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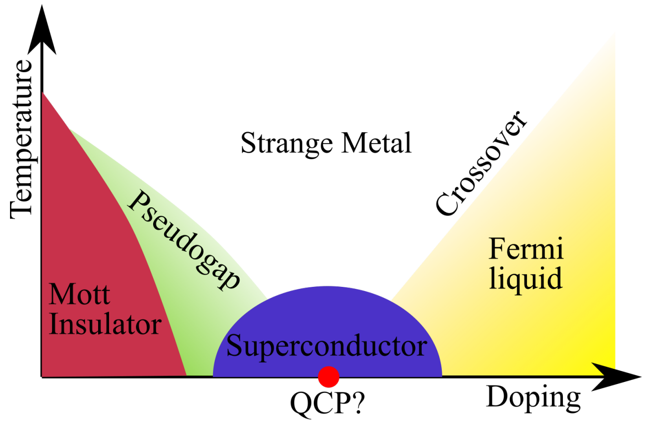
STRANGE METALS: STRONGLY CORRELATED QUANTUM MATTER WITH SPATIALLY RANDOM INTERACTIONS

A. A. P., Haoyu Guo, Ilya Esterlis, Subir Sachdev, arXiv:2203.04990
Haoyu Guo, A. A. P., Ilya Esterlis, Subir Sachdev, arXiv:2207.08841,
Ilya Esterlis, Haoyu Guo, A. A. P., Subir Sachdev, arXiv: 2103.08615,
Erik E. Aldape, Tessa Cookmeyer, A. A. P., Ehud Altman, arXiv: 2012.00763
And more to come…

AAVISHKAR PATEL
Center for Computational Quantum Physics, Flatiron Institute, New York

“**RECENT PROGRESS IN MANY BODY THEORIES XXI - KÜMMEL AWARD TALK**”, September 14 2022, Chapel Hill, NC

Strange Metals



- Several two-dimensional or layered materials with strongly correlated electrons display a ubiquitous and unusual metallic phase at intermediate values of carrier density.
- Unlike Landau Fermi liquids, which have a DC electrical resistivity that scales as T^2, these strange metals have a resistivity that scales linearly with temperature.
- This often occurs near a quantum critical point.

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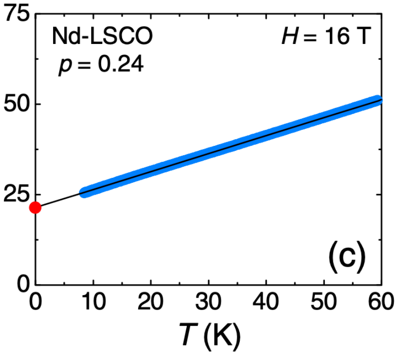
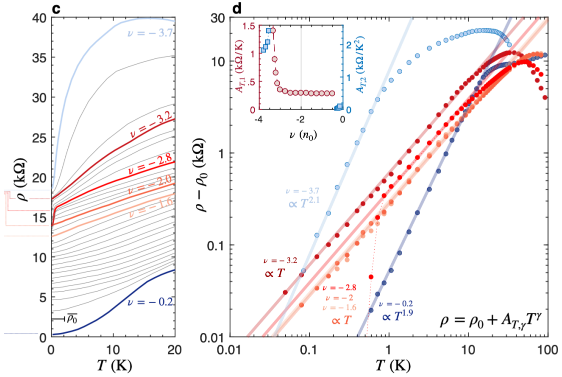
Strange Metals

- While, at high temperatures, *T*-linear resistivity might putatively be explained by phonons, at low temperatures one expects $\rho(T) = \rho(0) + aT^2$ for Landau Fermi liquids, with the temperature dependence arising from the parts of the quasiparticle decay process that involve momentum and current relaxing Umklapp scattering.
- This *T*-linear behavior is often accompanied by other signs of strong interactions - such as a large, *T*-dependent quasiparticle effective mass that shows up in specific heat measurements.



