

# Universal dynamics near non-thermal fixed points



Thomas Gasenzer  
Synthetic Quantum Systems  
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SynQS

Kirchhoff-Institut für Physik  
Ruprecht-Karls Universität Heidelberg  
Im Neuenheimer Feld 227 • 69120 Heidelberg • Germany



# Synthetic Quantum Systems

## Kirchhoff-Institut @ Uni Heidelberg



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Rocuzzo  
(not on ~~the~~ roof)

Philipp  
Heinen

Niklas  
Rasch

Aleksandr  
Mikheev

Martin  
Zboron

Sören  
Breidenbach

Ido  
Siovitz

TG

... not to forget former members



Isara  
Chantesana



Stefanie  
Czischek



Christian  
Schmied



Markus  
Karl  
...



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Rocuzzo  
(not on pie roof)

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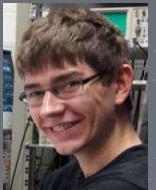
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Breidenbach

Ido  
Siovitz

TG

... and Heidelberg SynQS experiment



Stefan  
Lannig



Max  
Prüfer



Jan  
Dreher



Yannik  
Deller



Helmut  
Strobel



Markus  
Oberthaler



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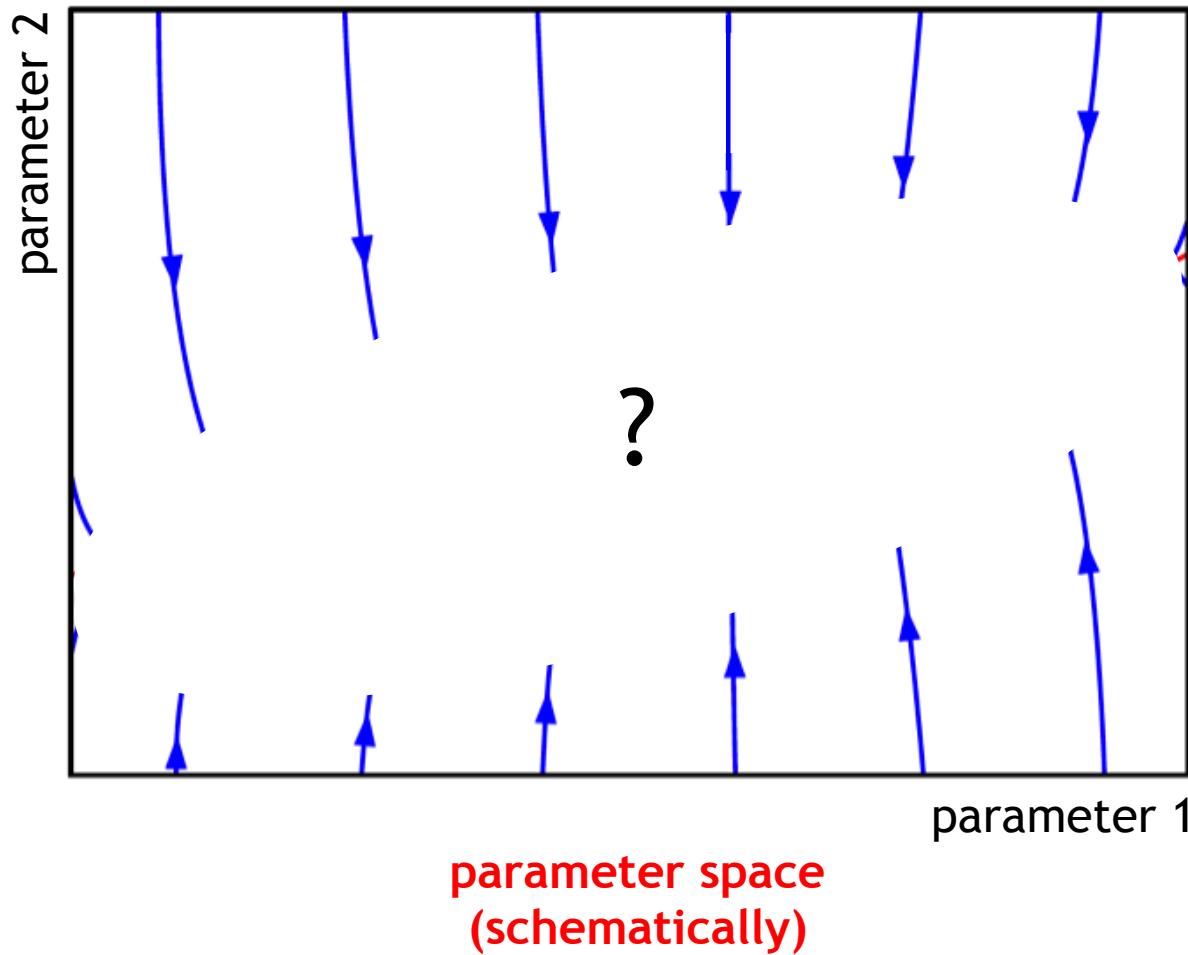
Sören  
Breidenbach

