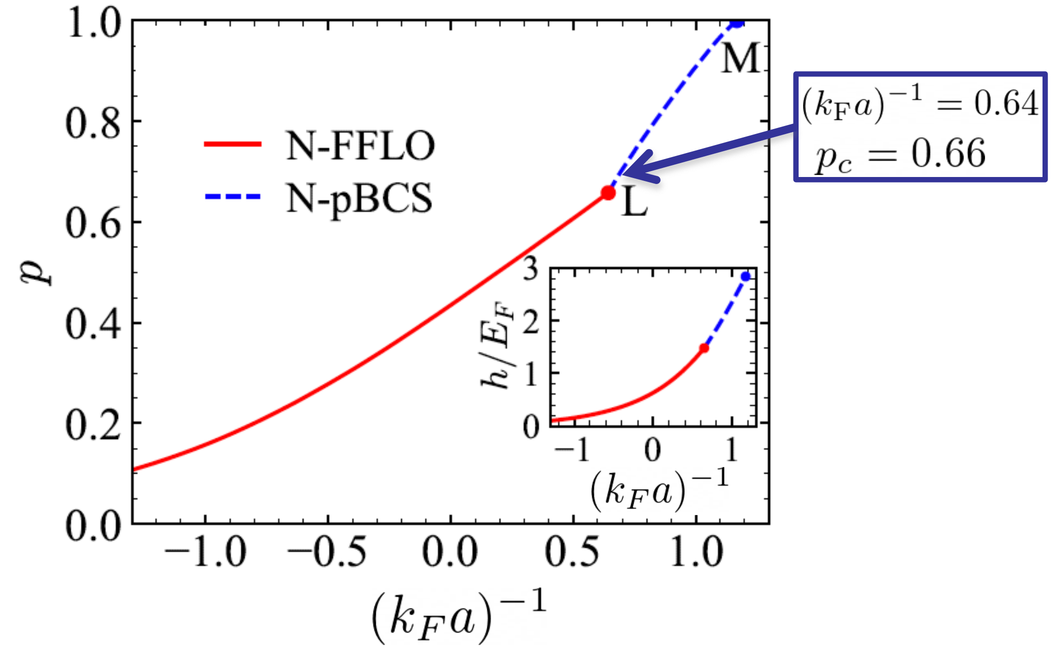
We vary dimensionless coupling $(k_F a)^{-1}$ (with $k_\mathrm{F} \equiv (3\pi^2 n)^{1/3}$, $n = n_\uparrow + n_\downarrow$) and determine critical polarization p_c [with $p = (n_\uparrow - n_\downarrow)/(n_\uparrow + n_\downarrow)$].

Second order phase transition determined by:

$$\left[\Gamma(|\mathbf{Q}| = Q_0, i\Omega = 0)|_{p=p_c}\right]^{-1} = 0 \longleftrightarrow \text{diverging pairing susceptibility } \chi_\mathrm{pair}(Q_0),$$

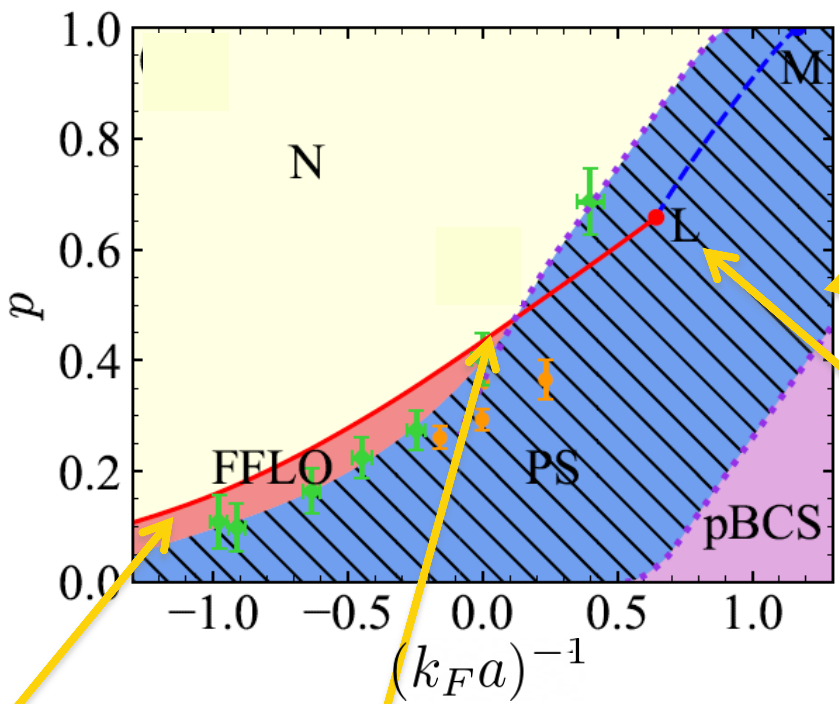
where Q_0 is the value of $|\mathbf{Q}|$ minimizing $\Gamma(|\mathbf{Q}|, i\Omega = 0)^{-1}$





