Lattice simulations for the nuclear many body problem

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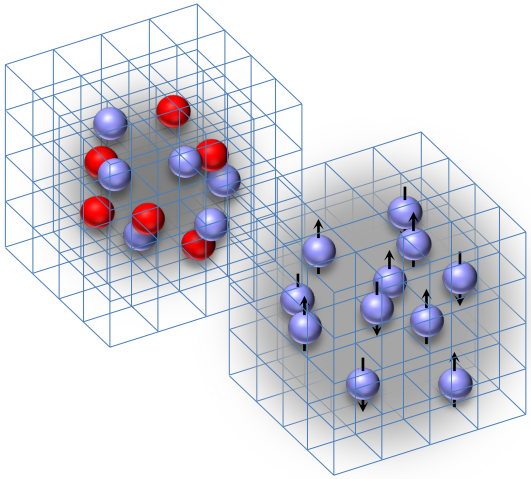

International Conference on
Recent Progress in Many-Body Theories XXI
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Chapel Hill, NC
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Outline

Lattice effective field theory

Spin-isospin exchange symmetry

Essential elements for nuclear binding

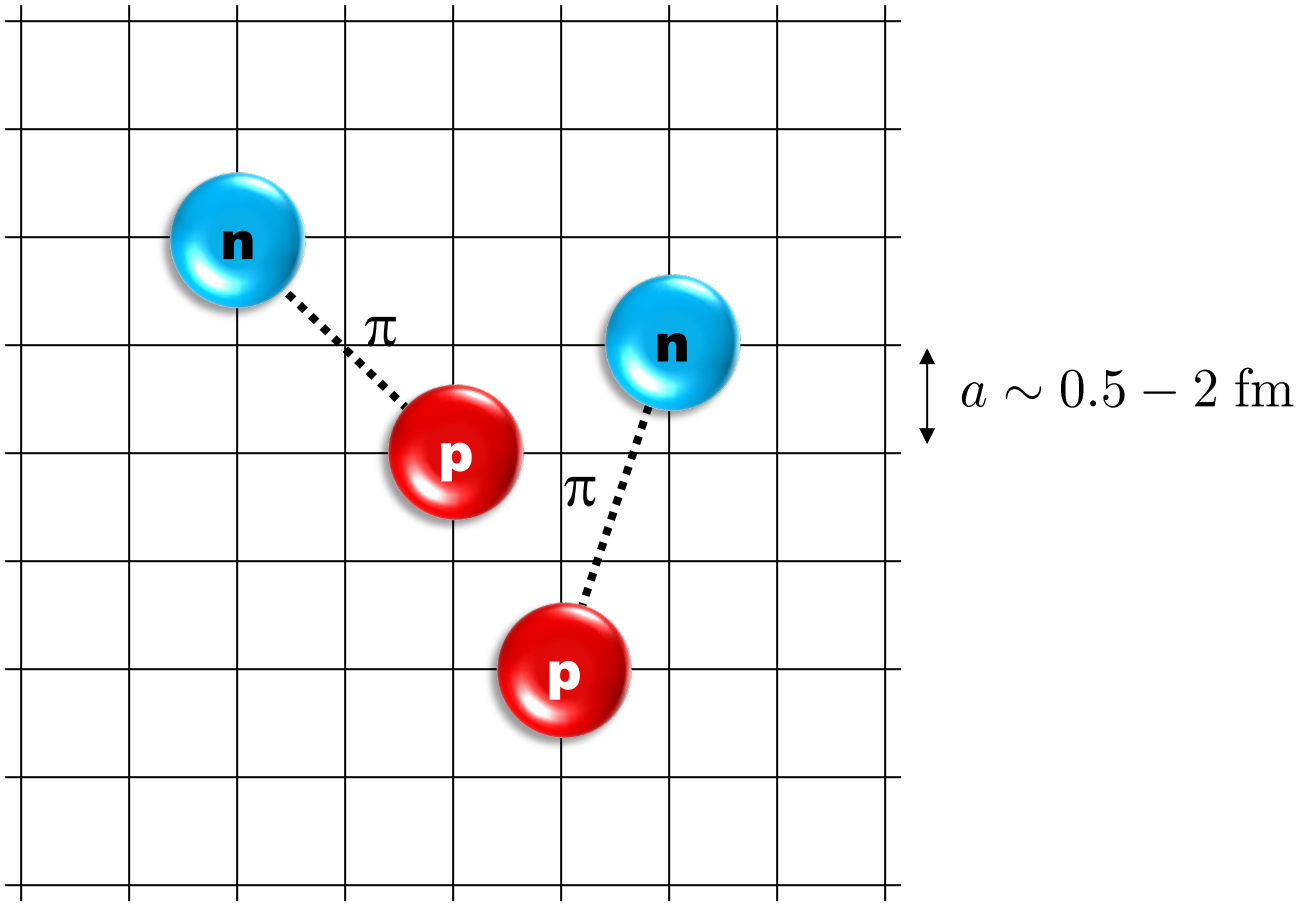
Pinhole algorithm

Structure of the carbon-12 states

Wave function matching

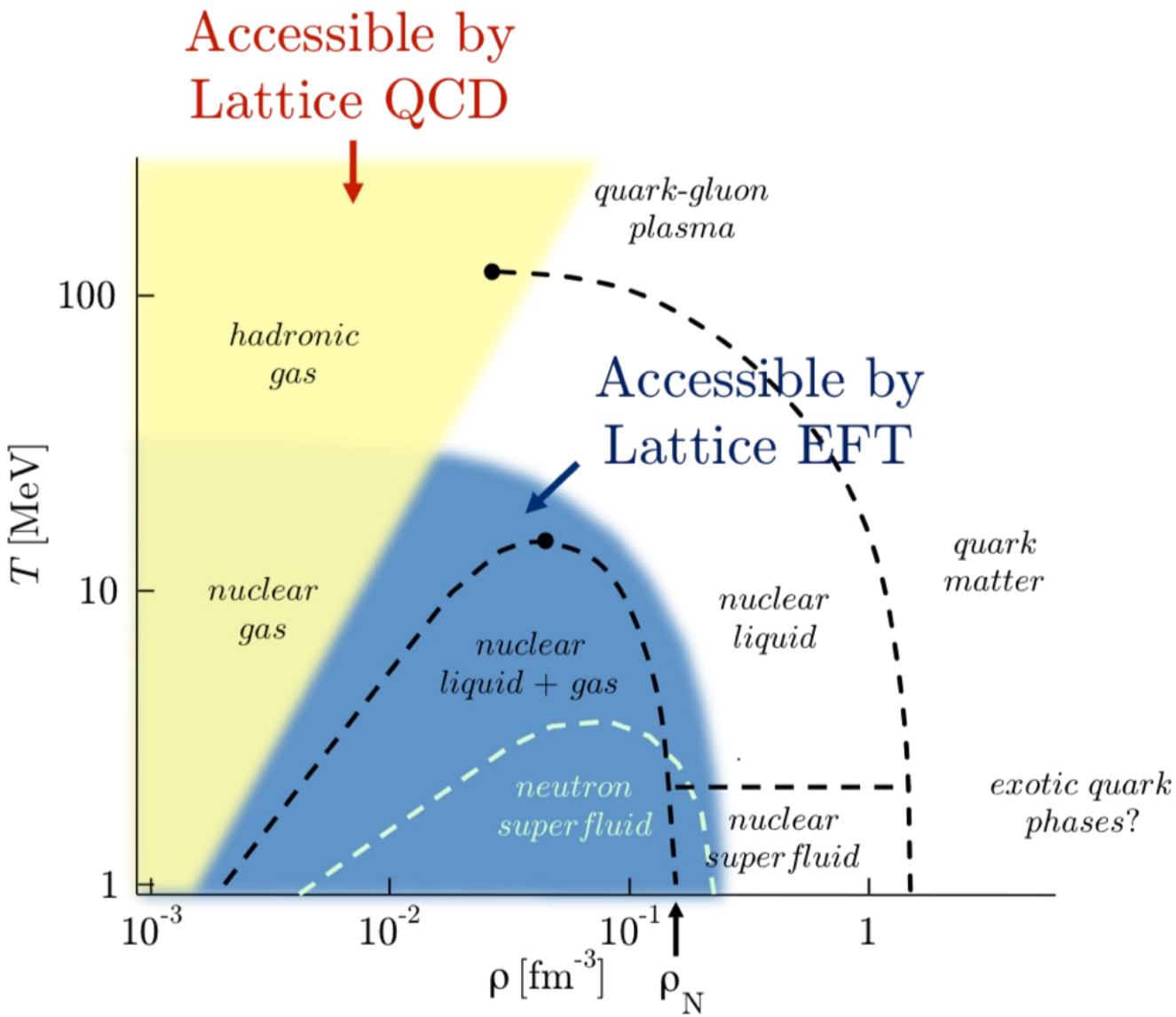
Summary

Lattice effective field theory



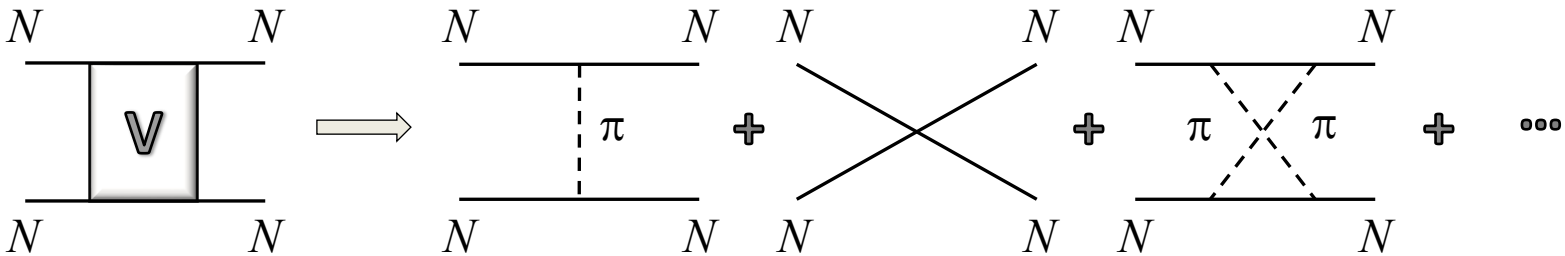
D.L, Prog. Part. Nucl. Phys. 63 117-154 (2009)

Lähde, Meißner, Nuclear Lattice Effective Field Theory (2019), Springer


Accessible by
Lattice QCD
quark-gluon
plasma
hadronic
gas
Accessible by
Lattice EFT
quark
matter
nuclear
gas
nuclear
liquid
nuclear
liquid + gas
neutron
superfluid
nuclear
superfluid
exotic quark
phases?
T [MeV]
100
10
1
10^-3
10^-2
10^-1
1
ρ [fm^-3]
ρ_N