Michon et al, PRX **8**, 041010 (2018)



Jaoui et al, arXiv:2108.07753 (magic angle twisted bilayer graphene)

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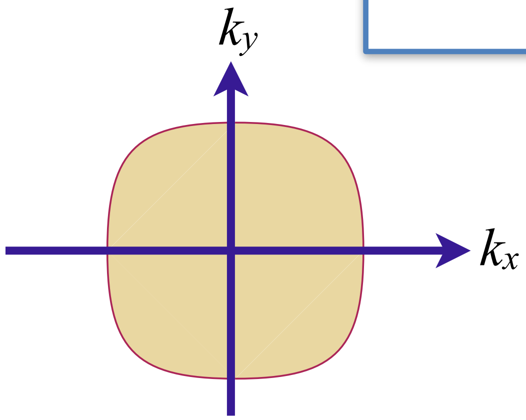
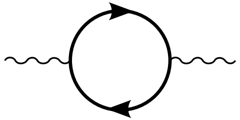
Landau Fermi Liquid Metal

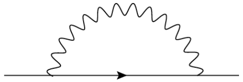
$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}} + \mathcal{L}_{\text{int}} = \psi_{\mathbf{r}}^\dagger \psi_{\mathbf{r}} U(\mathbf{r}-\mathbf{r}') \psi_{\mathbf{r}'}^\dagger \psi_{\mathbf{r}'}$$

$$\boxed{\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}} + \mathcal{L}_\phi = \frac{1}{2}\left[(\partial_\tau \phi)^2 + (\nabla \phi)^2 + m^2 \phi^2 \right] + \text{“Yukawa” coupling:} \quad g \int d^2 r d\tau \, \psi^\dagger(r,\tau) \psi(r,\tau) \phi(r,\tau)}$$



Screening leads to massive boson

$m^2 \sim$ 

 $\mathrm{Im}[\Sigma_R(\omega,T)] \sim \max(\omega^2, T^2)$

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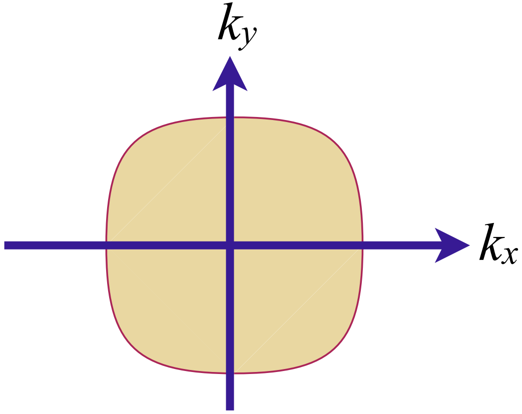
non-Fermi Liquid Metal

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}} \quad + \quad \mathcal{L}_\phi = \frac{1}{2} \left[(\partial_\tau \phi)^2 + (\boldsymbol{\nabla} \phi)^2 + (\lambda - \lambda_c)\phi^2 \right]$$

"Yukawa" coupling: $g \int d^2 r d\tau \, \psi^\dagger(r,\tau)\psi(r,\tau)\phi(r,\tau)$



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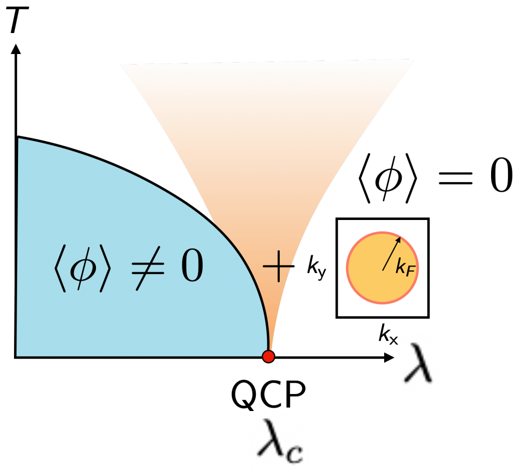


non-Fermi Liquid Metal

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}} \quad + \quad \mathcal{L}_\phi = \frac{1}{2} \left[(\partial_\tau \phi)^2 + (\boldsymbol{\nabla} \phi)^2 + (\lambda - \lambda_c) \phi^2 \right]$$

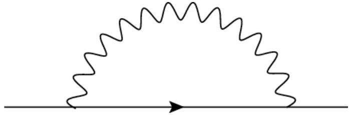
$$\text{"Yukawa" coupling:} \quad g \int d^2 r d\tau \, \psi^\dagger(r,\tau) \psi(r,\tau) \phi(r,\tau)$$

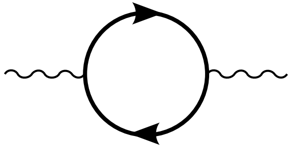
Eliashberg solution for electron (G) and boson (D) Green's functions at small ω:

P.A. Lee (1989)

$$\Sigma(\hat{\boldsymbol{k}}, i\omega) \sim -i\,\mathrm{sgn}(\omega) |\omega|^{2/3}, \quad G(\boldsymbol{k}, i\omega) = \frac{1}{i\omega - \varepsilon(\boldsymbol{k}) - \Sigma(\hat{\boldsymbol{k}}, i\omega)}, \quad D(\boldsymbol{q}, i\Omega) = \frac{1}{\Omega^2 + q^2 + \gamma |\Omega| / q}$$

(in two spatial dimensions)





(Boson is massless but damped at QCP)

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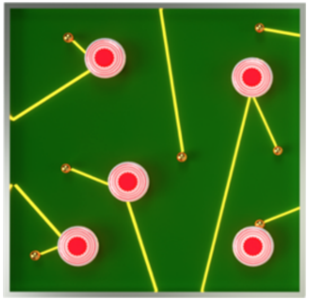
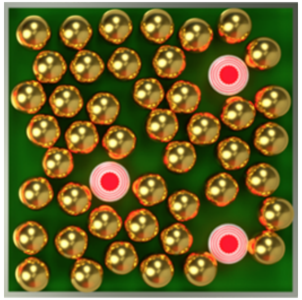
non-Fermi Liquid Metal

Because of the conserved total momentum due to translational invariance, and a finite charge density that prevents excitation of currents without excitation of momentum, such a metal has an infinite DC conductivity (up to some weak Umklapp processes on a lattice), as a finite DC conductivity requires current, and therefore momentum, to relax.





Presence of impurities (red bumpers) is required to degrade momentum and therefore current, irrespective of whether quasiparticles are well-defined or not.

All these 2D or quasi-2D materials with low-temperature strange metal behavior have plenty of disorder (dopants, twist angle mismatch etc).

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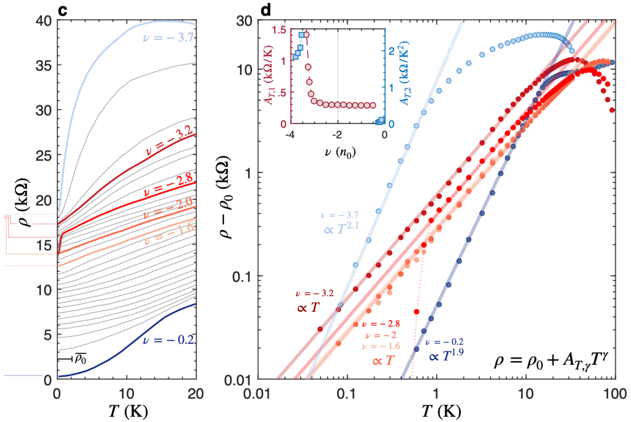
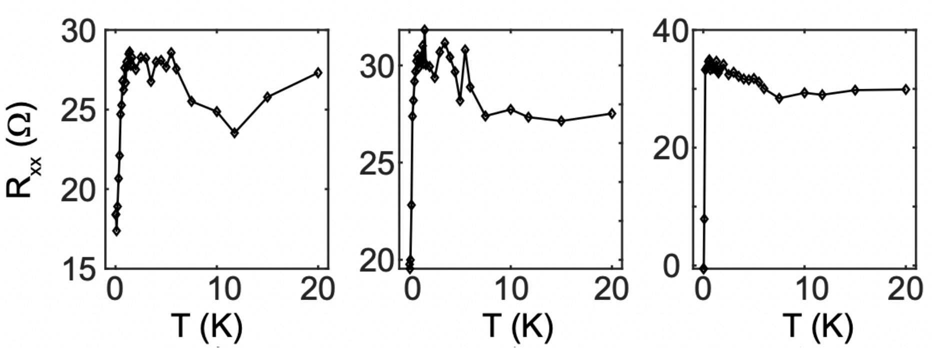
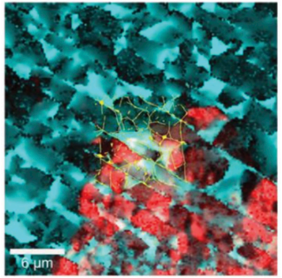
Disorder vs. no Disorder in Experiment

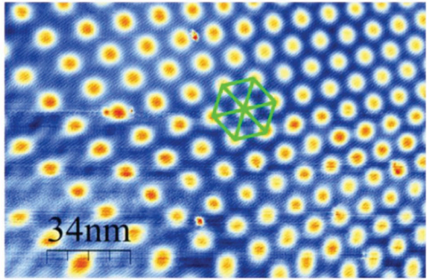


Jaoui et al, arXiv:2108.07753 (magic angle twisted bilayer graphene)

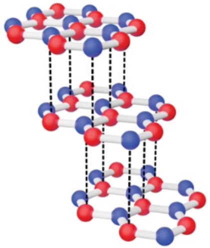


Zhou et al, arXiv:2106.07640 (**non-twisted** rhombohedral tri-layer graphene)





Significant amounts of twist angle disorder



Very regular system with no twist angle disorder; no T-linear resistivity despite SC etc.