At the Lifshitz point (L) the transition changes from N-FFLO to N-pBCS where pBCS is a polarized SF with standard BCS pairing $(Q_0 = 0)$.

Phase diagram at T=0: including phase separation



PS region combines exp. data data from Shin et al., 2008 and Olsen et al., 2015 (symbols with error bars) with QMC results from Pilati et al., 2008 (dotted lines)

Lifshitz point is hidden by phase separation region. However, the L point could be reached in principle following a metastable N phase (we found a positive definite compressibility matrix).

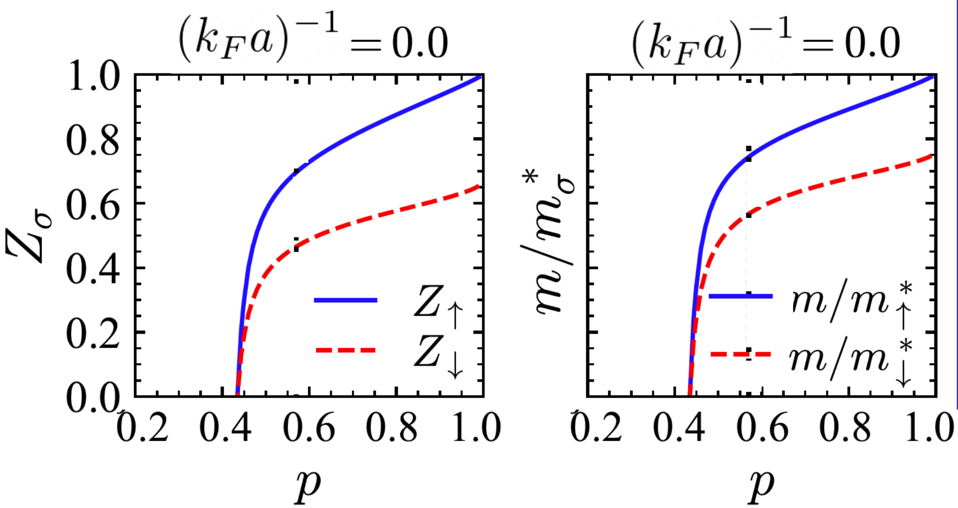
Relatively narrow region for observability of FFLO, still larger than within mean-field. It includes a polarization range at unitarity. Need boxlike trap for observability.

At unitarity, critical polarization to FFLO: ~ 0.435

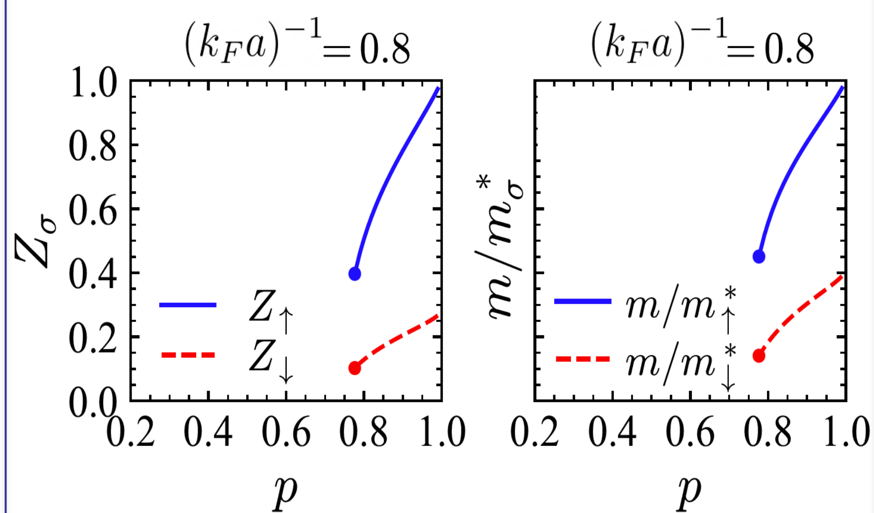
Quasi-particle residue and effective mass