Ido  
Siovitz

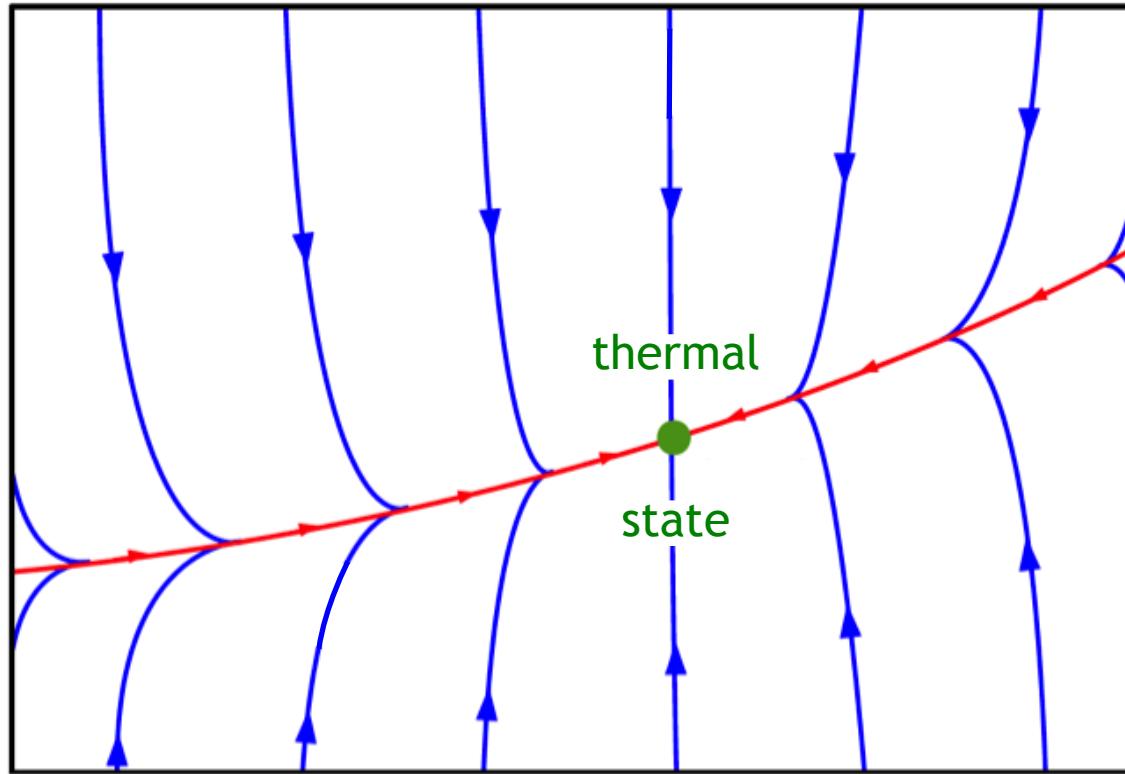
TG



Suppose we start from some non-equilibrium state  
– what can we expect?

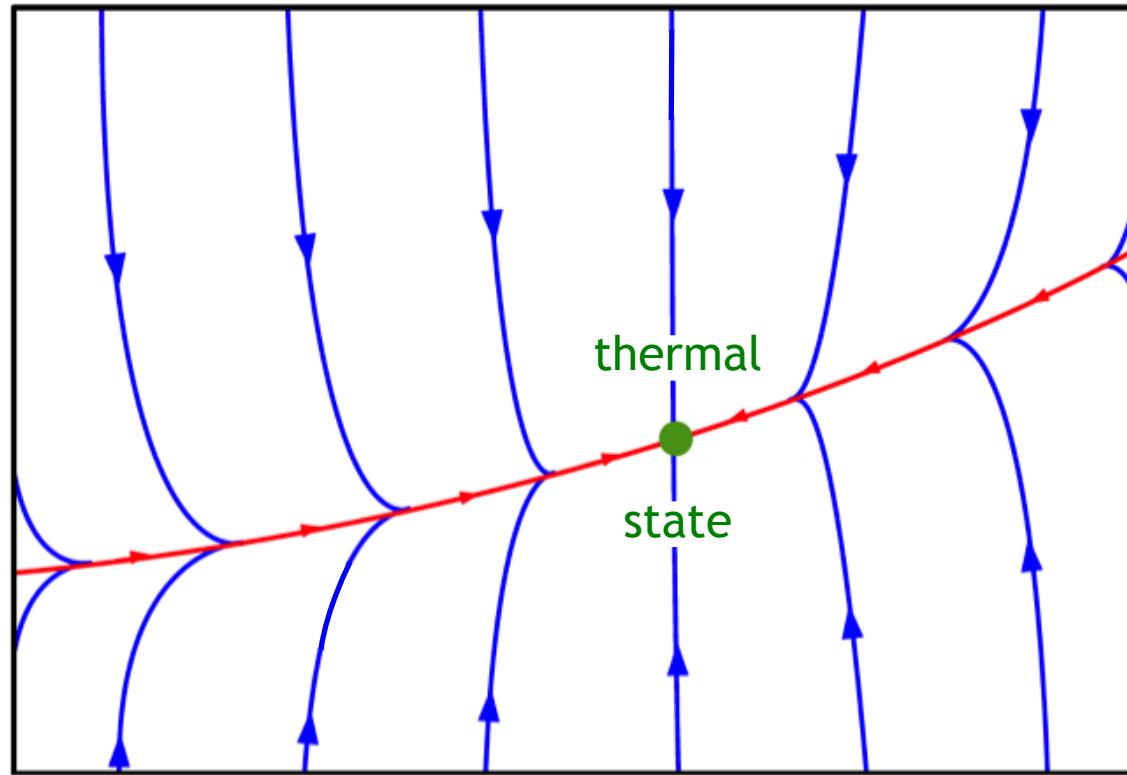


# Thermalisation...



**loss of information about initial conditions**

...and (partially) universal quantum dynamics



[Sketch after C. Wetterich]

**partial loss of information about initial conditions**

e.g. in terms of density matrix: loss of off-diagonal phase relations  
(between non-degenerate levels)