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non-Fermi Liquid Metal with Disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}} \quad + \quad \mathcal{L}_\phi = \frac{1}{2} \left[(\partial_\tau \phi)^2 + (\boldsymbol{\nabla} \phi)^2 + (\lambda - \lambda_c)\phi^2 \right]$$

"Yukawa" coupling: $\quad g \int d^2r d\tau\, \psi^\dagger(r,\tau)\psi(r,\tau)\phi(r,\tau)$

Random potential $\quad \int d^2r d\tau\, v(r)\, \psi^\dagger(r,\tau)\psi(r,\tau)$

Spatially random potential $v(r)$ with $\overline{v(r)} = 0$, $\overline{v(r)v(r')} = v^2\delta(r - r')$

Boson self energy: $\Pi \sim -\dfrac{g^2}{v^2}|\Omega|$, $\qquad D(q, i\Omega) = \dfrac{1}{q^2 + \gamma|\Omega|}$

Fermion self energy: $\Sigma(i\omega) \sim -iv^2 \mathrm{sgn}(\omega) - i\dfrac{g^2}{v^2}\omega \ln(1/|\omega|)$; $\quad \dfrac{1}{\tau(\varepsilon)} \sim |\varepsilon|$ (in two spatial dimensions)

Marginal Fermi liquid self energy and $T \ln(1/T)$ specific heat

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non-Fermi Liquid Metal with Disorder

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Marginal Fermi liquid self energy and $T\ln(1/T)$ specific heat

However, because the fermion-boson interactions largely represent forward scattering (boson propagator is peaked at *q = 0*), the marginal Fermi liquid contribution does not contribute to the resistivity (which remains a constant at low T).

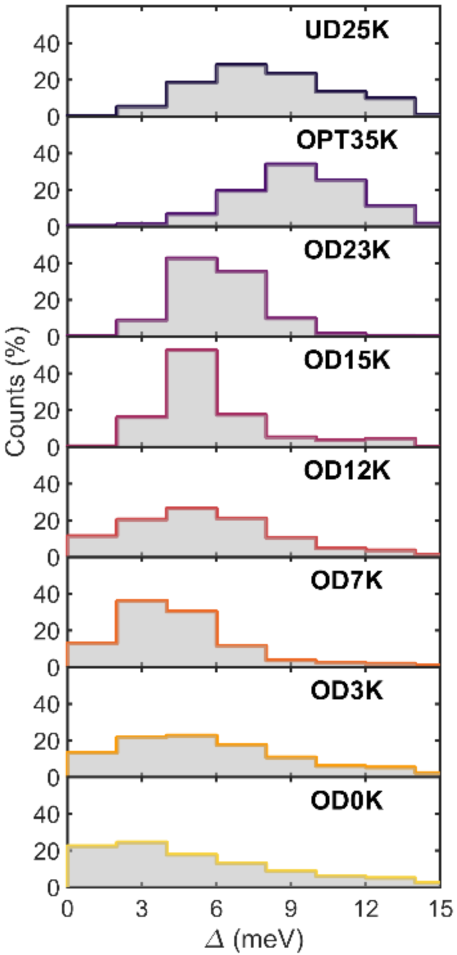
Spatially random interactions!