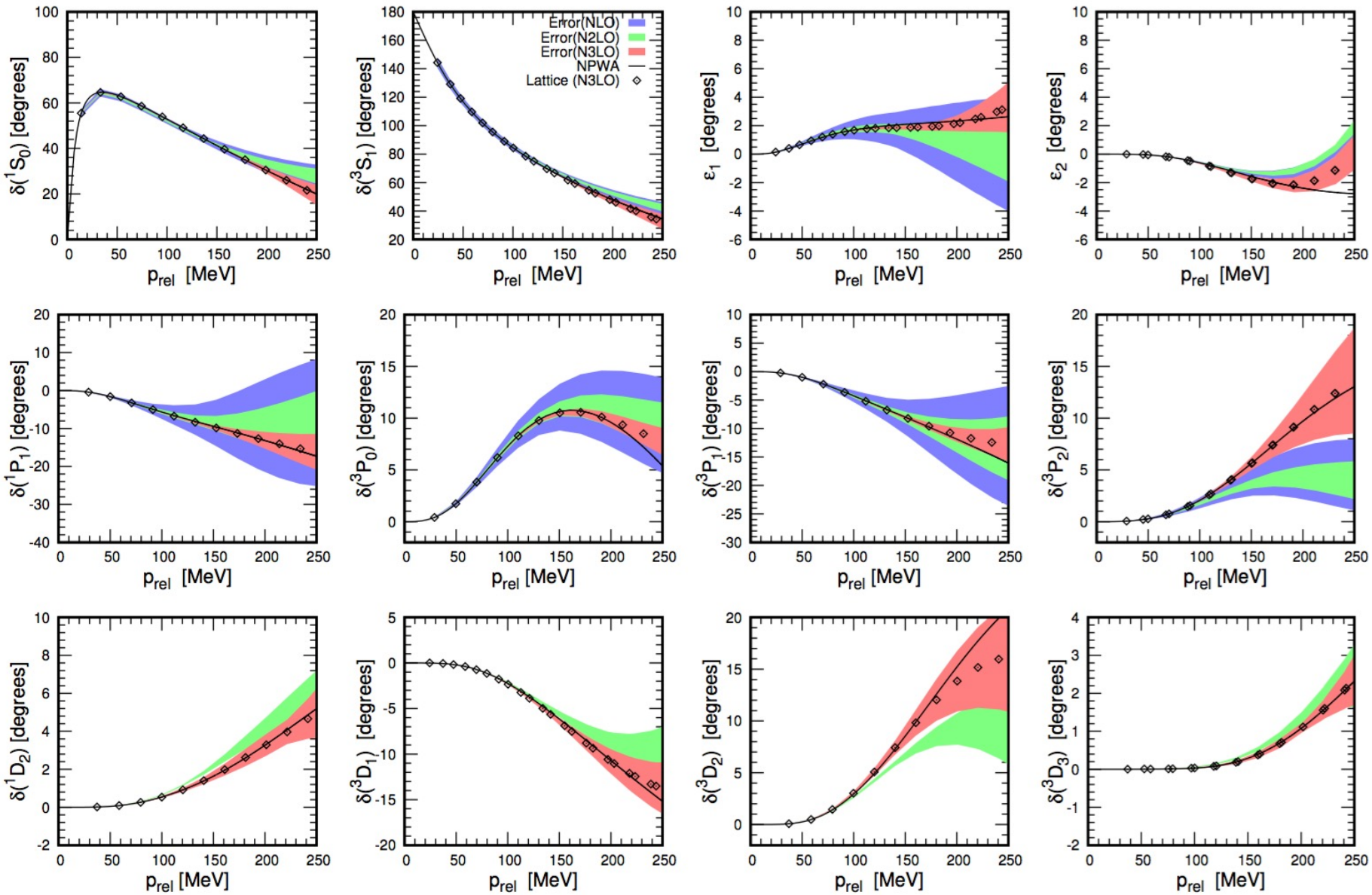
Chiral effective field theory

Construct the effective potential order by order



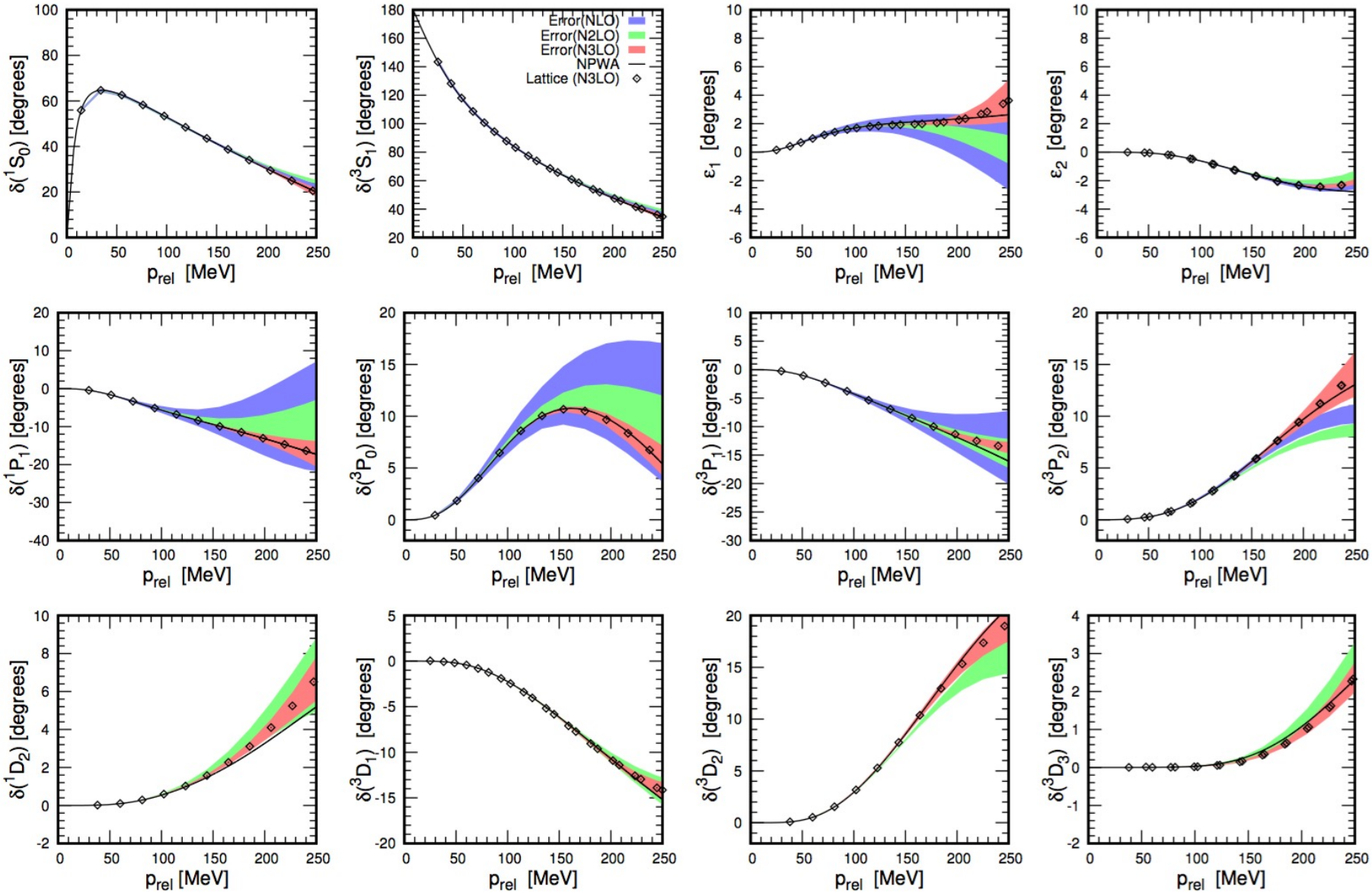
$$\underbrace{V^{\mathrm{OPEP}}\ \text{Contact interactions}}_{\text{Leading order (LO)}}\quad \underbrace{V^{\mathrm{TPEP}}\qquad\qquad\qquad}_{\text{Next-to-leading order (NLO)}}$$

$a = 1.315\,\mathrm{fm}$



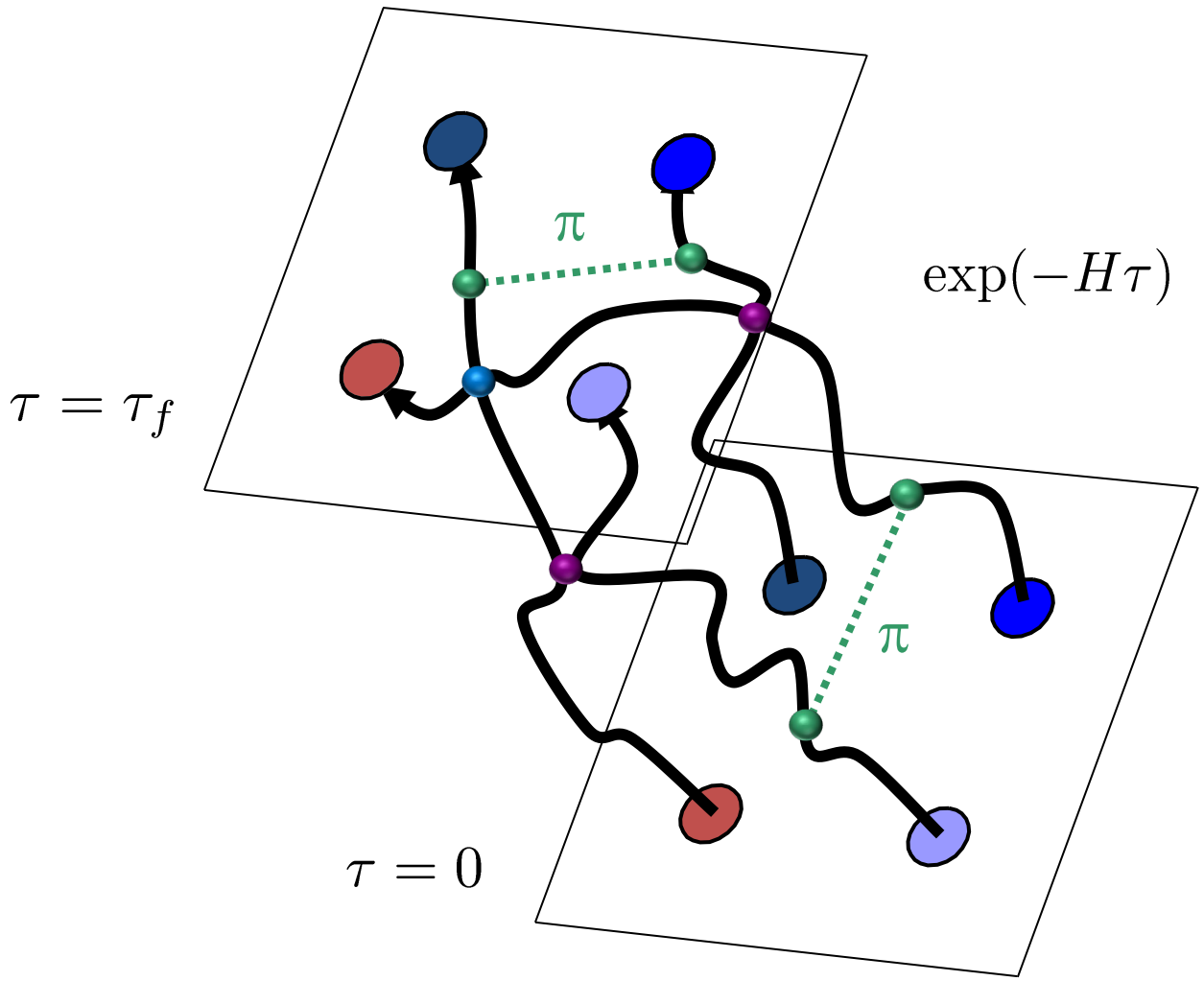
Li, Elhatisari, Epelbaum, D.L., Lu, Meißner, Phys. Rev. C 98, 044002 (2018)

$a = 0.987\,\mathrm{fm}$



Li, Elhatisari, Epelbaum, D.L., Lu, Meißner, Phys. Rev. C 98, 044002 (2018)

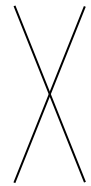
Euclidean time projection



Auxiliary field method

We can write exponentials of the interaction using a Gaussian integral identity

$$\exp\left[-\frac{C}{2}(N^\dagger N)^2\right]$$

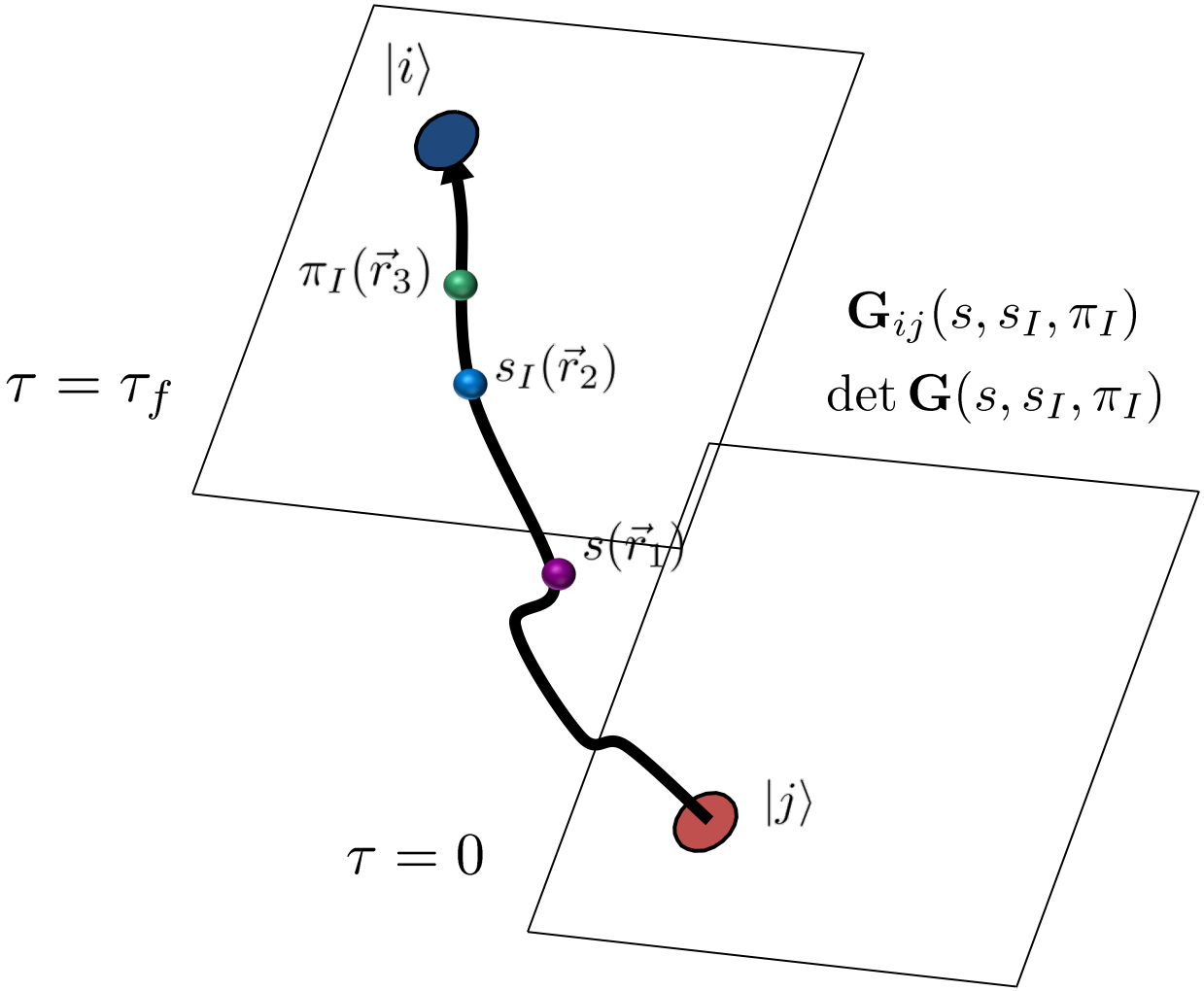


$$(N^\dagger N)^2$$

$$= \sqrt{\frac{1}{2\pi}}\int_{-\infty}^{\infty} ds \exp\left[-\frac{1}{2}s^2 + \sqrt{-C}\, s(N^\dagger N)\right]$$

$$sN^\dagger N$$

We remove the interaction between nucleons and replace it with the interactions of each nucleon with a background field.



$$\mathbf{G}_{ij}(s, s_I, \pi_I)$$

$$\det \mathbf{G}(s, s_I, \pi_I)$$

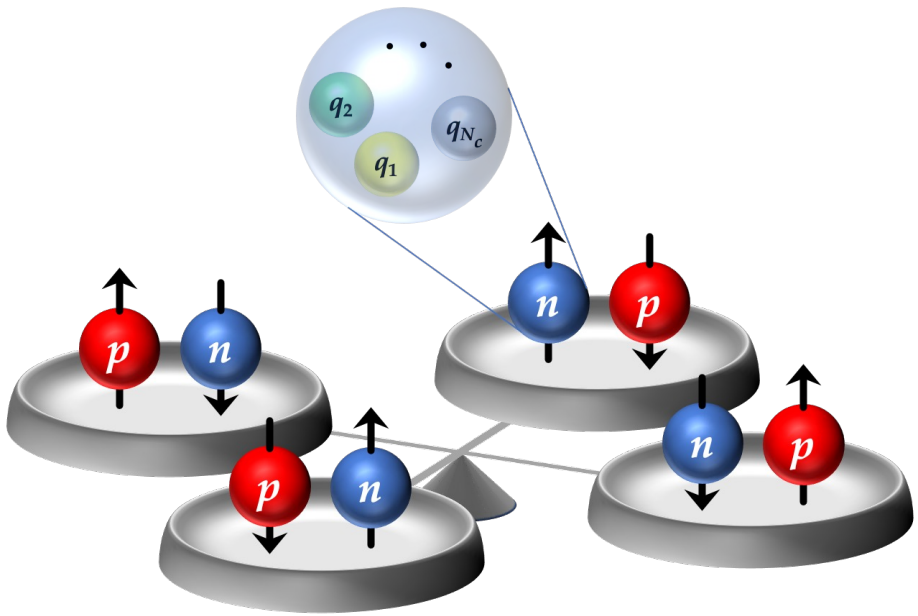
Spin-isospin exchange symmetry

Kaplan, Savage, PLB 365, 244 (1996)