# That's all?

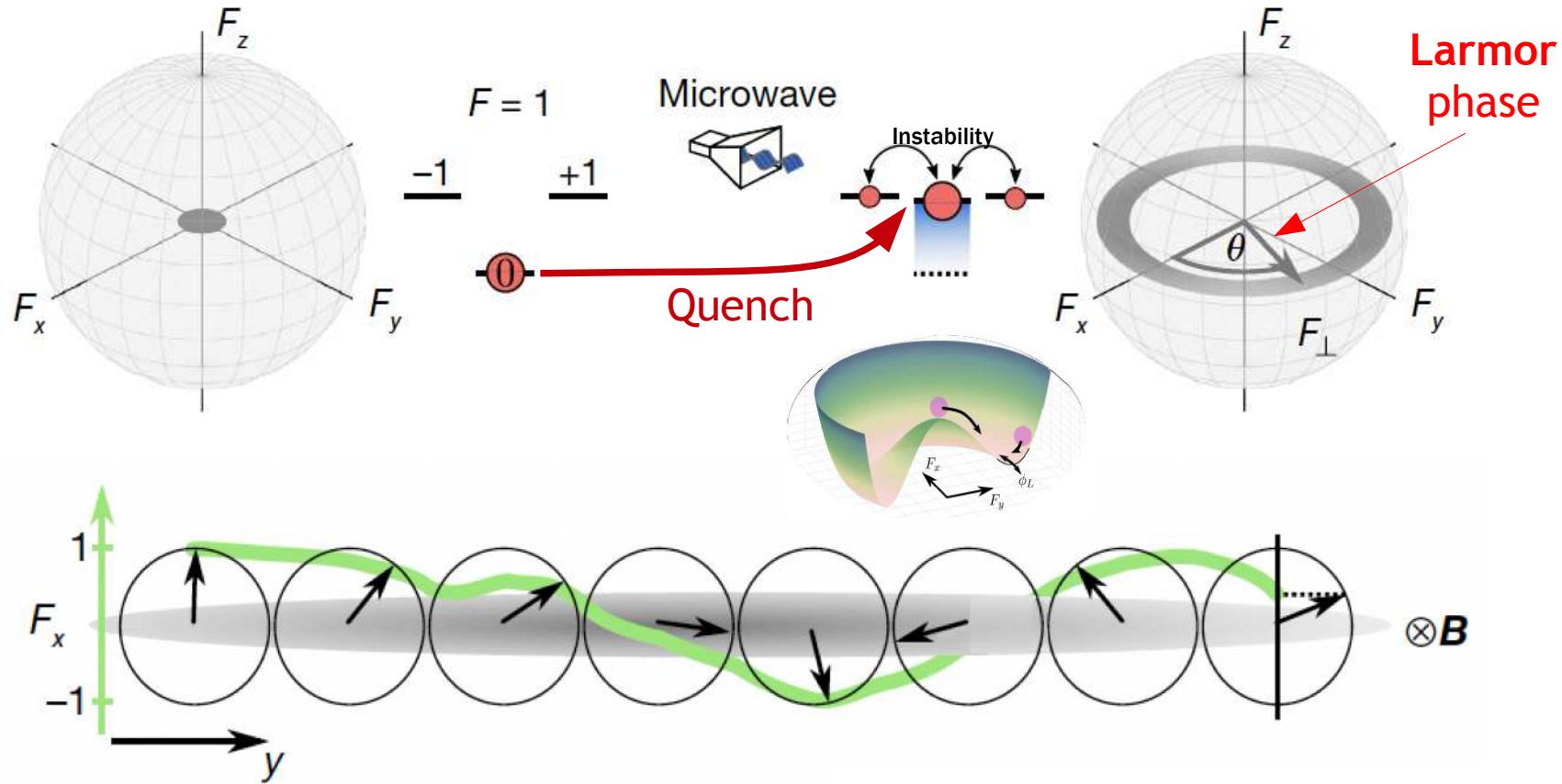
# Far from equilibrium in experiment

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$^{87}\text{Rb}$  BEC in a quasi 1D trap:

Spin-1 BEC ( $F = 1$  hyperfine state with magnetic sublevels  $m_F = 0, \pm 1$ )