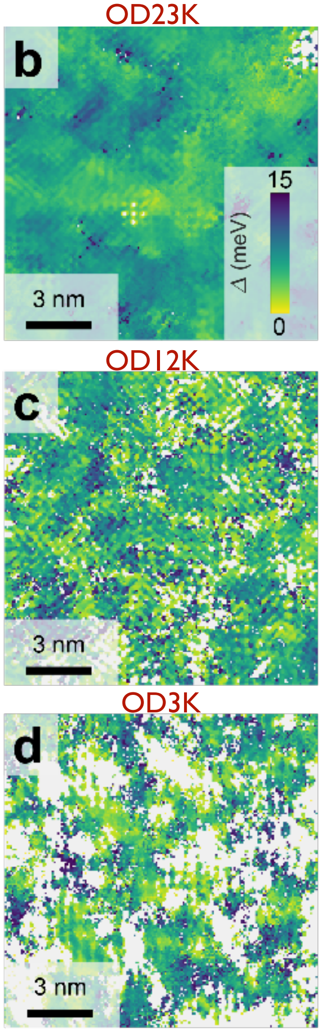
Puddle formation, persistent gaps, and non-mean-field breakdown of superconductivity in overdoped $(Pb,Bi)_2Sr_2CuO_{6+\delta}$

Willem O. Tromp, Tjerk Benschop, Jian-Feng Ge, Irene Battisti, Koen M. Bastiaans, Damianos Chatzopoulos, Amber Vervloet, Steef Smit, Erik van Heumen, Mark S. Golden, Yinkai Huang, Takeshi Kondo, Yi Yin, Jennifer E. Hoffman, Miguel Antonio Sulangi, Jan Zaanen, Milan P. Allan

Our scanning tunneling spectroscopy measurements in the overdoped regime of the $(Pb,Bi)_2Sr_2CuO_{6+\delta}$ high- temperature superconductor show the emergence of puddled superconductivity, featuring nanoscale superconducting islands in a metallic matrix

arXiv:2205.09740





Spatially random interactions!

Randomness in hopping t_{ij} leads to randomness in exchange interactions t_{ij}^2/U. The interaction associated with the ϕ collective mode has the schematic form

$$-\int d^2r d\tau \, J(r)\psi^\dagger\psi^\dagger\psi\psi$$

where we have omitted a local 'form factor' for the interaction, and the random strength of the overall interaction is determined by the coupling $J(r)$. Upon decoupling

$$\int d^2r d\tau \left[\frac{\phi^2}{2J(r)} - \phi\psi^\dagger\psi\right]$$

This as a random 'mass' in the boson and is strongly relevant.

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This as a random 'mass' in the boson and is strongly relevant. A key idea is that we should account for the relevant disorder exactly by rescaling the field ϕ in a r-dependent manner so that

$$\boxed{\int d^2r d\tau \left[\frac{\phi^2}{2} - \sqrt{J(r)}\phi\psi^\dagger\psi\right]}$$

The disorder is in the boson-fermion coupling, and can be accounted for systematically.

non-Fermi Liquid Metal with Disordered interactions

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}} \quad + \quad \mathcal{L}_\phi = \frac{1}{2}\left[(\partial_\tau \phi)^2 + (\boldsymbol{\nabla}\phi)^2 + (\lambda - \lambda_c)\phi^2 \right]$$

Random potential $\int d^2 r d\tau \, v(r) \, \psi^\dagger(r,\tau)\psi(r,\tau)$

"Yukawa" coupling: $\int d^2 r d\tau \; [g + \underline{g'(r)}] \, \psi^\dagger(r,\tau)\psi(r,\tau)\phi(r,\tau)$

Spatially random Yukawa coupling $g'(r)$ with $\overline{g'(r)} = 0$, $\overline{g'(r)g'(r')} = g'^2\delta(r-r')$

Fermion self energy: $\Sigma = \Sigma_v + \Sigma_g + \Sigma_{g'}$

$\Sigma_v(i\omega) \sim -iv^2 \mathrm{sgn}(\omega)$, $\quad \Sigma_g(i\omega) \sim -i\dfrac{g^2}{v^2}\omega \ln(1/|\omega|)$, $\quad \Sigma_{g'}(i\omega) \sim -ig'^2\omega \ln(1/|\omega|)$ (in two spatial dimensions)

Boson self energy: $\Pi = \Pi_g + \Pi_{g'}$

$\Pi_g(i\Omega) \sim -\dfrac{g^2}{v^2}|\Omega|$, $\quad \Pi_{g'}(i\Omega) \sim -g'^2|\Omega|$, $\quad D(q, i\Omega) = \dfrac{1}{q^2 + \gamma|\Omega|}$