Kaplan, Manohar, PRC 56, 76 (1997)

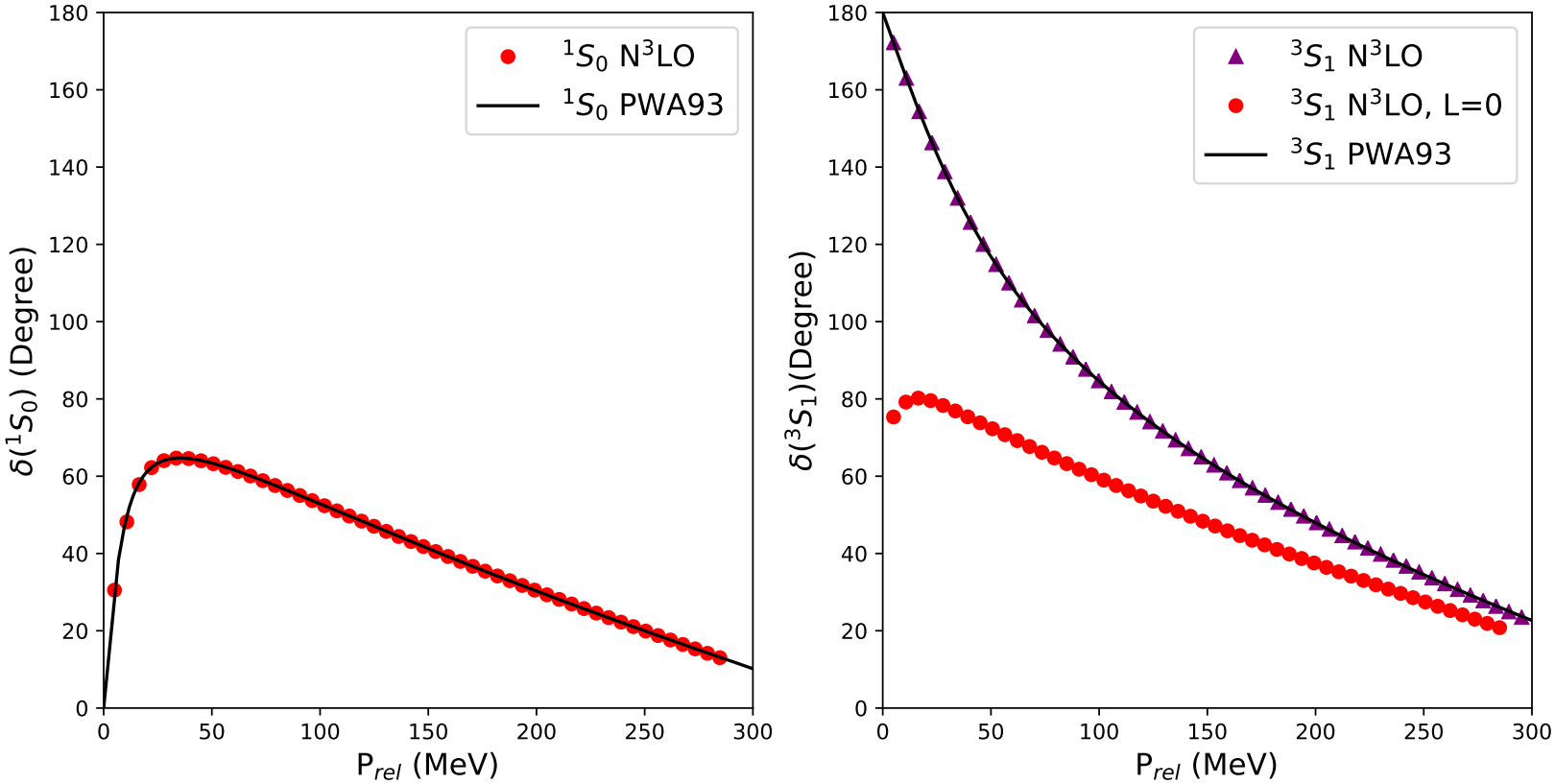
Calle Gordon, Arriola, PRC 80, 014002 (2009)

$$V^{2N}_{\text{large}-N_c} = V_C + \vec{\sigma}_1 \cdot \vec{\sigma}_2 \vec{\tau}_1 \cdot \vec{\tau}_2 W_S + (3\hat{r} \cdot \vec{\sigma}_1 \hat{r} \cdot \vec{\sigma}_2 - \vec{\sigma}_1 \cdot \vec{\sigma}_2)\vec{\tau}_1 \cdot \vec{\tau}_2 W_T$$



D.L., Bogner, Brown, Elhatisari, Epelbaum, Hergert, Hjorth-Jensen, Krebs, Li, Lu, Meißner, Phys. Rev. Lett. 127, 062501 (2021)

$$\Lambda_{\text{large}-N_c} \sim 500 \text{ MeV}$$



D.L., Bogner, Brown, Elhatisari, Epelbaum, Hergert, Hjorth-Jensen, Krebs, Li, Lu, Meißner, Phys. Rev. Lett. 127, 062501 (2021)

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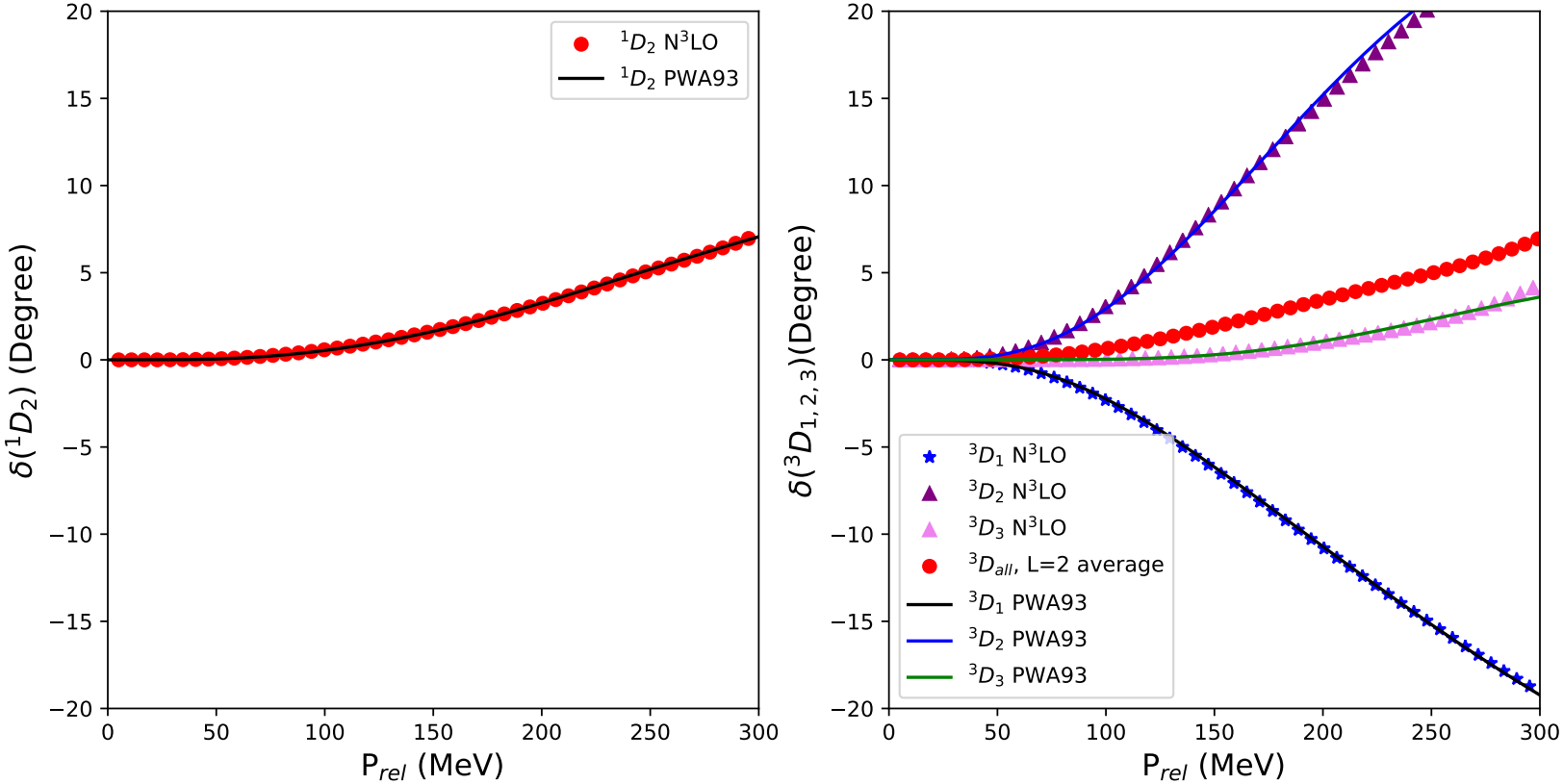


D.L., Bogner, Brown, Elhatisari, Epelbaum, Hergert, Hjorth-Jensen, Krebs, Li, Lu, Meißner, Phys. Rev. Lett. 127, 062501 (2021)

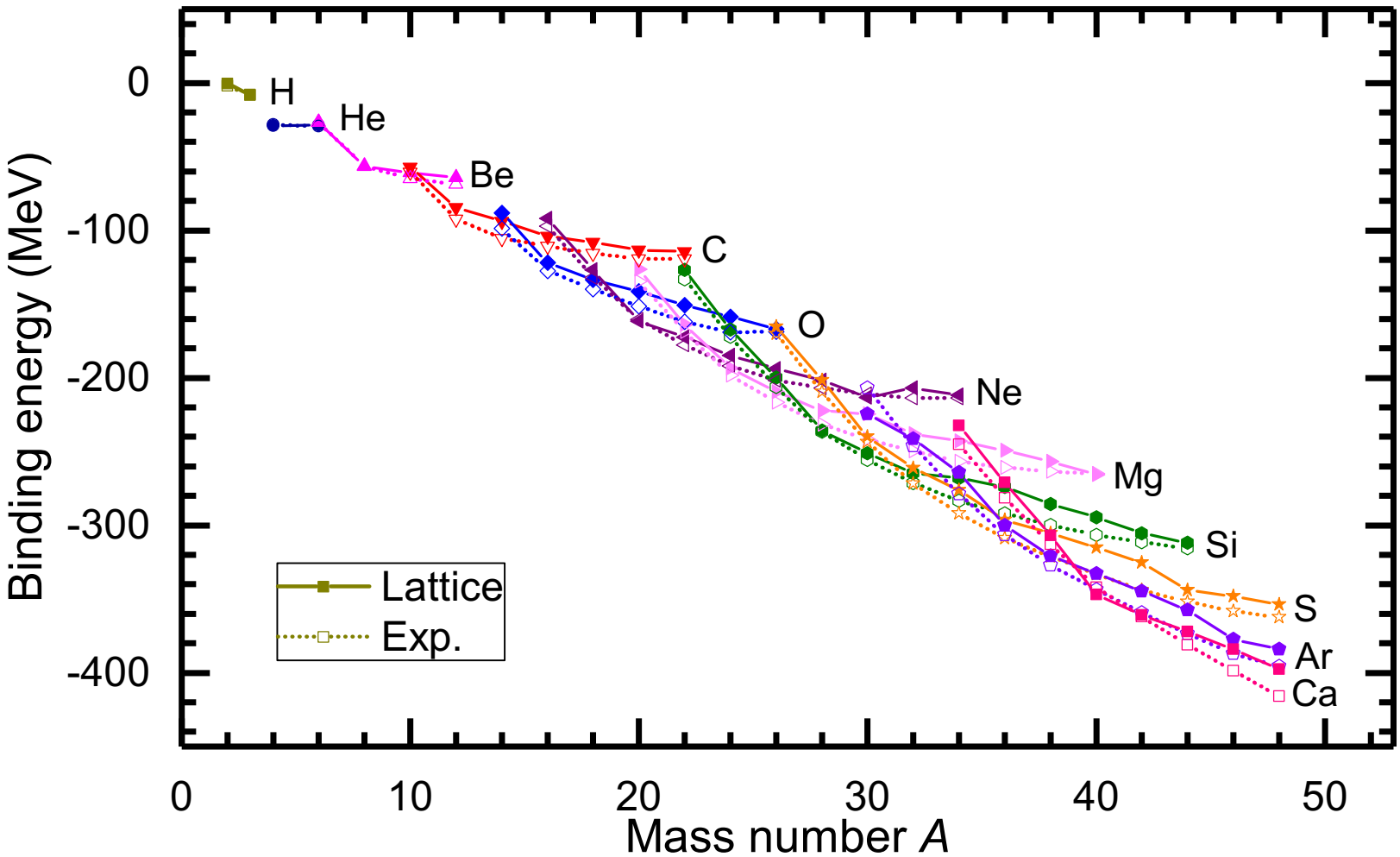
Essential elements for nuclear binding

What is the minimal nuclear interaction that can reproduce the ground state properties of light nuclei, medium-mass nuclei, and neutron matter simultaneously with no more than a few percent error in the energies and charge radii?

We construct an interaction with only four parameters.

1. Strength of the two-nucleon S-wave interaction
2. Range of the two-nucleon S-wave interaction
3. Strength of three-nucleon contact interaction
4. Range of the local part of the two-nucleon interaction

Except for the Coulomb potential, the interaction is invariant under Wigner's SU(4) symmetry.