M. Prüfer, et al., Nature 563, 217 (18); arXiv:1805.11881 [cond-mat.quant-gas]

# Far from equilibrium in experiment

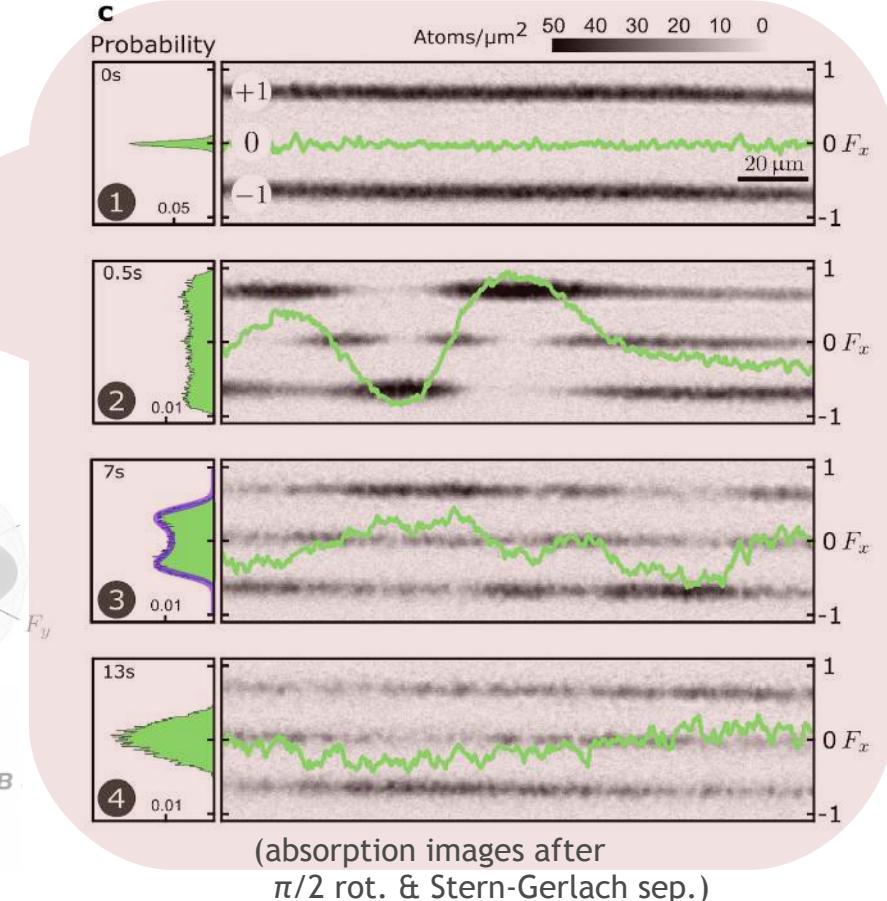
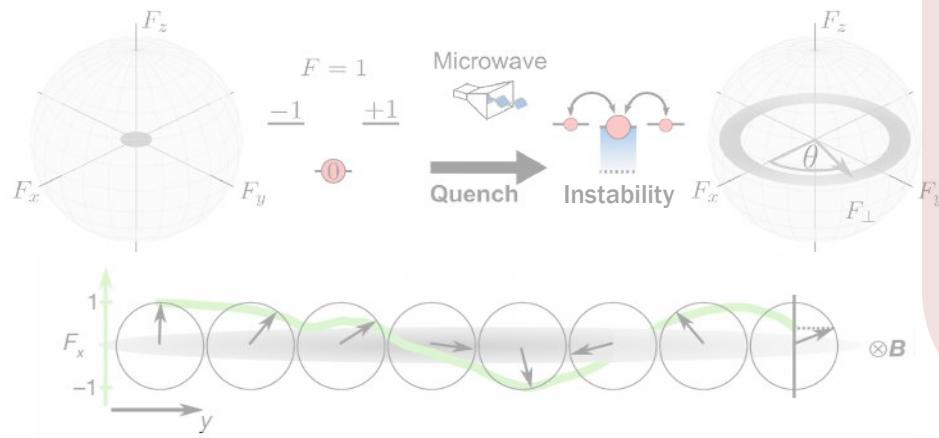
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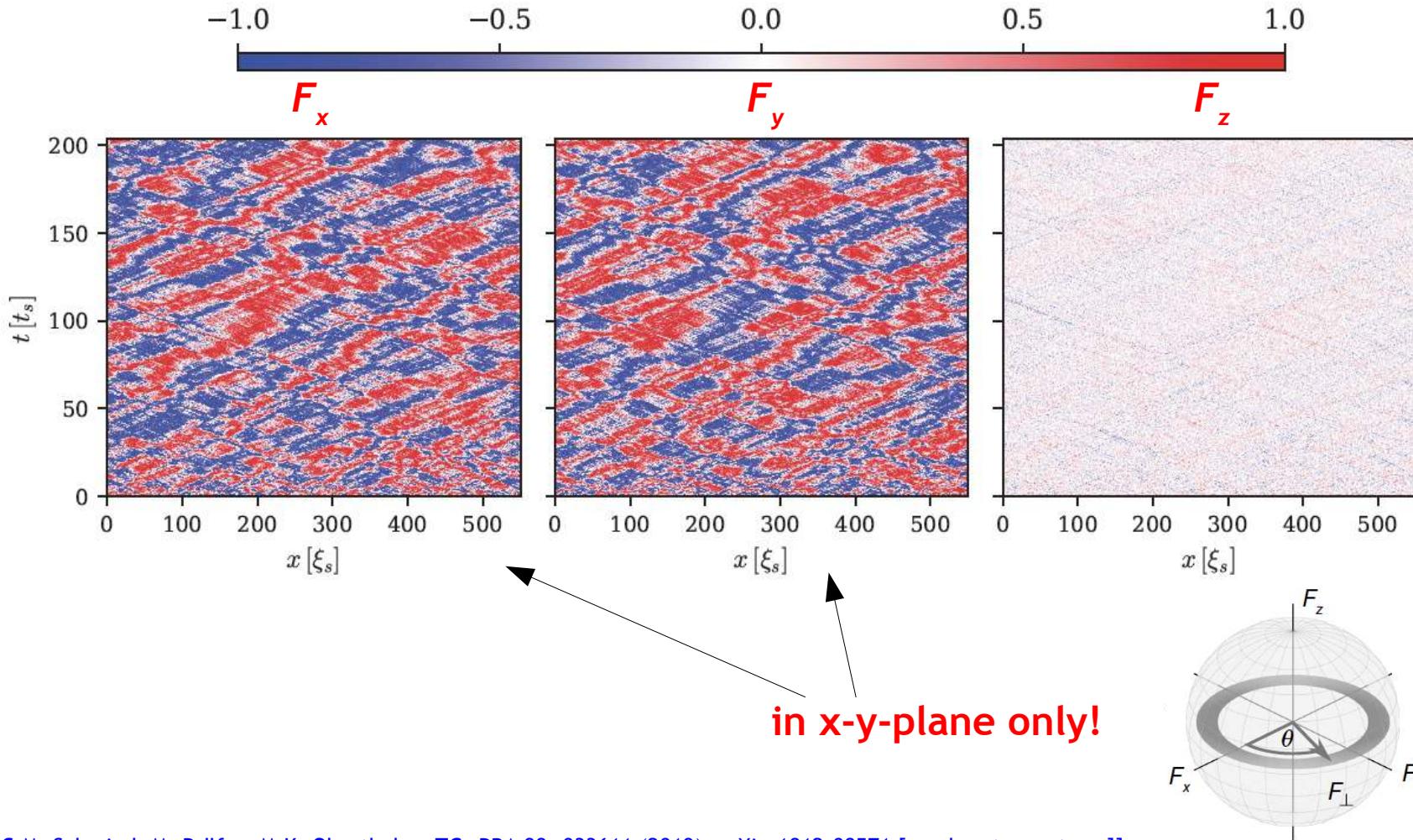
→ 4 snapshots in time



M. Prüfer, et al., Nature 563, 217 (18); arXiv:1805.11881 [cond-mat.quant-gas]

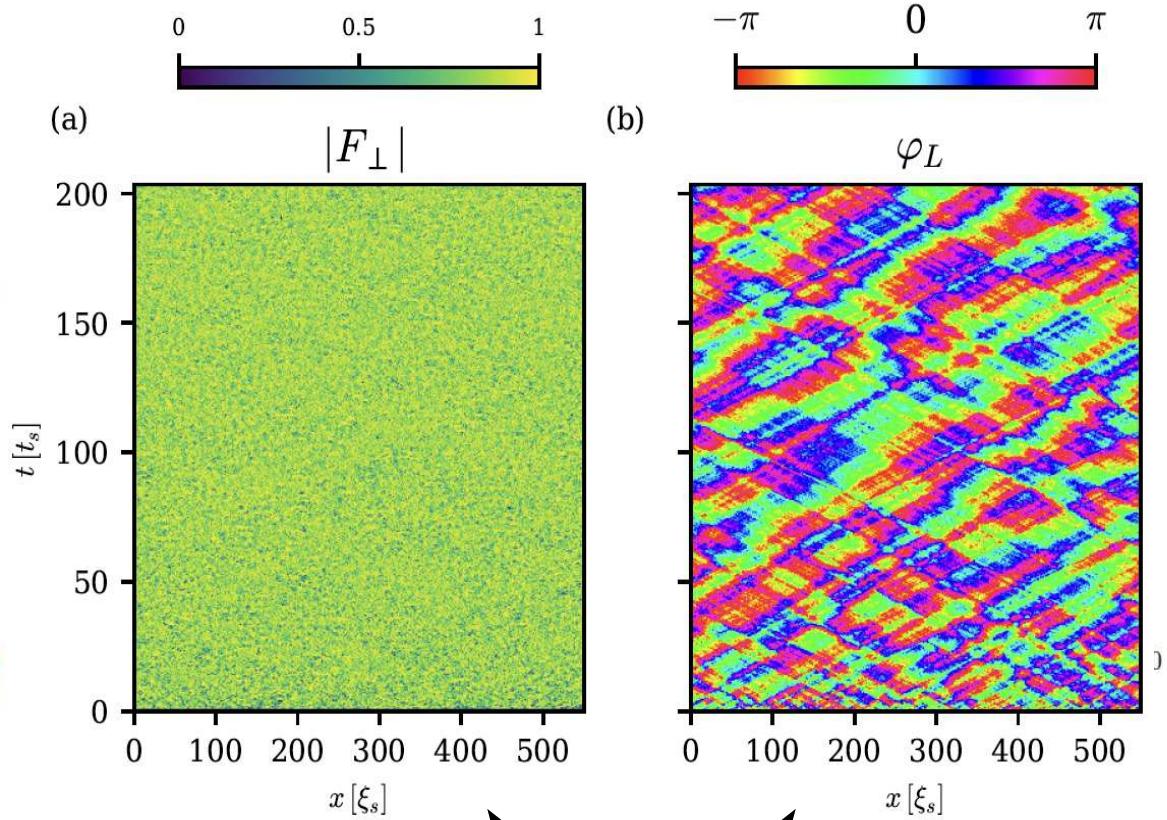
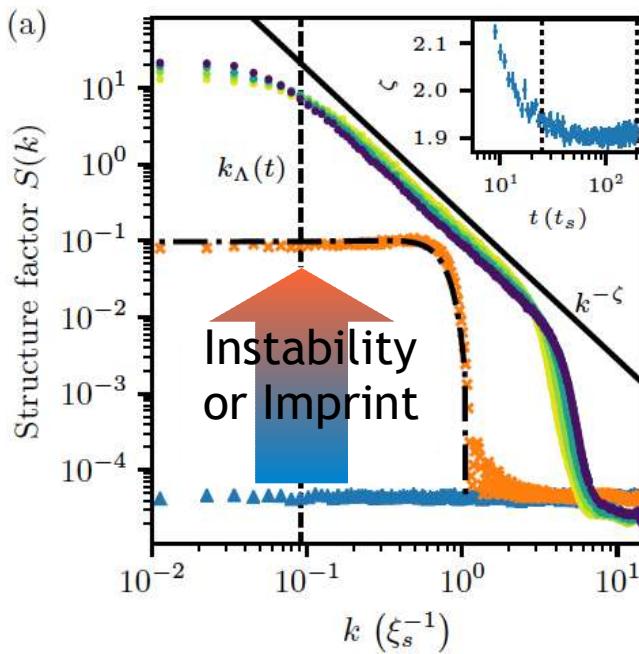
# Simulation: Pattern formation