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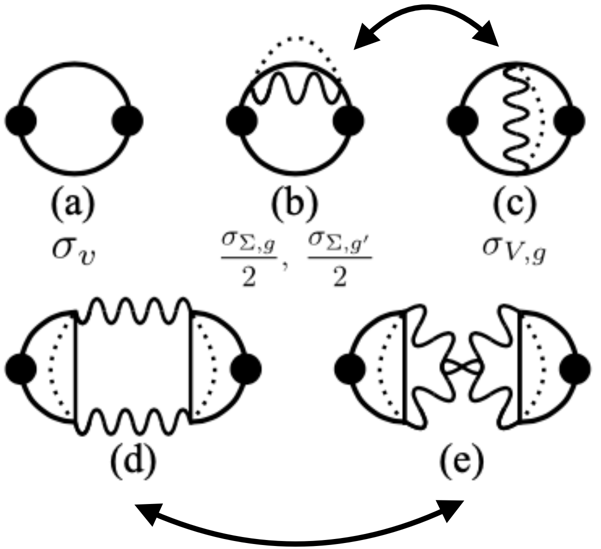
non-Fermi Liquid Metal with Disordered interactions

Cancel for *g* (forward scattering) but **not** *g'* (large angle and momentum non-conserving scattering).



(+ all rainbows/ladders/boxes)

Leading contributions cancel in the large k_F limit; only sub-leading ω^2/E_F terms survive.

non-Fermi Liquid Metal with Disordered interactions

Fermion self energy: $\Sigma = \Sigma_v + \Sigma_g + \Sigma_{g'}$

$$\Sigma_v(i\omega) \sim -iv^2 \mathrm{sgn}(\omega), \quad \Sigma_g(i\omega) \sim -i\frac{g^2}{v^2}\omega \ln(1/|\omega|), \quad \Sigma_{g'}(i\omega) \sim -ig'^2\omega \ln(1/|\omega|)$$ (in two spatial dimensions)

Boson self energy: $\Pi = \Pi_g + \Pi_{g'}$

$$\Pi_g(i\Omega) \sim -\frac{g^2}{v^2}|\Omega|, \qquad \Pi_{g'}(i\Omega) \sim -g'^2|\Omega|, \qquad D(q, i\Omega) = \frac{1}{q^2 + \gamma|\Omega|}$$

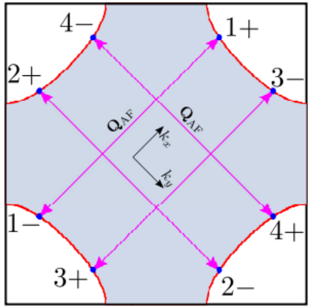
The g^2 log term does not contribute to transport but the g'^2 log term does!

Conductivity: $\sigma(\omega) \sim [1/\tau_{\rm trans}(\omega) - i\omega\, m^*(\omega)/m]^{-1}$

$$\frac{1}{\tau_{\rm trans}(\omega)} \sim v^2 + g'^2|\omega| \quad ; \quad \frac{m^*(\omega)}{m} \sim \frac{2g'^2}{\pi}\ln(\Lambda/\omega)$$

Residual resistivity is determined by v^2; Linear-in-T resistivity determined by g'^2.

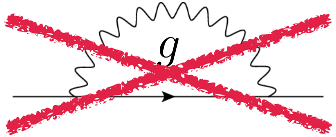
Other Order Parameters

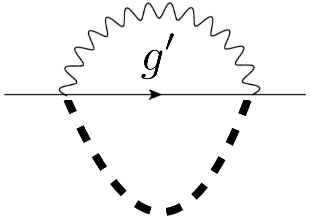


- Finite-wavevector (e.g. antiferromagnetic) quantum criticality

$$H_{\text{int}} = (g + g'(r))(e^{iQ\cdot r}\psi_r^\dagger \psi_r \phi_r + \text{H.c.})$$

g interactions are off shell for the parts of the Fermi surface away from the measure zero "hot spots", but g' affects the entire Fermi surface. The transport scattering rate is thus set by g' for almost all of the fermions, giving rise to strange metal behavior in 2D at the QCP.





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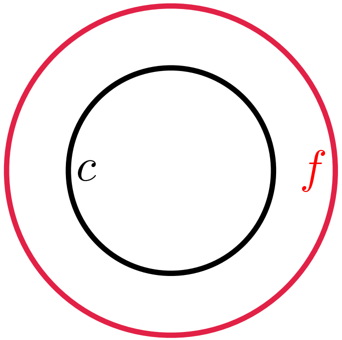
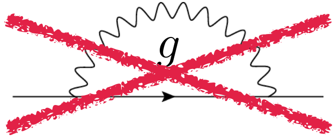
Other Order Parameters

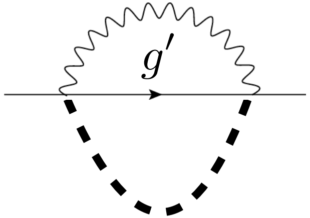


- Two-band quantum criticality of hybridization

$$H_{\text{int}} = (g + g'(r))(c_r^\dagger f_r b_r + \text{H.c.})$$

g interactions are off shell as *c* and *f* Fermi surfaces do not match, but g' affects the entire Fermi surfaces. The transport scattering rate is again thus set by g' for almost all of the fermions, giving rise to strange metal behavior in 2D at the QCP.