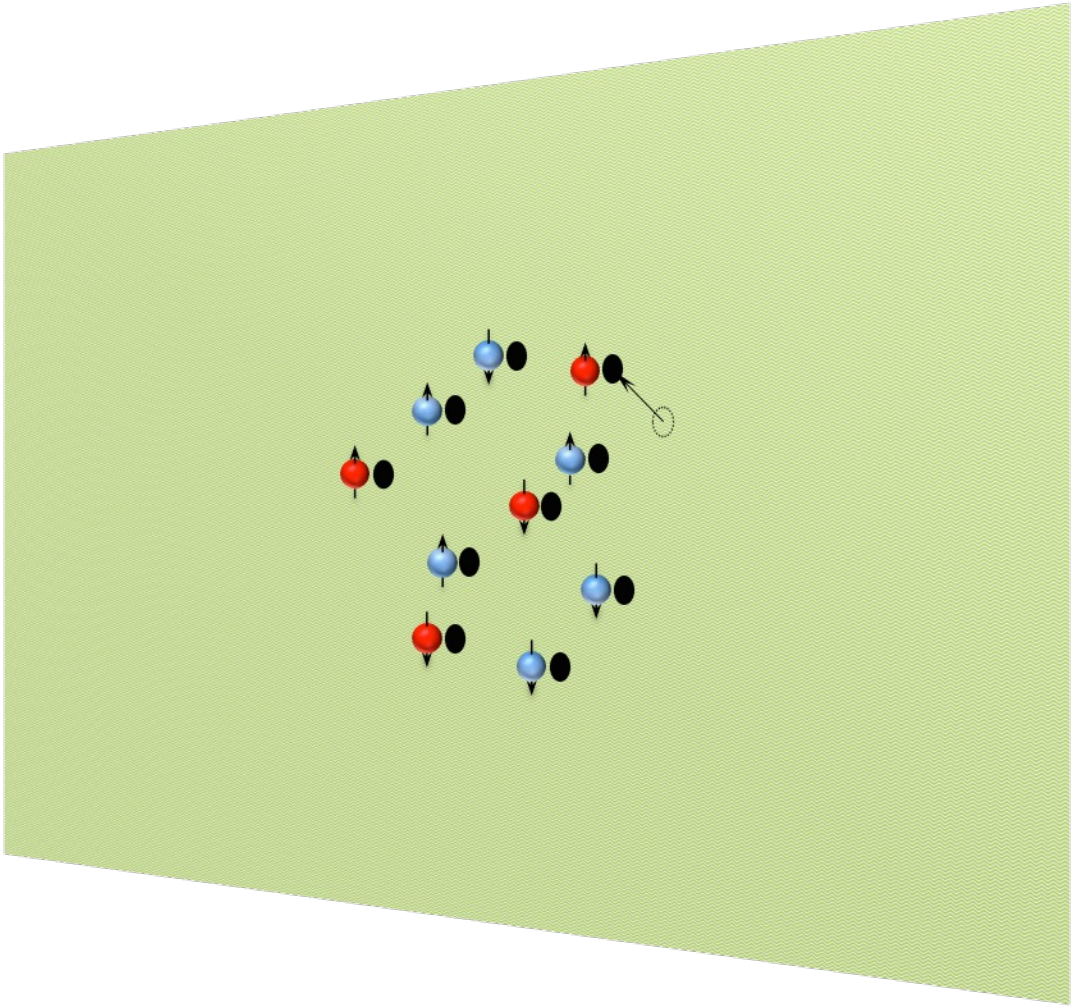


Lu, Li, Elhatisari, D.L., Epelbaum, Meißner, Phys. Lett. B 797, 134863 (2019)

	B	Exp.	R_{ch}	Exp.
^{3}H	8.48(2)(0)	8.48	1.90(1)(1)	1.76
^{3}He	7.75(2)(0)	7.72	1.99(1)(1)	1.97
^{4}He	28.89(1)(1)	28.3	1.72(1)(3)	1.68
^{16}O	121.9(1)(3)	127.6	2.74(1)(1)	2.70
^{20}Ne	161.6(1)(1)	160.6	2.95(1)(1)	3.01
^{24}Mg	193.5(02)(17)	198.3	3.13(1)(2)	3.06
^{28}Si	235.8(04)(17)	236.5	3.26(1)(1)	3.12
^{40}Ca	346.8(6)(5)	342.1	3.42(1)(3)	3.48

Lu, Li, Elhatisari, D.L., Epelbaum, Meißner, Phys. Lett. B 797, 134863 (2019)

Pinhole algorithm



Seeing Structure with Pinholes

Consider the density operator for nucleon with spin i and isospin j

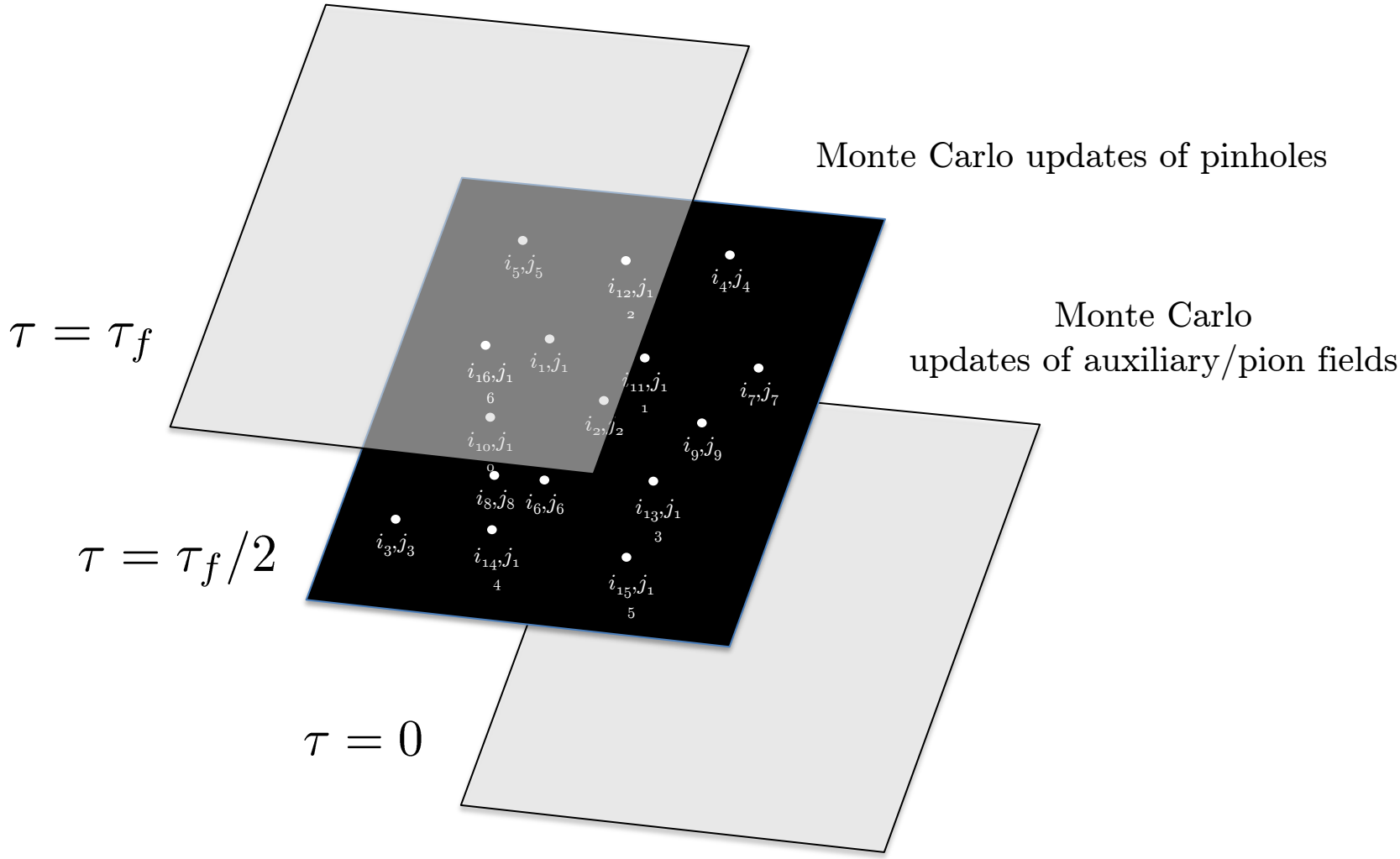
$$\rho_{i,j}(\mathbf{n}) = a^\dagger_{i,j}(\mathbf{n}) a_{i,j}(\mathbf{n})$$

We construct the normal-ordered A-body density operator

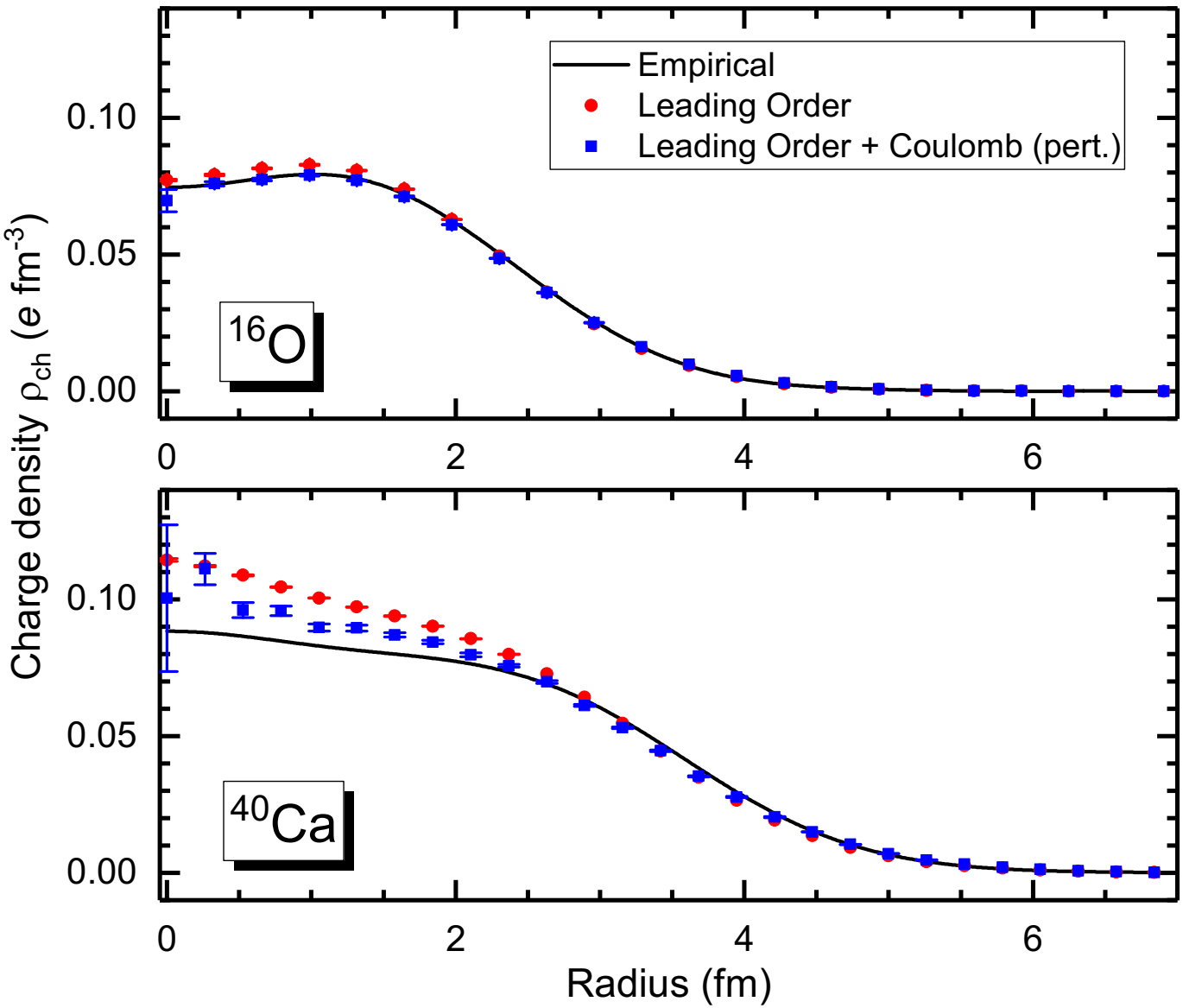
$$\rho_{i_1,j_1,\cdots i_A,j_A}(\mathbf{n}_1,\cdots \mathbf{n}_A) = :\rho_{i_1,j_1}(\mathbf{n}_1)\cdots\rho_{i_A,j_A}(\mathbf{n}_A):$$

In the simulations we do Monte Carlo sampling of the amplitude

$$A_{i_1,j_1,\cdots i_A,j_A}(\mathbf{n}_1,\cdots \mathbf{n}_A,t) = \langle\Psi_I|e^{-Ht/2}\rho_{i_1,j_1,\cdots i_A,j_A}(\mathbf{n}_1,\cdots \mathbf{n}_A)e^{-Ht/2}|\Psi_I\rangle$$



Elhatisari, Epelbaum, Krebs, Lähde, D.L., Li, Lu, Meißner, Rupak, Phys. Rev. Lett. 119, 222505 (2017)