Quasi-particle residue and effective mass at $|\mathbf{k}| = k_{F\sigma}$ can be calculated directly on the imaginary frequency axis:

$$Z_\sigma = \left[1 - \frac{\partial\, \mathrm{Im}\Sigma_\sigma(k_{F\sigma}, i\omega)}{\partial \omega}\Big|_{\omega=0}\right]^{-1} \qquad \frac{m}{m^*_\sigma} = Z_\sigma\left[1 + \frac{m}{k_{F\sigma}} \frac{\partial \Sigma_\sigma(\mathbf{k}, i\omega = 0)}{\partial |\mathbf{k}|}\Big|_{|\mathbf{k}|=k_{F\sigma}}\right]$$

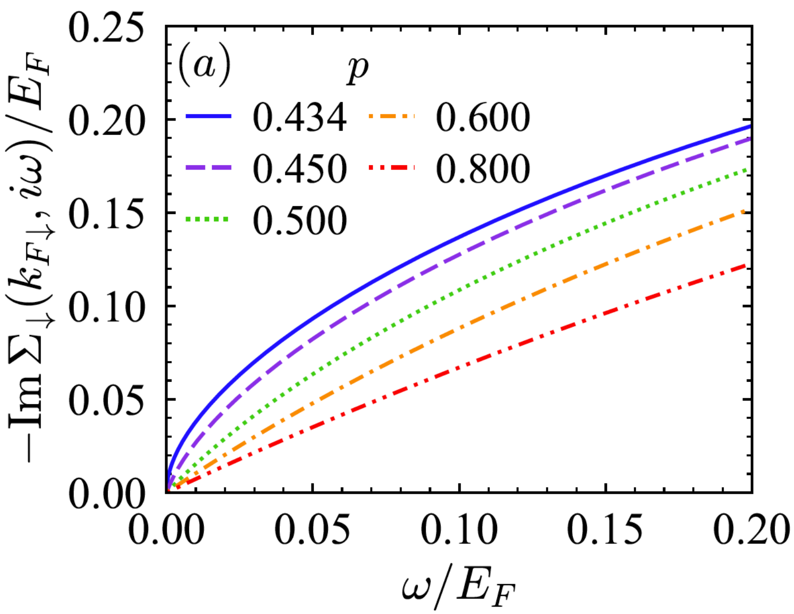


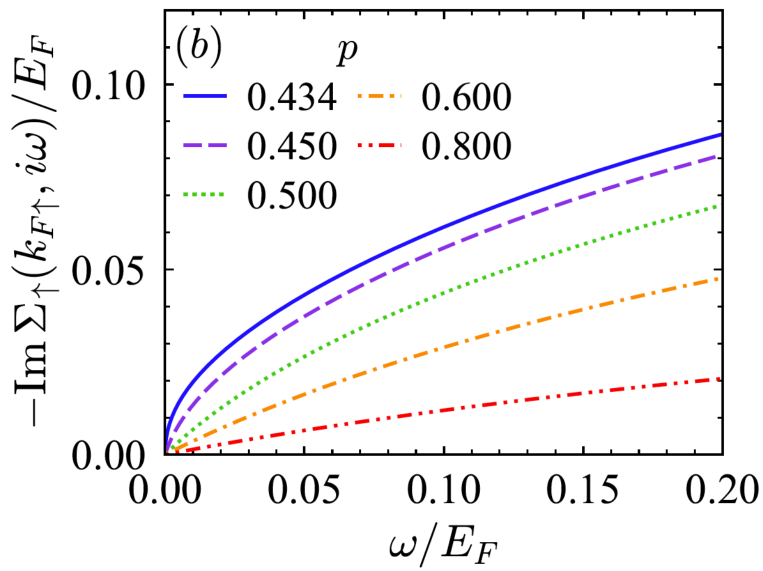
Vanishing quasi-particle residue and diverging effective mass at FFLO QCP: breakdown of FL properties analogous to what is found in heavy-fermions in the proximity of an AFM QCP.



When QCP is to standard BCS pairing, no breakdown of FL properties at QCP.