## Cartesian Spin components



C.M. Schmied, M. Prüfer, M.K. Oberthaler, TG, PRA 99, 033611 (2019); arXiv:1812.08571 [cond-mat.quant-gas]  
I. Siovitz, C.M. Schmied, S. Lannig, H. Strobel, M.K. Oberthaler, TG et al., unpublished

# Pattern formation & Coarsening



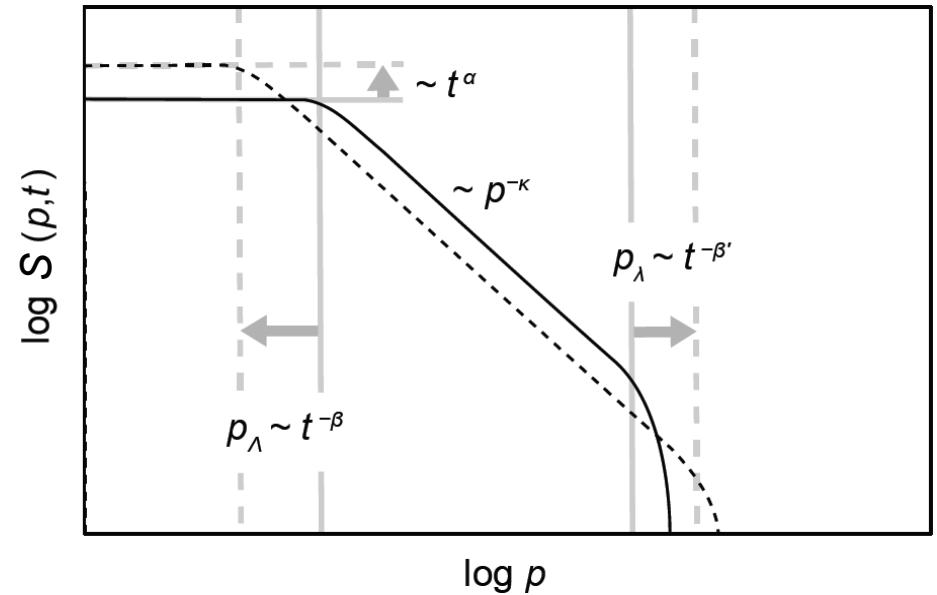
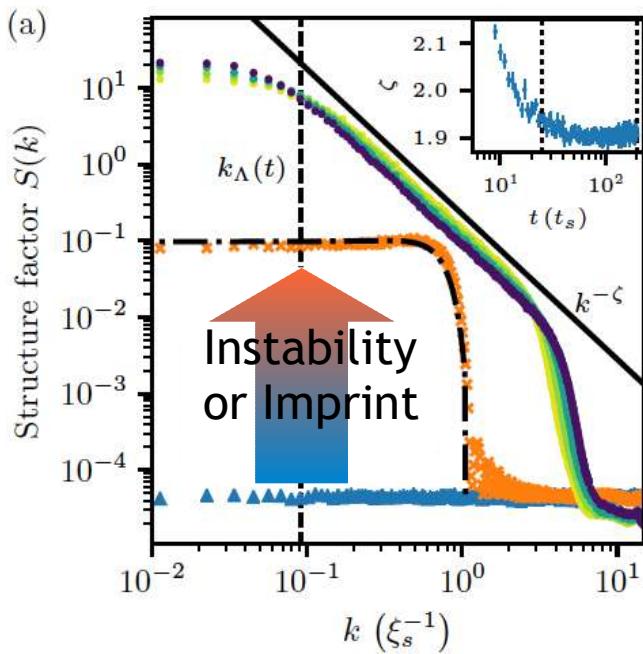
Structure factor ... of ...      Amplitude & Larmor phase

$$S(k, t) = \langle |F_{\perp}(k, t)|^2 \rangle \quad F_{\perp} = F_x + iF_y = |F_{\perp}|e^{i\varphi_L}$$

C.M. Schmied, M. Prüfer, M.K. Oberthaler, TG, PRA 99, 033611 (2019); arXiv:[1812.08571 \[cond-mat.quant-gas\]](https://arxiv.org/abs/1812.08571)  
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# Universal rescaling dynamics!