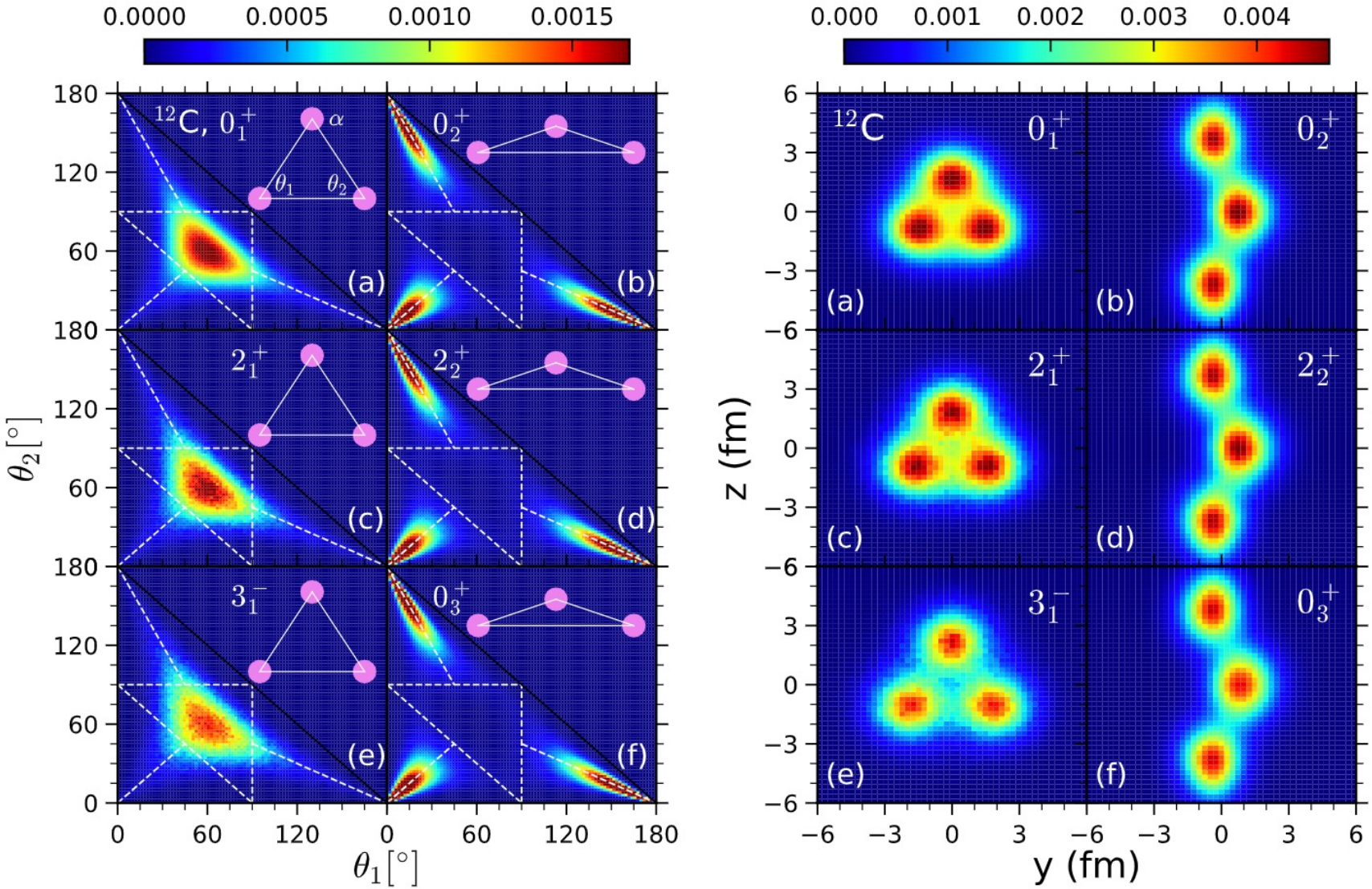


Lu, Li, Elhatisari, D.L., Epelbaum, Meißner, Phys. Lett. B 797, 134863 (2019)

Structure and spectrum of ^{12}C

Shen, Lähde, D.L. Meißner, Eur. Phys. J. A 57, 276 (2021)

State	$a = 1.97$ fm	$a = 1.64$ fm	Experiment
0_1^+	$-92.15(3)$	$-92.12(4)$	-92.162
2_1^+	$-88.87(4)$	$-88.19(17)$	-87.722
0_2^+	$-85.20(15)$	$-85.23(22)$	-84.508
3_1^-	$-84.9(2)$	$-83.3(5)$	$-82.521(5)$
2_2^+	$-83.5(2)$	$-83.1(5)$	$-82.29(6)$
0_3^+	$-80.0(3)$	$-79.2(6)$	$-81.9(3)$
1_1^-	$-81.5(4)$	$-79.7(4)$	$-81.315(4)$
2_1^-	$-78.6(2)$	$-76.1(2)$	$-80.326(4)$
1_1^+	$-79.67(11)$	$-78.14(24)$	$-79.452(6)$
4_1^-	$-78.1(2)$	$-75.5(5)$	$-78.846(20)$
4_1^+	$-80.99(11)$	$-79.1(6)$	$-78.083(5)$
2_3^+	$-79.9(4)$	$-77.9(2)$	-76.056
0_4^+	$-79.25(11)$	$-76.94(18)$	-74.402



Shen, Lähde, D.L. Meißner, arXiv:2202.13596

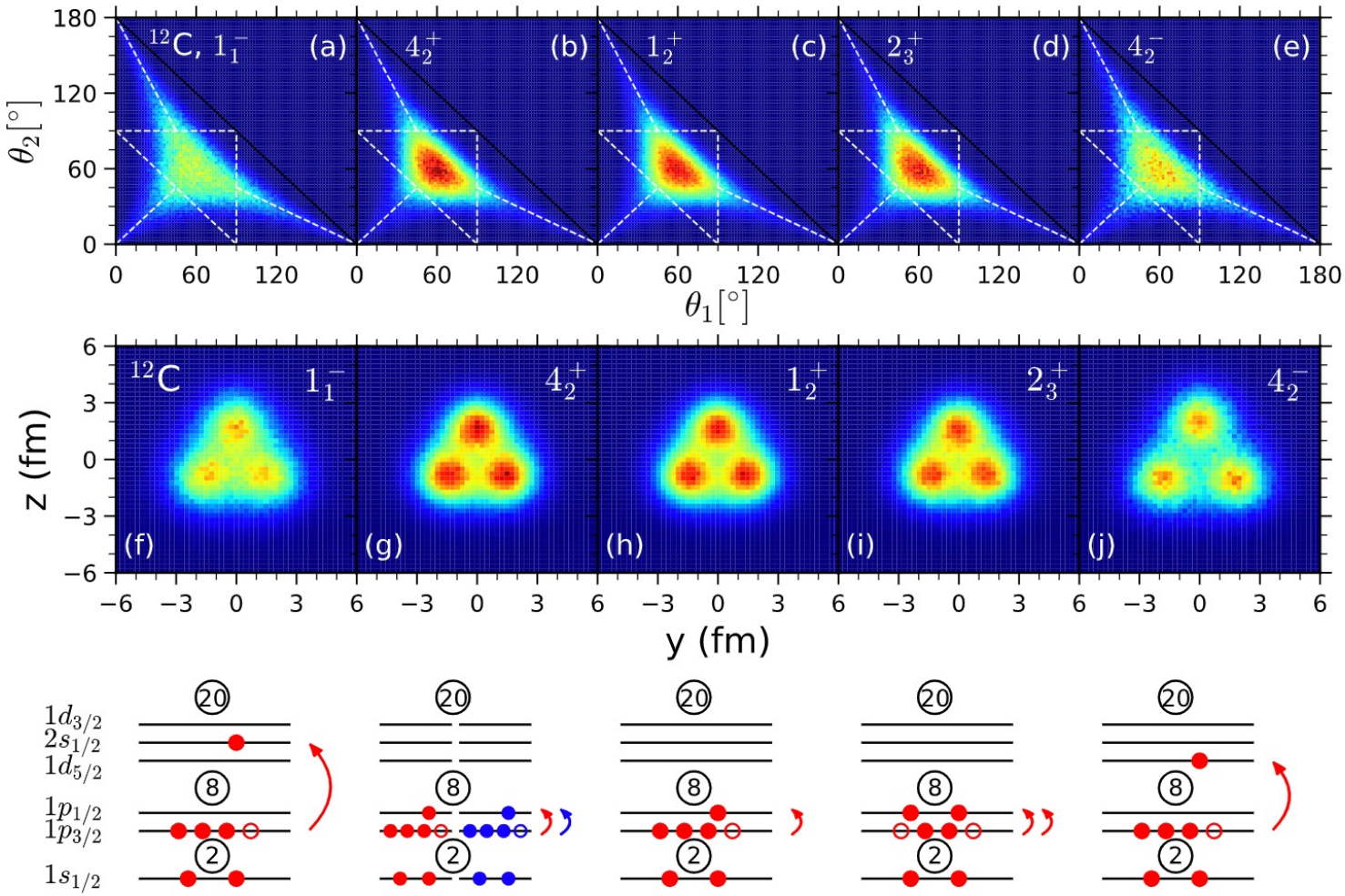


Figure S3: **Top Panel:** Density distribution for the two inner angles of the triangle formed by the three alpha clusters. **Middle Panel:** Tomographic projection of the nuclear density. **Lower Panel:** Sketch of the orbitals for the shell model initial states used in each of these calculations.

Shen, Lähde, D.L. Meißner, arXiv:2202.13596

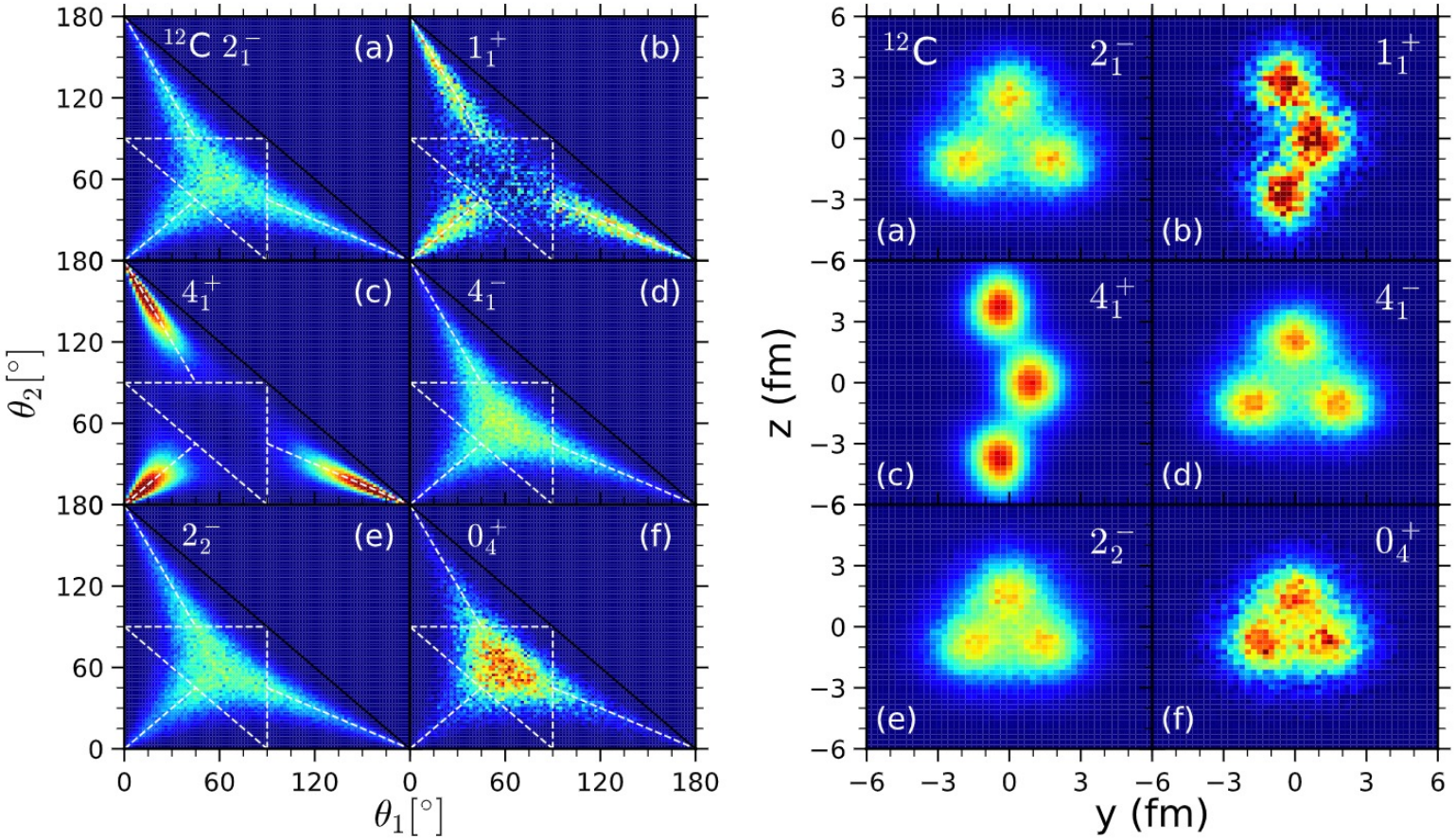
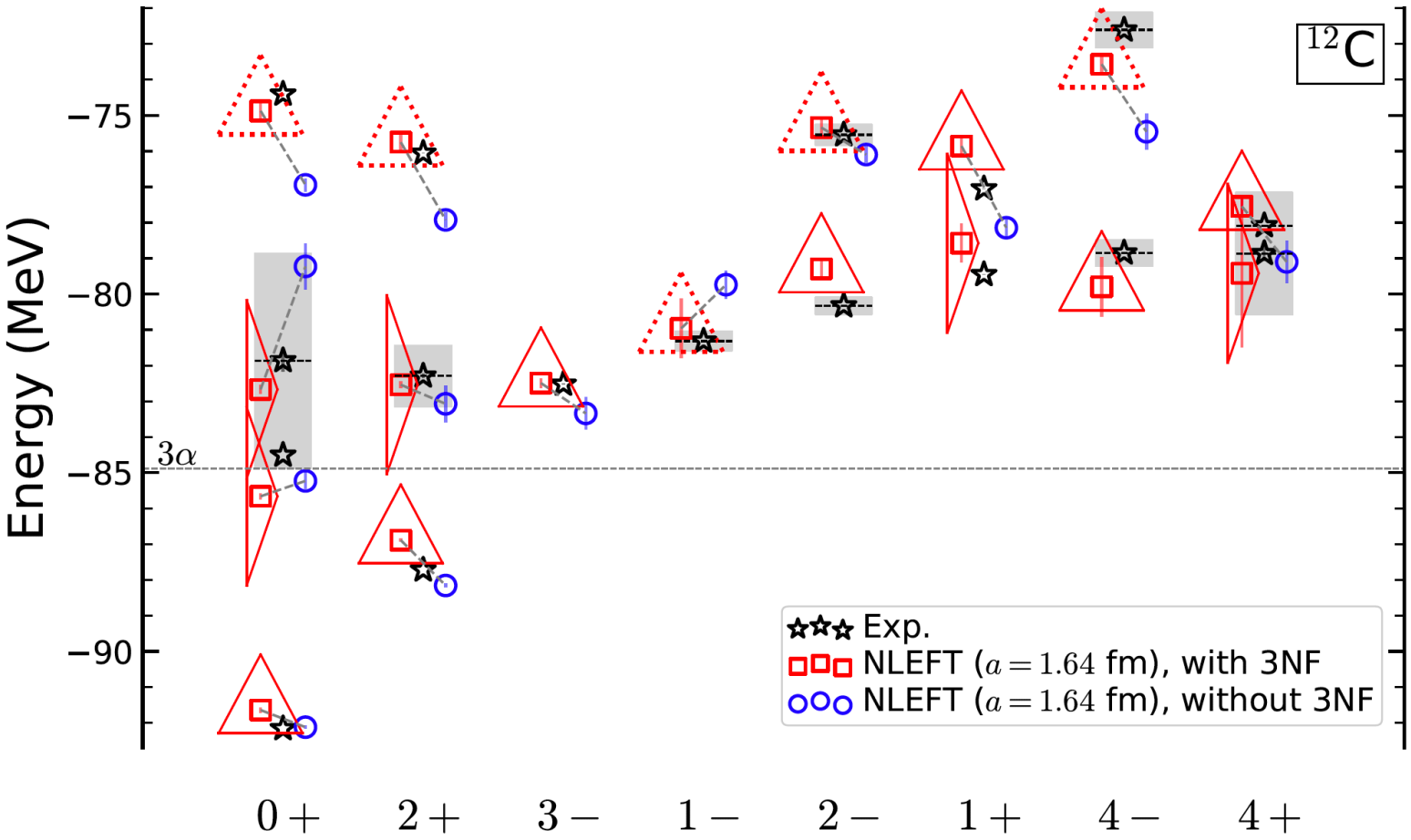