NFL behavior of self-energy (along imaginary axis)

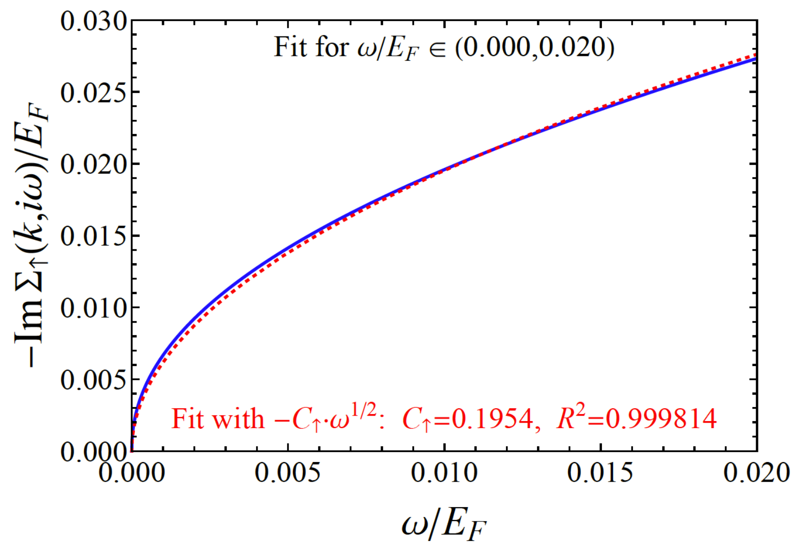




At FFLO QCP:

$$\mathrm{Im}\Sigma_\sigma(k_{F\sigma}, i\omega) = -C_\sigma \omega^{1/2}$$

$$\mathrm{Re}\Sigma_\sigma(k_{F\sigma}, i\omega) = \Sigma_\sigma(k_{F\sigma}, 0) + B_\sigma \omega^{1/2}$$



NFL behavior of self-energy (along real frequency axis) and dynamical critical exponent

Extending analytically small $i\omega$ behavior to small z in the upper complex plane:

$$\Sigma_\sigma(k_{F\sigma}, z) = \Sigma_\sigma(k_{F\sigma}, 0) + (B_\sigma - iC_\sigma)\sqrt{-iz}$$

yielding for $z = \tilde{\omega} + i0^+$ the retarded self-energy:

$$\mathrm{Re}\Sigma_\sigma^{\mathrm{R}}(k_{F\sigma}, \tilde{\omega}) = \Sigma_\sigma(k_{F\sigma}, 0) + [B_\sigma - C_\sigma \mathrm{sgn}(\tilde{\omega})]\sqrt{|\tilde{\omega}|/2}$$

$$\mathrm{Im}\Sigma_\sigma^{\mathrm{R}}(k_{F\sigma}, \tilde{\omega}) = -[C_\sigma + B_\sigma \mathrm{sgn}(\tilde{\omega})]\sqrt{|\tilde{\omega}|/2}$$

manifestly NFL

Spectral-weight function at Fermi surface: $A_\sigma(k_{F\sigma}, \tilde{\omega}) = \dfrac{D_{\sigma\pm}}{|\tilde{\omega}|^{1/2}}$

According to general consideration (Senthil, 2008) at QCP where QP residue vanishes:

$A_\sigma(k_{F\sigma}, \tilde{\omega}) \sim \dfrac{1}{|\tilde{\omega}|^{1/z}}$ where z is the **dynamical critical exponent**

We found for the dynamical pairing susceptibility: $\chi_{\mathrm{pair}}(\mathbf{Q}, i\Omega) \simeq \dfrac{(mk_{\mathrm{F}})^{-1}}{\epsilon + b(|\mathbf{Q}| - Q_0)^2 - (d_1 + id_2)i\Omega}$