$$\beta \approx 1/4, \quad \alpha \approx d\beta \approx 1/4$$

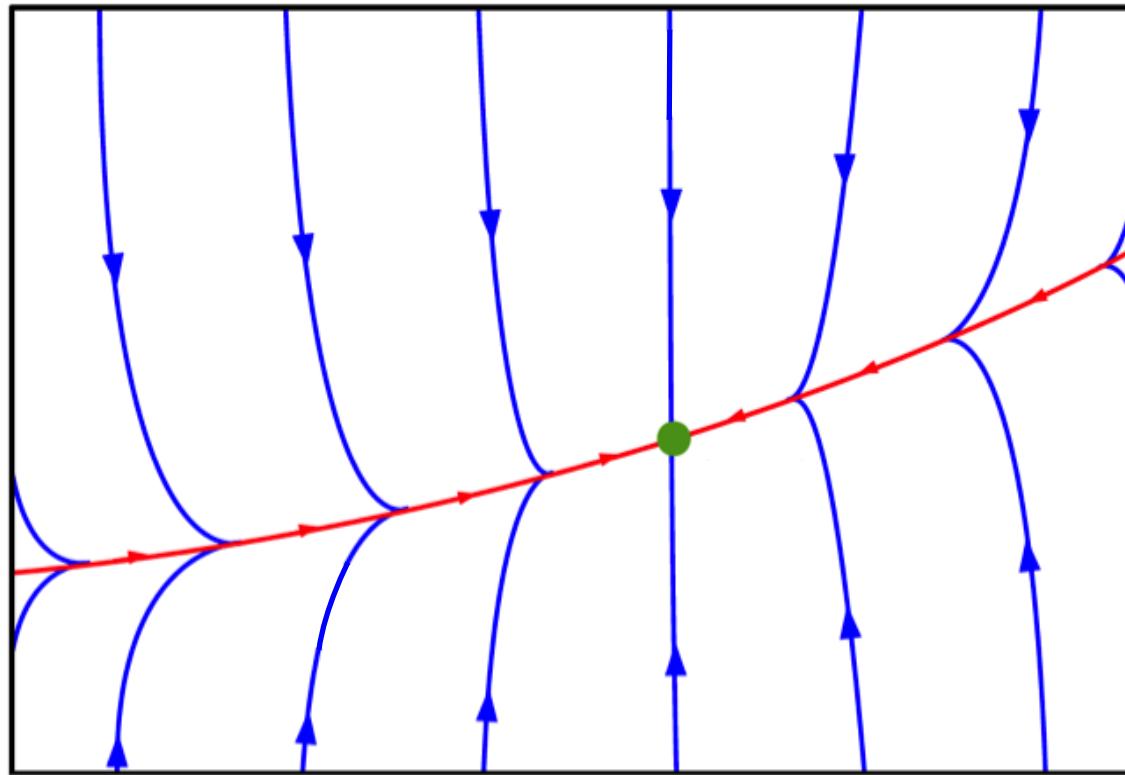


Structure factor

$$S(k, t) = \langle |F_\perp(k, t)|^2 \rangle \quad F_\perp = F_x + iF_y = |F_\perp|e^{i\varphi_L}$$

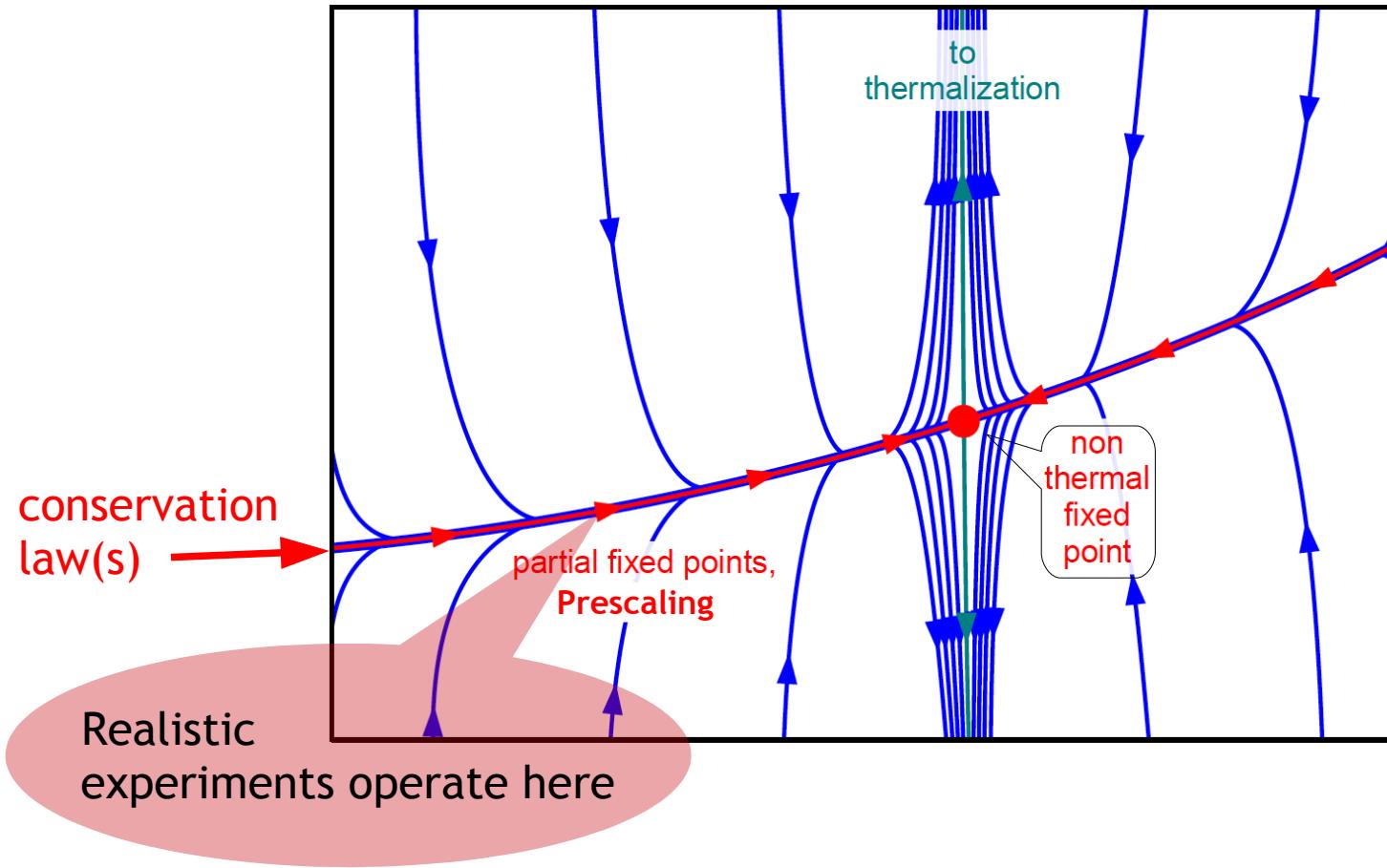
C.M. Schmied, M. Prüfer, M.K. Oberthaler, TG, PRA 99, 033611 (2019); arXiv:[1812.08571 \[cond-mat.quant-gas\]](https://arxiv.org/abs/1812.08571)  
I. Siovitz, C.M. Schmied, S. Lannig, H. Strobel, M.K. Oberthaler, TG et al., unpublished