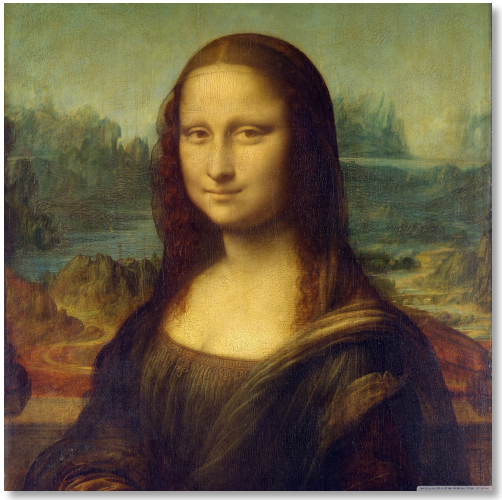
Figure S4: **Left Panel:** Density distribution for the two inner angles of the triangle formed by the three alpha clusters. **Right Panel:** Tomographic projection of the nuclear density. From (a) to (f), the selected states are ordered by their energies from low to high.

Shen, Lähde, D.L. Meißner, arXiv:2202.13596



Shen, Lähde, D.L. Meißner, arXiv:2202.13596

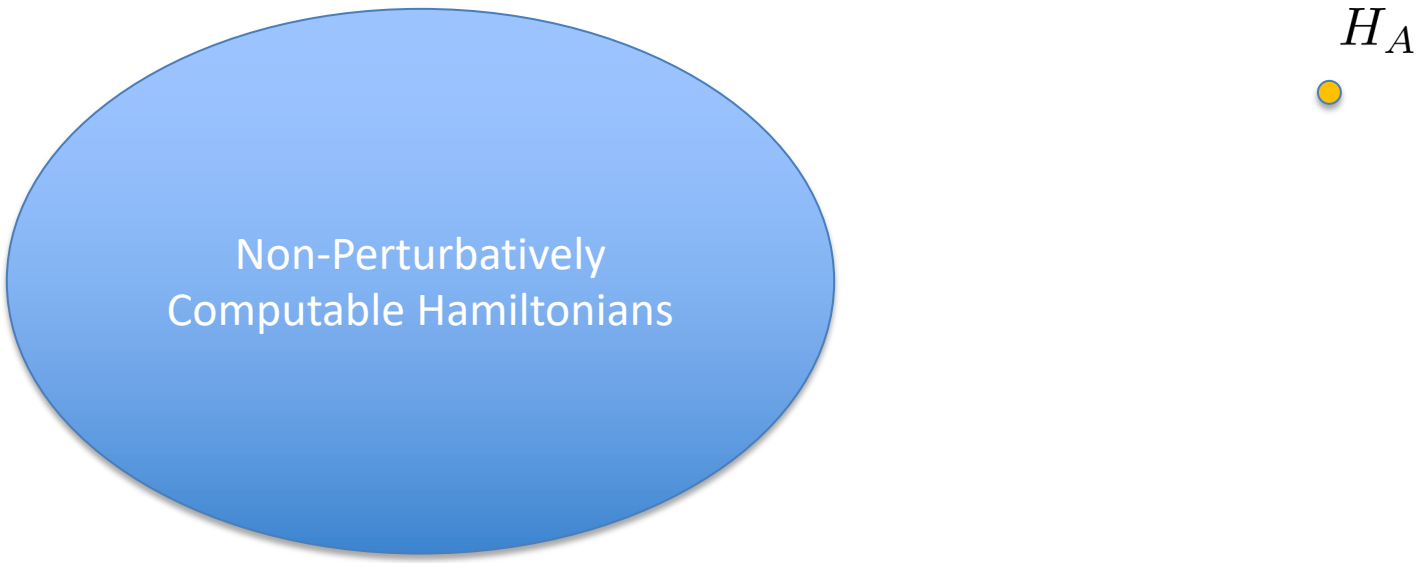
Wave function matching

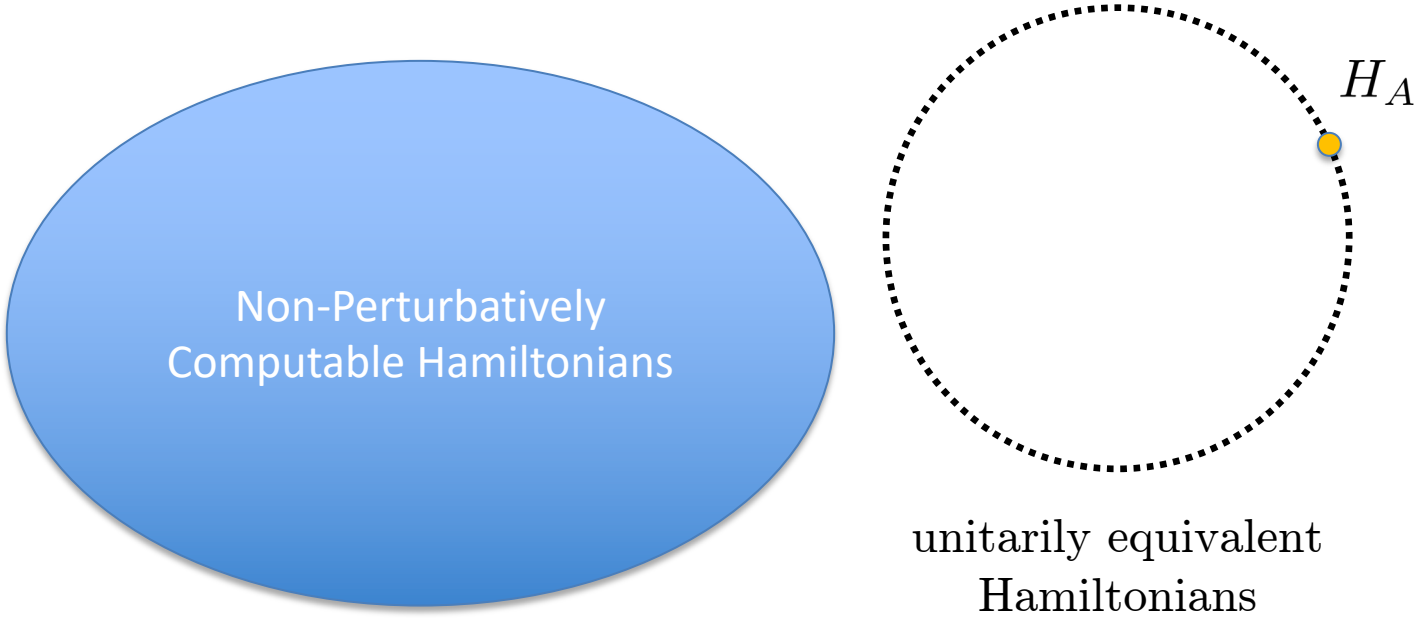


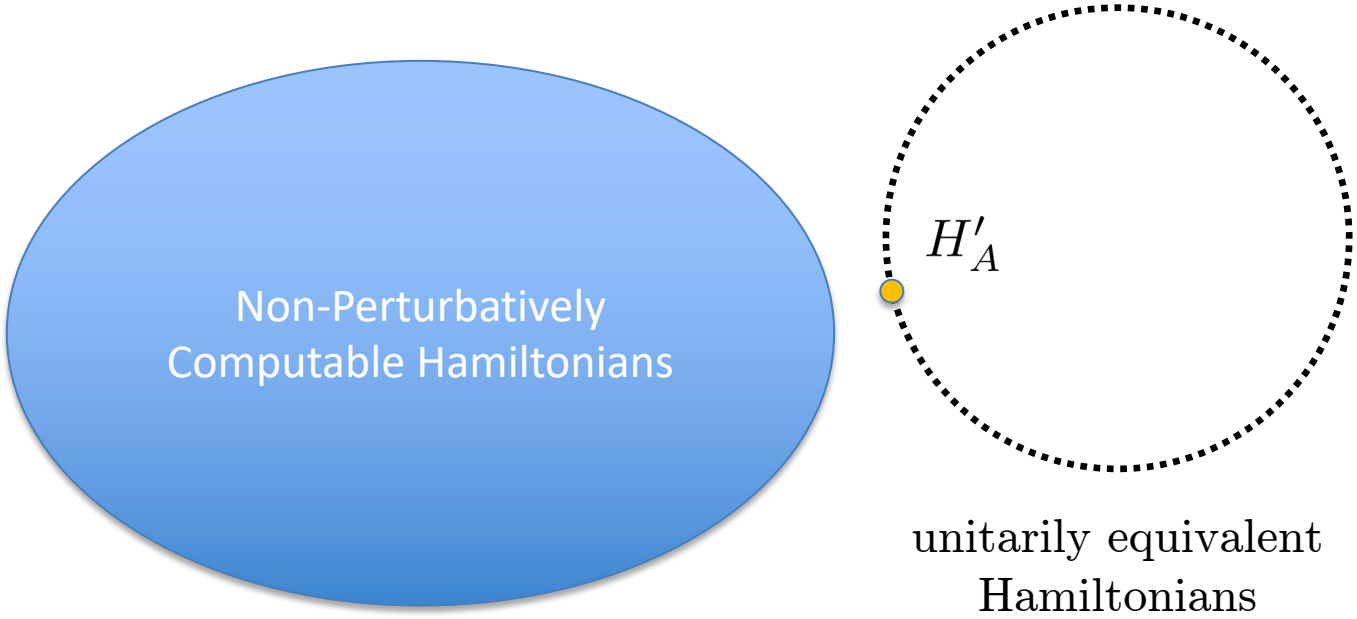
Work in progress: Elhatisari, Bovermann, Epelbaum, Frame, Hildenbrand, Krebs, Lähde, D.L., Li, Lu, M. Kim, Y. Kim, Ma, Meißner, Rupak, Shen, Song, Stellin

Lattice Monte Carlo simulations can compute highly nontrivial correlations in nuclear many-body systems. Unfortunately, sign oscillations prevent direct simulations using a high-fidelity Hamiltonian based on chiral effective field theory due to short-range repulsion.

Wave function matching solves this problem by means of unitary transformations and perturbation theory. By using unitary transformations, we construct a high-fidelity Hamiltonian that can be reached by perturbation theory, starting from a Hamiltonian without a sign problem.

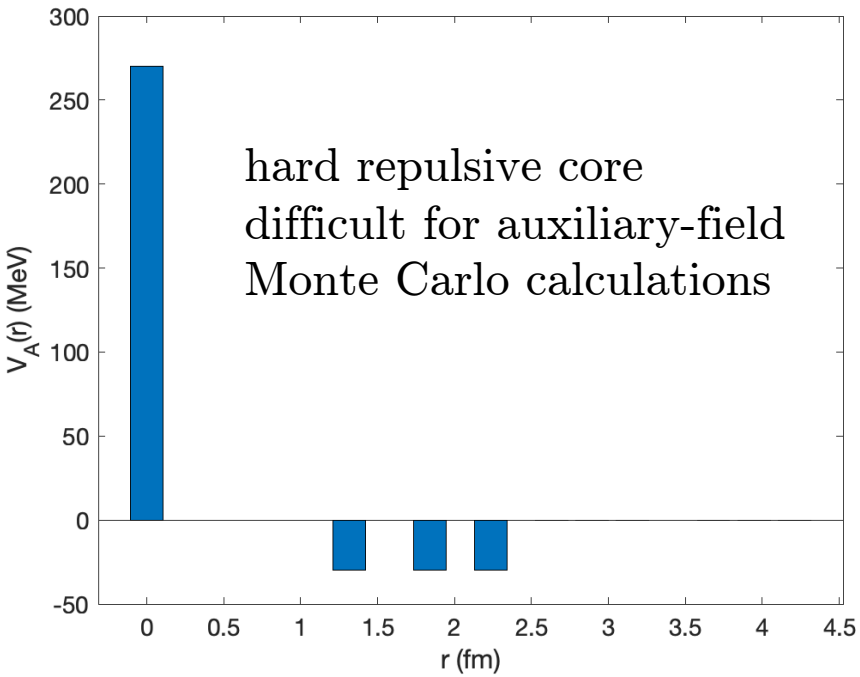

Non-Perturbatively
Computable Hamiltonians
H_A


Non-Perturbatively
Computable Hamiltonians
H_A
unitarily equivalent
Hamiltonians

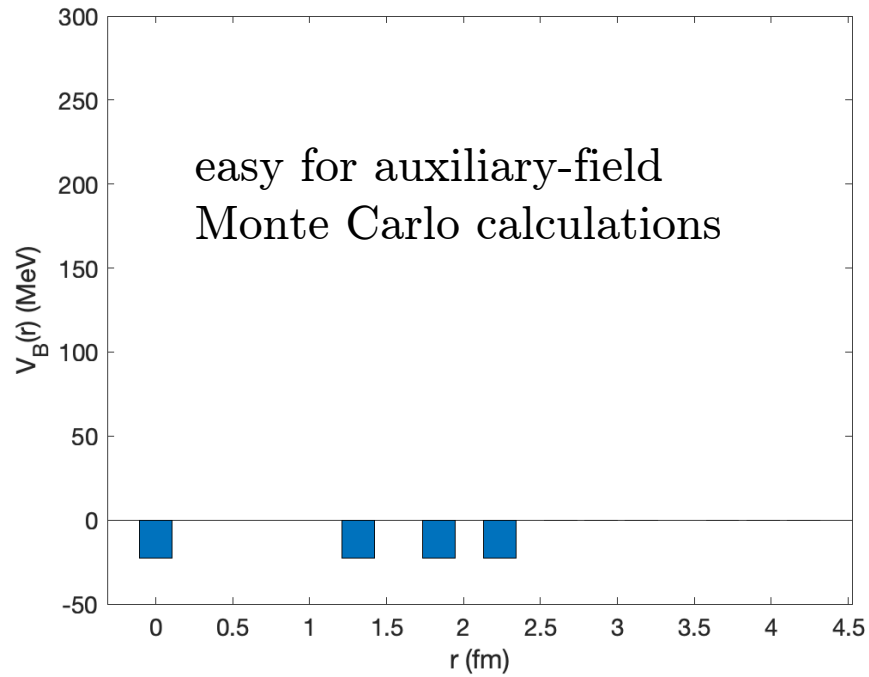

Non-Perturbatively
Computable Hamiltonians
H'_A
unitarily equivalent
Hamiltonians