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Other Order Parameters

- The g' mechanism for strange metals in 2D also works for quantum critical random pairing, pair density wave etc.
- This universal behavior independent of order parameter type simply occurs because the disordered g' always couples the fermions to the *local* fluctuations of 2D quantum critical bosons in a current and momentum-relaxing manner. The *local* boson fluctuations have the "marginal" susceptibility

$$\chi(\omega) \sim \langle \phi_r(\omega)\phi_r(-\omega)\rangle \sim \int \frac{d^2q}{q^2+|\omega|} \sim \ln\left(\frac{\Lambda^2}{|\omega|}\right).$$

- When combined with the constant density of states near the Fermi surface, this gives *T*-linear resistivity. This universality is particularly appealing given the diversity of quantum criticalities leading to 2D strange metals in experiment.

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Why Does All This Work?

- This is not all just uncontrolled perturbation theory. The diagrams we sum correspond to an exact large *N* saddle point of a certain type of model;
- Promote fermions and bosons to *N* flavors per site: $\psi \to \psi_j, \quad \phi \to \phi_j$, *j* = {1…*N*}.
- Make the couplings g and g' random in the flavors (in addition to g' being random in space). $g \to g_{ijk}, \quad H_{\text{int}} \sim \sum_{ijk} g_{ijk} \psi_i^\dagger \psi_j \phi_k \qquad \langle g_{ijk} \rangle = 0, \quad \langle |g_{ijk}|^2 \rangle = g^2/N^2, \quad g_{ijk}^* = g_{jik}.$
- This is like the Sachdev-Ye-Kitaev (SYK) model, but with fermions and bosons instead of only fermions. Now called the Yukawa-SYK model.

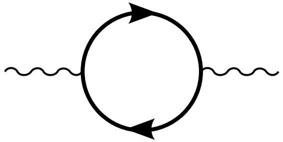
A. A. P. and S. Sachdev, PRB **98**, 125134 (2018)

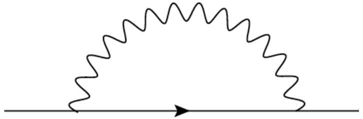
I. Esterlis and J. Schmalian PRB **100**, 115132 (2019)

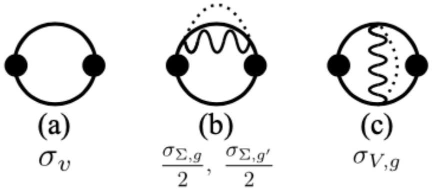
Y. Wang, PRL **124**, 017002 (2020)

(Initial papers on Yukawa-SYK, construction but in 0+1 dimensions)

Why Does All This Work?

$\Pi =$ 

$\Sigma =$ 



- The large *N* limit of the Yukawa-SYK construction leads to exact 1-loop self-consistent Eliashberg equations, whose solution matches the perturbative solutions discussed earlier.
- The series of diagrams summed for the current correlation function also matches the series discussed earlier.

(+ all rainbows/ladders/boxes)

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Why Does All This Work?

- The Green's functions are defined as a saddle point of the large *N* theory like in SYK.
- However, unlike SYK, *1/N* fluctuations about this saddle point are much weaker due to the absence of any soft modes… (see arXiv:2103.08615).

$$\delta S = N \int_{1,2;3,4} \delta G_{1,2} \left(K^{-1}_{1,2;3,4} - 1 \right) \delta G_{3,4}$$

(No unit eigenvalue of *K*, therefore no soft fluctuating mode unlike SYK)

- Therefore, corrections to the large-*N* saddle point, and therefore theory defined by the diagrams mentioned in this talk are "weak".