# Recall: Partially universal quantum dynamics



**partial loss of information about initial conditions**

# Prescaling and Non-Thermal Fixed Point



Non-thermal Fixed Points:

Berges, Rothkopf, Schmidt: PRL 101, 041603 (08), Hoffmeister, Sixty, Schlichting, Piñeiro Orioli, Boguslavski, ... Berges (09-) Scheppach, Berges, TG: PRA 81, 033611 (10), Nowak, Sixty, Schole, Schmidt, Erne, Karl, Schmied, ... TG (11-)

Kinetic theory summary:  
Low-energy effective theory:

Chantesana, Piñeiro Orioli, TG: PRA 99, 043620 (19);  
Mikheev, Schmied, TG: PRA 99, 063622 (19)

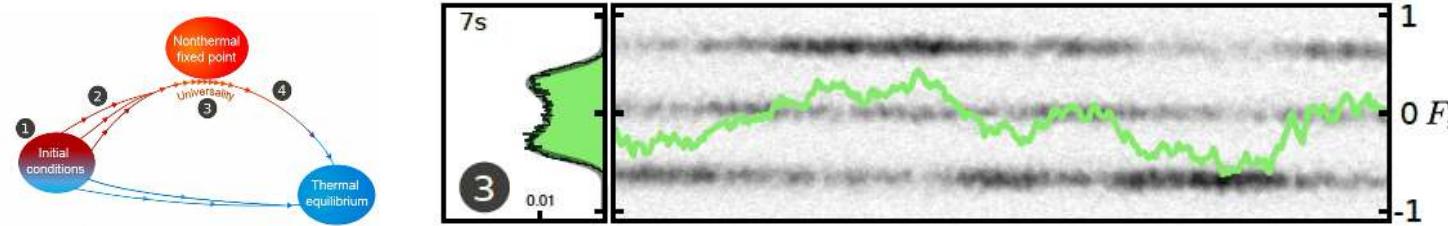
Review article: Schmied, Mikheev, TG, arXiv:1810.08143

# Universal dynamics in experiment

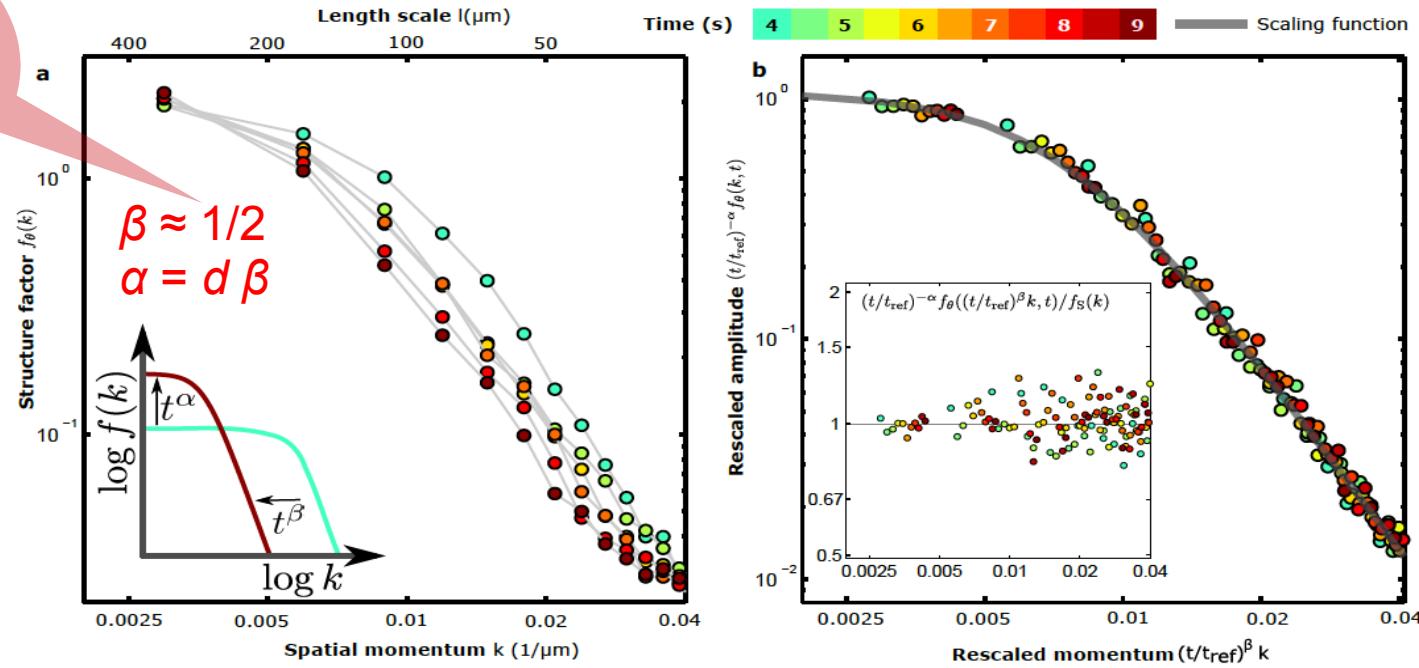
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Experimental identification of a non-thermal fixed point in a Spin-1 Bose gas