Wave function matching

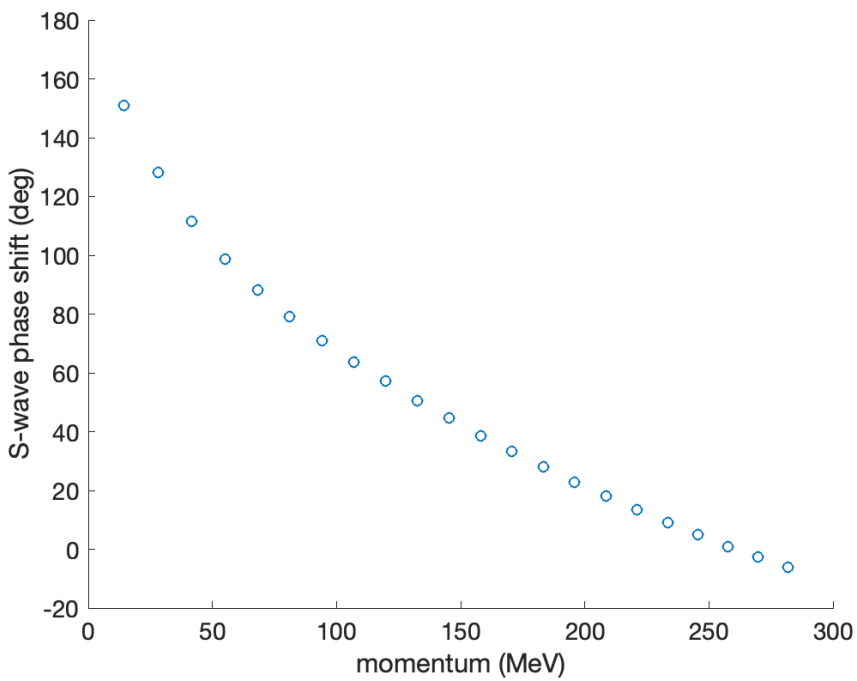
$V_A(r)$



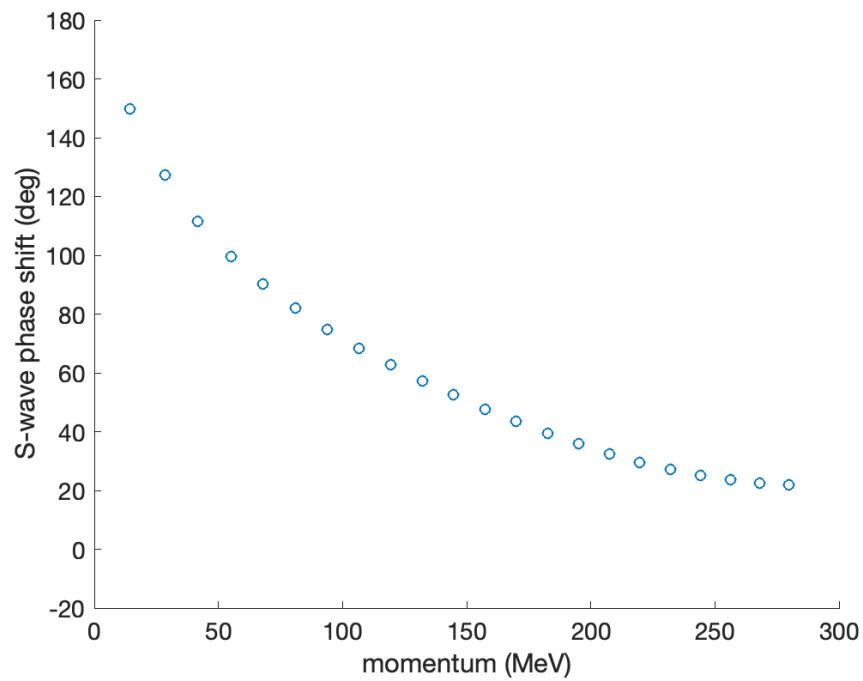
$V_B(r)$



$V_A(r)$


S-wave phase shift (deg)
momentum (MeV)
180
160
140
120
100
80
60
40
20
0
-20
0
50
100
150
200
250
300

$V_B(r)$


S-wave phase shift (deg)
momentum (MeV)
180
160
140
120
100
80
60
40
20
0
-20
0
50
100
150
200
250
300

Let us write the eigenenergies and eigenfunctions for the two interactions as

$$H_A \left|\psi_{A,n}\right\rangle = \left(K + V_A\right) \left|\psi_{A,n}\right\rangle = E_{A,n} \left|\psi_{A,n}\right\rangle$$
$$H_B \left|\psi_{B,n}\right\rangle = \left(K + V_B\right) \left|\psi_{B,n}\right\rangle = E_{B,n} \left|\psi_{B,n}\right\rangle$$

We would like to compute the eigenenergies of H_A starting from the eigenfunctions of H_B and using first-order perturbation theory.

Not surprisingly, this does not work very well. The interactions V_A and V_B are quite different.

$E_{A,n}$ (MeV)	$\langle\psi_{B,n}\|H_A\|\psi_{B,n}\rangle$ (MeV)
-1.2186	3.0088
0.2196	0.3289
0.8523	1.1275
1.8610	2.2528
3.2279	3.6991
4.9454	5.4786
7.0104	7.5996
9.4208	10.0674
12.1721	12.8799
15.2669	16.0458

Let P be a projection operator that is nonzero only for separation distances r less than R. We define a short-distance unitary operator U such that

$$U : P\left|\psi_B^0\right\rangle / \left\| P\left|\psi_B^0\right\rangle \right\| \to P\left|\psi_A^0\right\rangle / \left\| P\left|\psi_A^0\right\rangle \right\|$$

There are many possible choices for U. The corresponding action of U on the Hamiltonian is

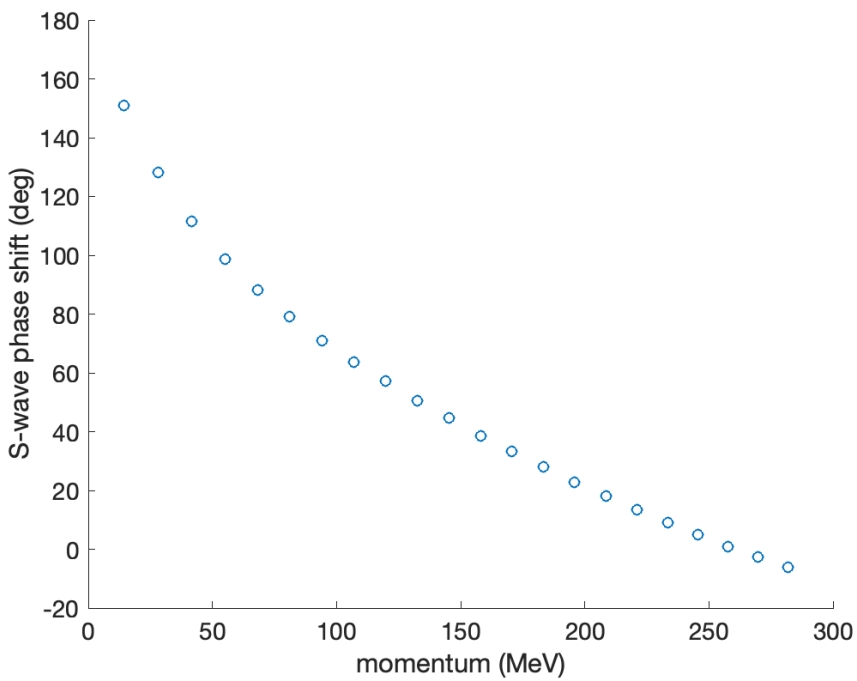
$$U : H_A \to H'_A = U^\dagger H_A U$$

and the resulting nonlocal interaction is

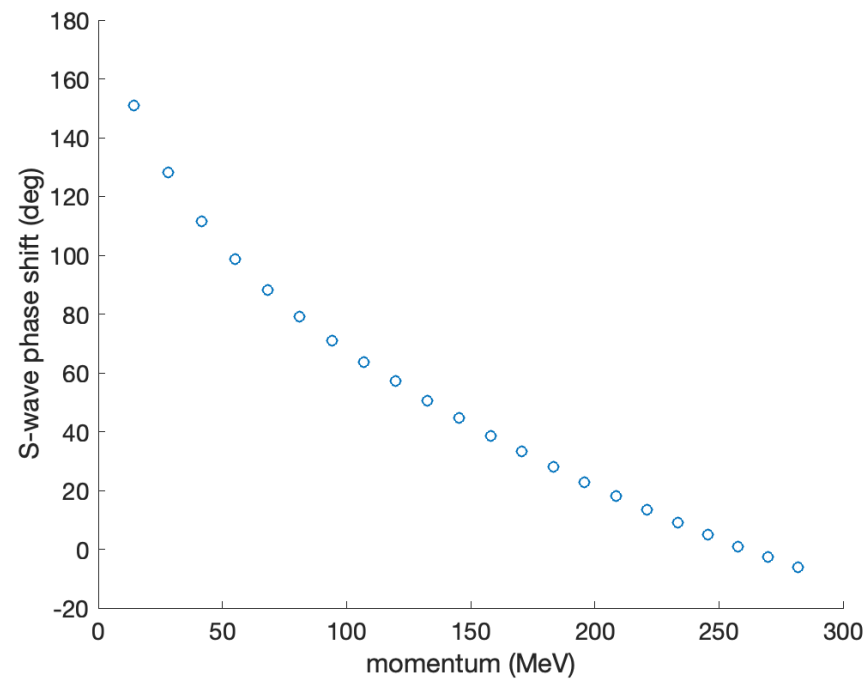
$$V'_A = H'_A - K = U^\dagger H_A U - K$$

Since they are unitarily equivalent, the phase shifts are exactly the same.