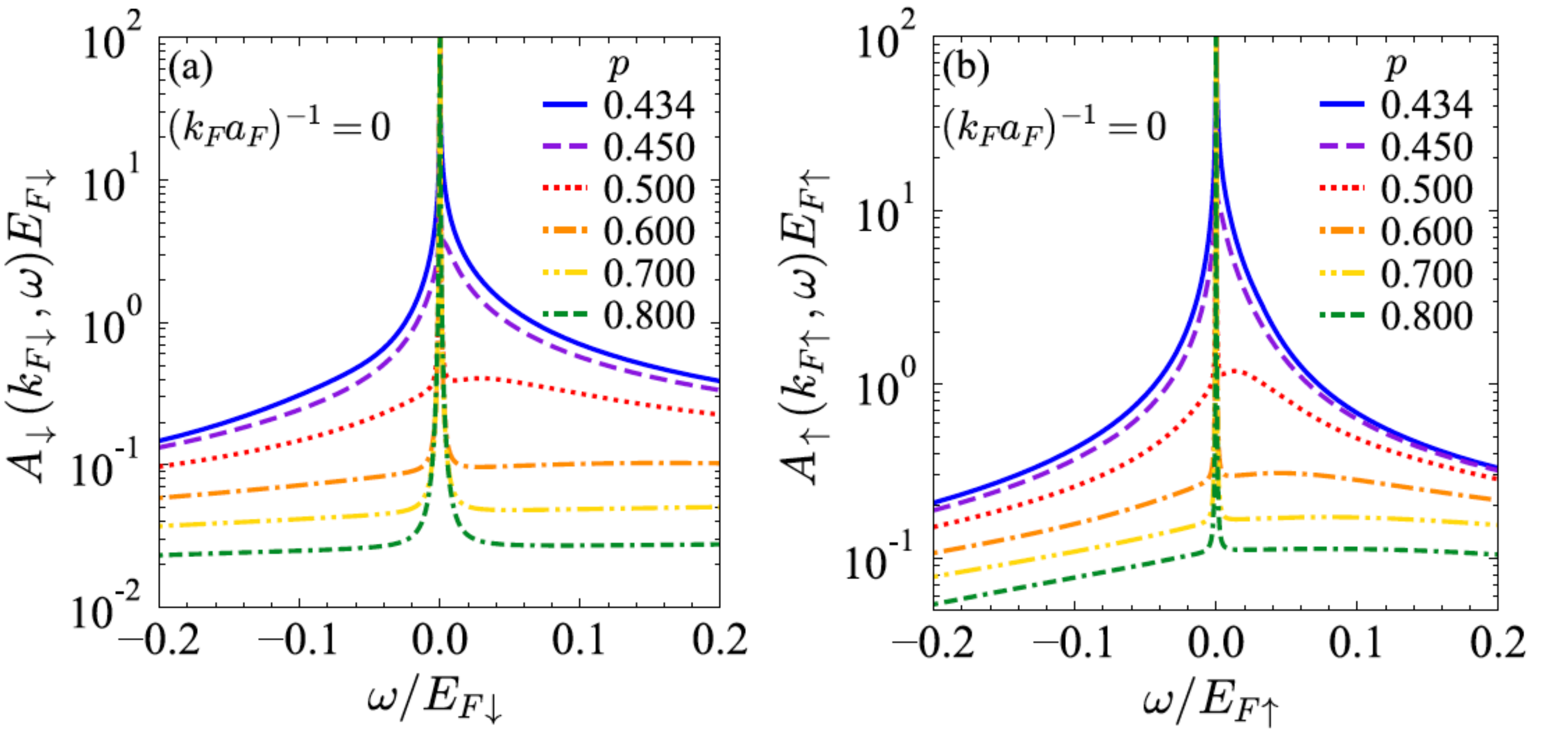
Yielding, on the real frequency axis, after setting $\xi = \sqrt{b/\epsilon}$, the scaling form:

$$\chi_{\mathrm{pair}}(\mathbf{Q}, \tilde{\Omega}; \xi) = \frac{\xi^2}{mk_{\mathrm{F}}b}\Phi_0[(|\mathbf{Q}| - Q_0)\xi, m_1\tilde{\Omega}\xi^2]$$

$\boxed{z = 2}$

$$m_1 = (d_1 + id_2)/b$$

Spectral weight function at Fermi surface, from large polarizations to QCP for the unitary Fermi gas



Analytical continuation performed numerically with Padé approximants. Away from the QCP, delta-like peak over incoherent background. At QCP, power law behavior.

Conclusions

- At T=0 region of stability for FFLO significantly larger than mean-field prediction. It includes a polarization range at unitarity.

- Lifshitz point on the BEC side at $(k_{\mathrm{F}}a)^{-1} = 0.64$ and $p_c = 0.66$, with sudden jump of pair-momentum from $Q_0 \neq 0$ to $Q_0 = 0$.

- At FFLO QCP: vanishing quasi-particle residue and diverging effective mass: breakdown of FL properties analogous to what is found in heavy-fermions at AFM QCP.

- Power law behaviors of retarded self-energy and spectral weight function at FFLO QCP are determined by dynamical critical exponent as argued by Senthil, 2008.

Thank you!