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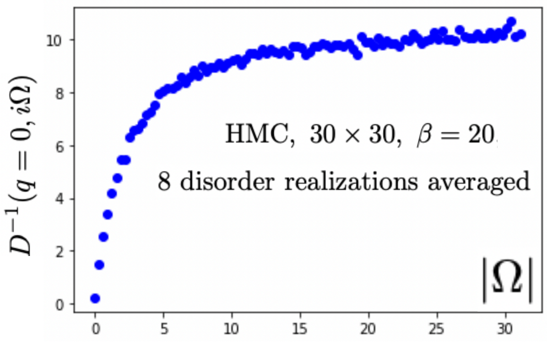
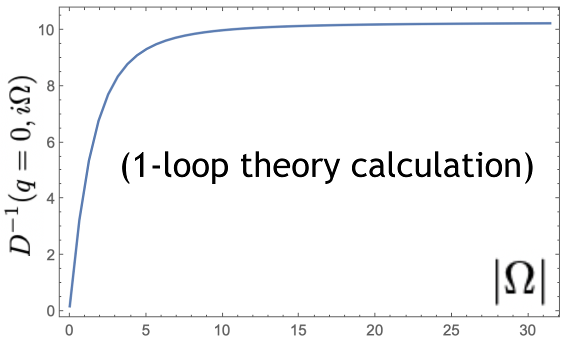
Quantum Monte Carlo

- Simulate sign-free two-band versions of the problem in 2D with random g' interactions using QMC techniques. No large *N* limit.
- Using Hybrid Monte Carlo (HMC) allows for larger spatial system sizes which aids in the self-averaging of disorder.
- Work in progress (with Peter Lunts and Michael Albergo), but very preliminary results show encouraging signs of $|\Omega|$ boson damping at low frequencies and therefore the *z* = 2 quantum criticality predicted by theory…





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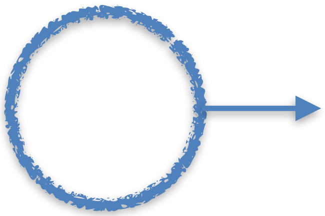
Dynamical Mean Field Theory

- Consider a situation with potential (v) disorder, and a purely disordered (g' but no g) interaction. The large N theory predicts strange metal behavior.



$$G_0(\mathbf{k}, i\omega) = \frac{1}{i\omega + i\Gamma\mathrm{sgn}(\omega) - v_F(k - k_F)} \approx \frac{1}{i\Gamma\mathrm{sgn}(\omega)} \quad (v_F(k-k_F) \ll \Gamma)$$

(Smeared FS in a disordered system)

- The Green's function near the FS is essentially local. The disordered g' interactions also couple fermions to local boson fluctuations as mentioned earlier. Almost all diagrams (including the large N saddle point diagrams) involve only local propagators.

Dynamical Mean Field Theory

- Because of the local propagators, this model is a good case for single-site DMFT even though it's a 2D system! An exact numerical solution can then be performed without invoking any large *N*.

$$S = \int d\tau d\tau' c^\dagger_\sigma(\tau)\left[\delta(\tau-\tau')\partial_\tau - \Delta(\tau-\tau')\right] c_\sigma(\tau') + g'^2 \int d\tau d\tau' D(\tau-\tau')\ \mathcal{O}(\tau)\cdot\mathcal{O}(\tau');$$

$$\Delta_\sigma(\tau-\tau') = \Gamma^2 G_\sigma(\tau-\tau'),\quad D(i\Omega) = \ln\left[\frac{\Lambda}{(\lambda-\lambda_c)-\Pi(i\Omega)}\right],\quad \Pi(\tau-\tau') = \langle\mathcal{O}(\tau)\cdot\mathcal{O}(\tau')\rangle;$$

$$O(\tau) = n(\tau),\ S_z(\tau),\ \vec{S}(\tau),\ \text{etc.}$$

(Can solve this using CTQMC methods such as CTSEG and obtain phase diagram, self-energies etc. Work in progress.)

Takeaways and future directions…

- Disorder in strongly correlated systems can manifest not just as potential disorder for the electrons, but also disorder in electron-electron interactions!
- Disorder is important to relax momentum strongly enough to lead to sizable resistivity at low temperatures below any phonon or Umklapp scales.
- The non-conserving inelastic scattering induced by interaction disorder leads to the low temperature strange metal behavior seen in experiments in essentially any 2D quantum critical itinerant electron system with a finite density of electrons.
- A combination of analytical and numerical techniques will lead to a full solution of all aspects of the disordered quantum critical problem, and allow for close and hopefully predictive connections to microscopics.

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Thank you for your attention!

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