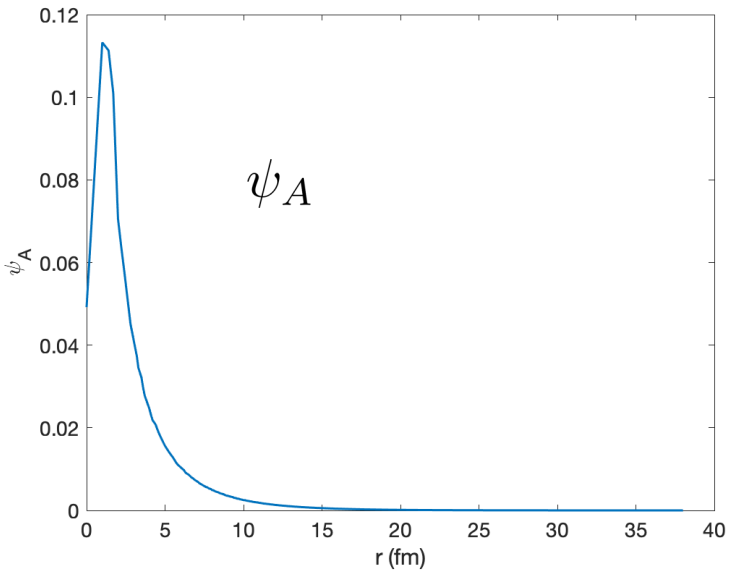
$V_A(r)$

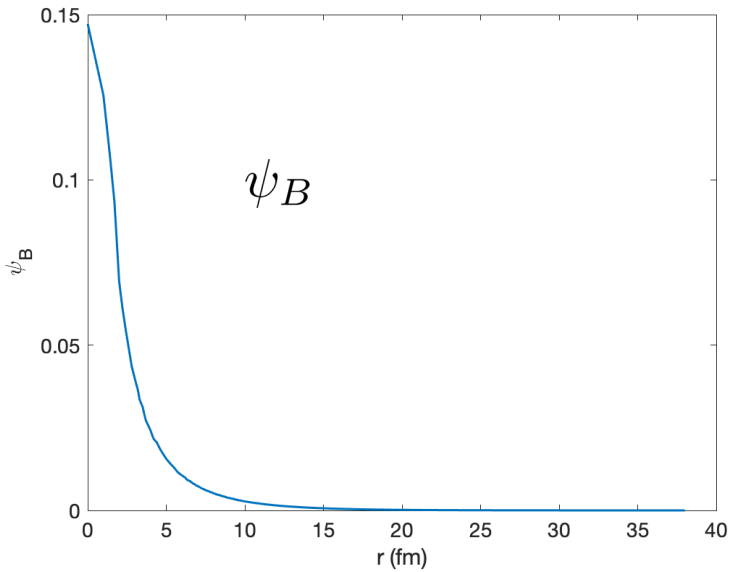


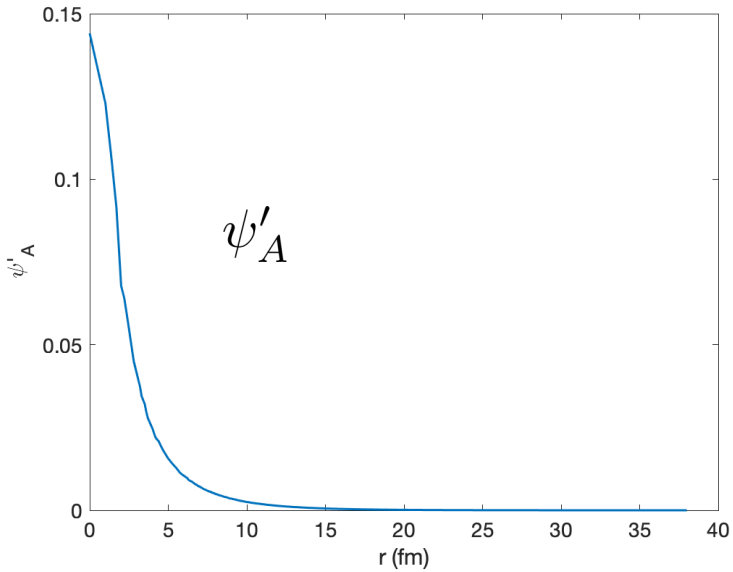
$V'_A(r, r')$



Ground state wave functions





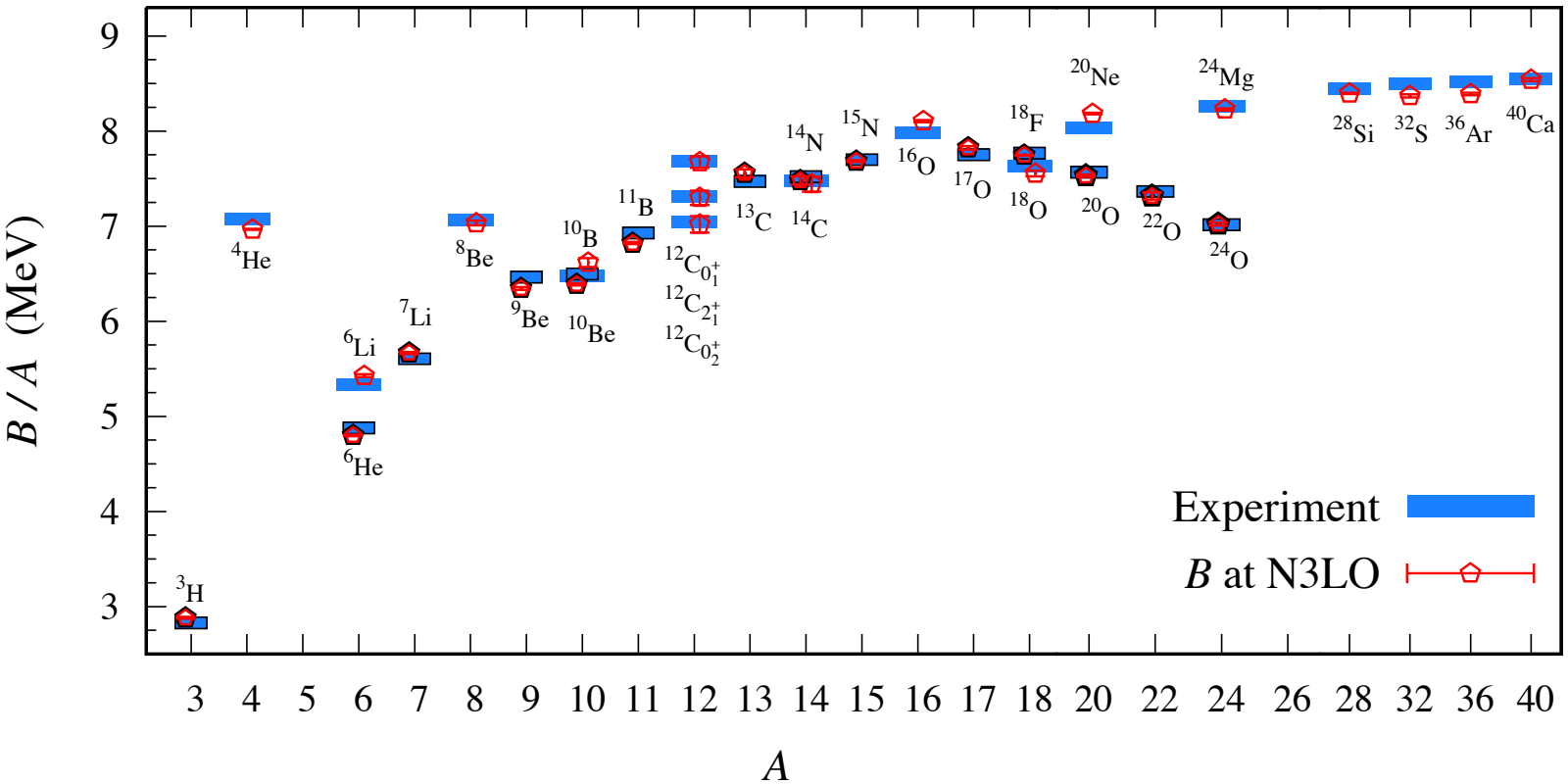


With wave function matching, we can now compute the eigenenergies starting from the eigenfunctions of H_B and using first-order perturbation theory.

$R = 2.6$ fm

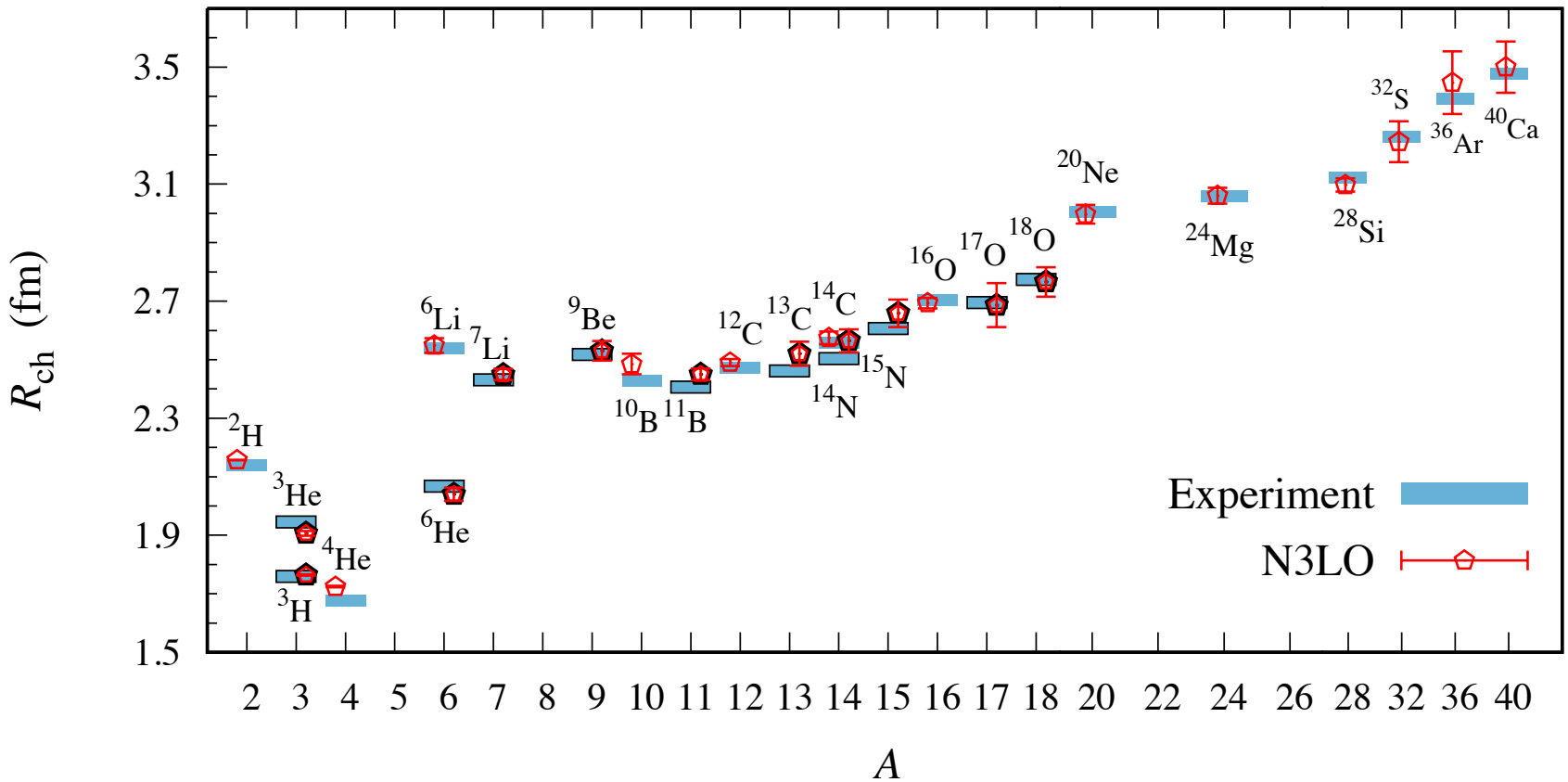
$E_{A,n} = E'_{A,n}$ (MeV)	$\langle\psi_{B,n}\|H_A\|\psi_{B,n}\rangle$ (MeV)	$\langle\psi_{B,n}\|H'_A\|\psi_{B,n}\rangle$ (MeV)
-1.2186	3.0088	-1.1597
0.2196	0.3289	0.2212
0.8523	1.1275	0.8577
1.8610	2.2528	1.8719
3.2279	3.6991	3.2477
4.9454	5.4786	4.9798
7.0104	7.5996	7.0680
9.4208	10.0674	9.5137
12.1721	12.8799	12.3163
15.2669	16.0458	15.4840

Binding energy per nucleon

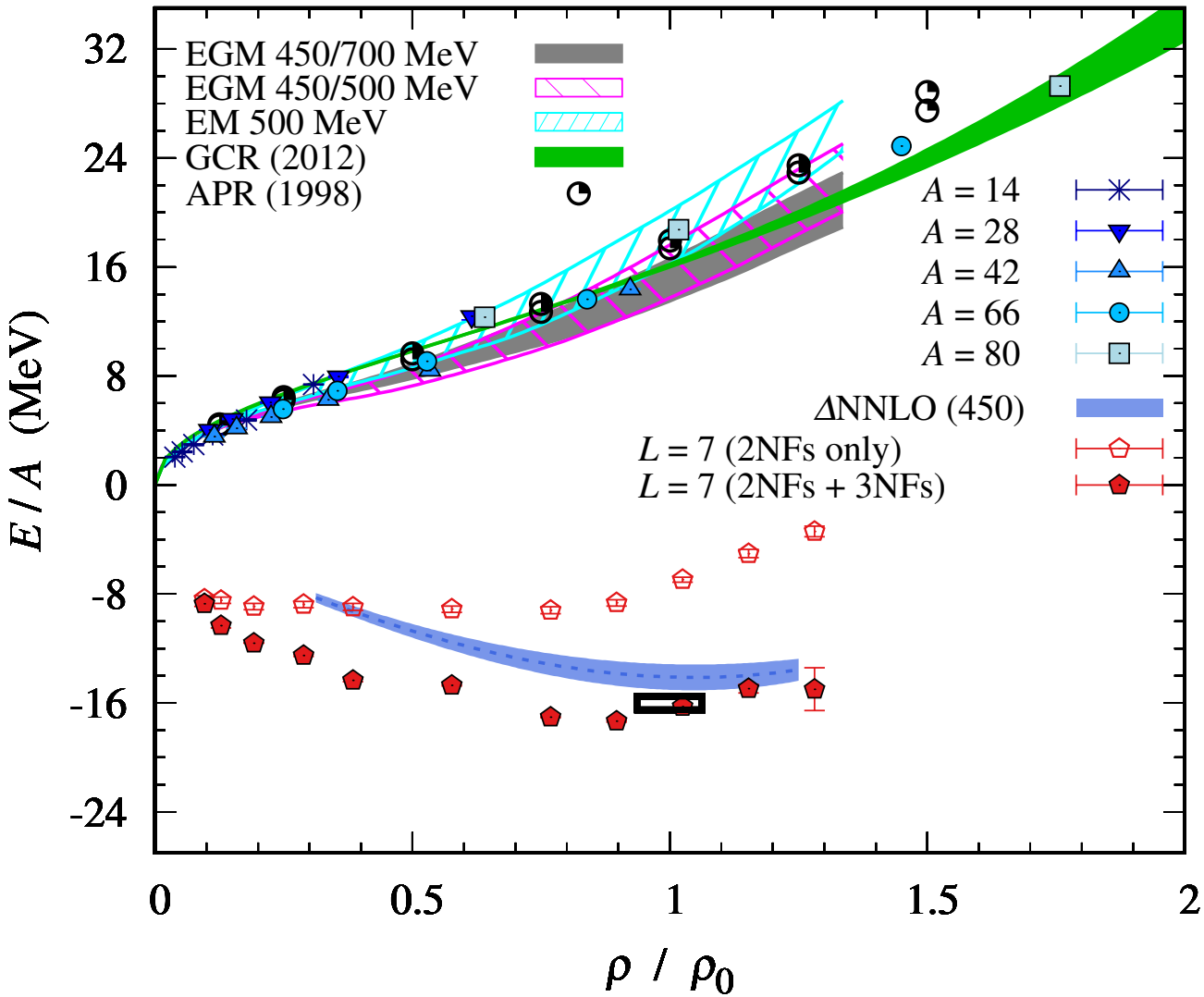


Work in progress: Elhatisari, Bovermann, Epelbaum, Frame, Hildenbrand, Krebs, Lähde, D.L., Li, Lu, M. Kim, Y. Kim, Ma, Meißner, Rupak, Shen, Song, Stellin

Charge radius



Work in progress: Elhatisari, Bovermann, Epelbaum, Frame, Hildenbrand, Krebs, Lähde, D.L., Li, Lu, M. Kim, Y. Kim, Ma, Meißner, Rupak, Shen, Song, Stellin



Work in progress: Elhatisari, Bovermann, et al.

Summary

We started with an introduction to lattice effective field theory. We presented large-N_c arguments and numerical evidence that nuclear physics is close to Wigner's SU(4) symmetric limit. We introduced the pinhole algorithm for seeing particle densities and correlations. We then studied the nuclear states of carbon-12 and found alpha cluster structures with two different geometries. We concluded with a discussion of a new method called wave function matching. This enables high-fidelity calculations of nuclear structure from *ab initio* lattice Monte Carlo simulations.