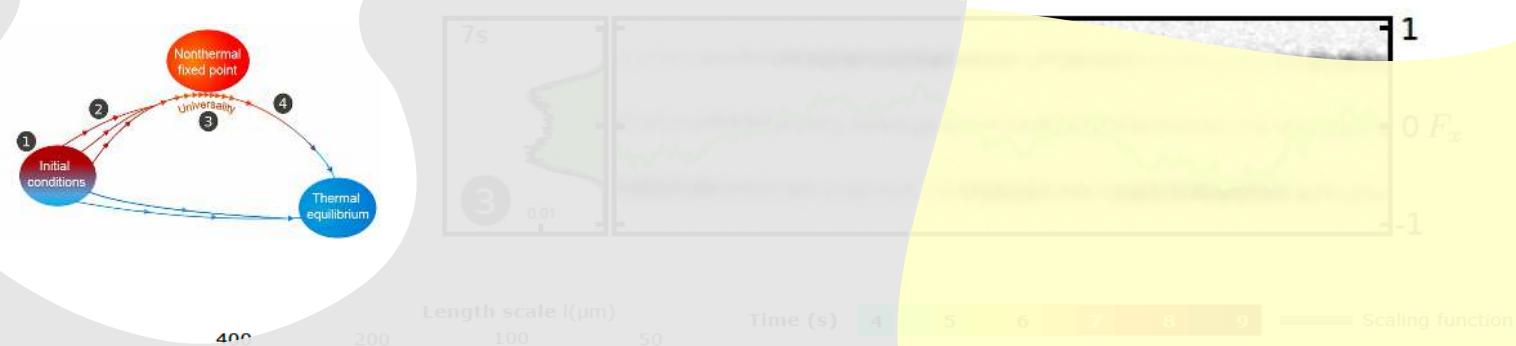
why  
not  $\frac{1}{4}$  ?!



M. Prüfer, et al., Nature 563, 217 (18); arXiv:1805.11881 [cond-mat.quant-gas]

# Universal dynamics in experiment

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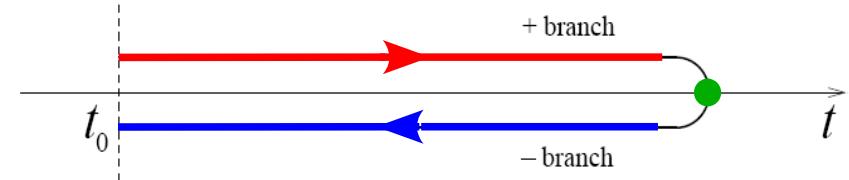
- M. Prüfer, P. Kunkel, H. Strobel, S. Lannig, D. Linnemann, C.-M. Schmied, J. Berges, TG,  
and M. K. Oberthaler,  
Nature 563, 217 (18); arXiv:[1805.11881](https://arxiv.org/abs/1805.11881) [cond-mat.quant-gas]
- C. Eigen, J. A. P. Glidden, R. Lopes, E. A. Cornell, R. P. Smith, and Z. Hadzibabic,  
Nature 563, 221 (18), arXiv:[1805.09892](https://arxiv.org/abs/1805.09892) [cond-mat.quant-gas]
- S. Erne, R. Bücker, TG, J. Berges, and J. Schmiedmayer,  
Nature 563, 225 (18); arXiv:[1805.12310](https://arxiv.org/abs/1805.12310) [cond-mat.quant-gas]
- N. Navon, C. Eigen, J. Zhang, R. Lopes, A. L. Gaunt, K. Fujimoto, M. Tsubota, R. P. Smith, and Z. Hadzibabic,  
Science 366, 382 (2019), arXiv:[1807.07564](https://arxiv.org/abs/1807.07564) [cond-mat.quant-gas]
- J. A. P. Glidden, C. Eigen, L. H. Dogra, T. A. Hilker, R. P. Smith, and Z. Hadzibabic,  
Nature Phys. 17, 457 (2021), arXiv:[2006.01118](https://arxiv.org/abs/2006.01118) [cond-mat.quant-gas]
- A. D. García-Orozco, L. Madeira, M. A. Moreno-Armijos, A. R. Fritsch, P. E. S. Tavares, P. C. M. Castilho, A. Cidrim,  
G. Roati, and V. S. Bagnato,  
arXiv:[2107.07421](https://arxiv.org/abs/2107.07421) [cond-mat.quant-gas].

M. Prüfer, et al., Nature 563, 217 (18); arXiv:[1805.11881](https://arxiv.org/abs/1805.11881) [cond-mat.quant-gas]

# Non-equilibrium Greens Functions

“Baym-Kadanoff-Schwinger-Mahanthappa-Bakshi-Keldysh formulation” (P.C. Martin, 1999)  
of real-time path integrals.

$$\langle t | O | t \rangle = \text{Tr} [ \rho(t_0) U^\dagger(t) O U(t) ] \sim$$



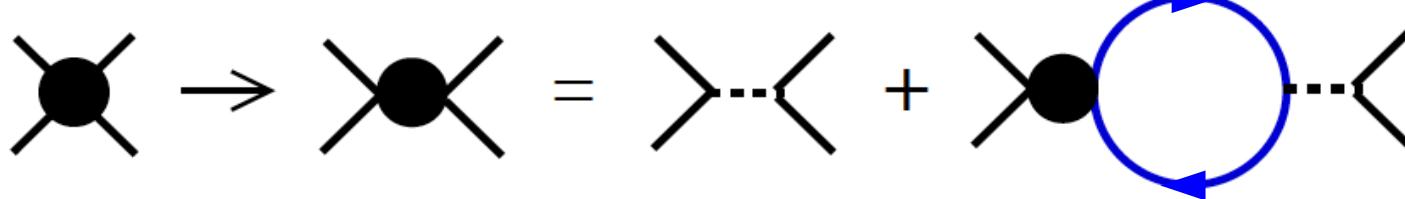
Kadanoff-Baym equation from 2PI (two-particle-irreducible) effective action

$$\partial_t n(p) = \text{---} \bullet \text{---} ,$$

$$\Sigma_{ab}(x,y) = \text{---} \bullet \text{---} \quad a \quad b$$

s-channel resummed 4-vertex (= NLO 1/N expansion / GW):

Luttinger, Ward, Kadanoff, Baym,... (60s)  
Cornwall, Jackiw, Tomboulis (70s)



J. Berges, G. Aarts, J. Serreau, ... (01-)

TG, M. Seco, M. Schmidt, J. Berges (05, 07)

J. Berges, A. Rothkopf, J. Schmidt, PRL 101 (08)

J. Berges, G. Hoffmeister, NPB 813 (09)

C. Scheppach, J. Berges, TG, PRA 81 (10)

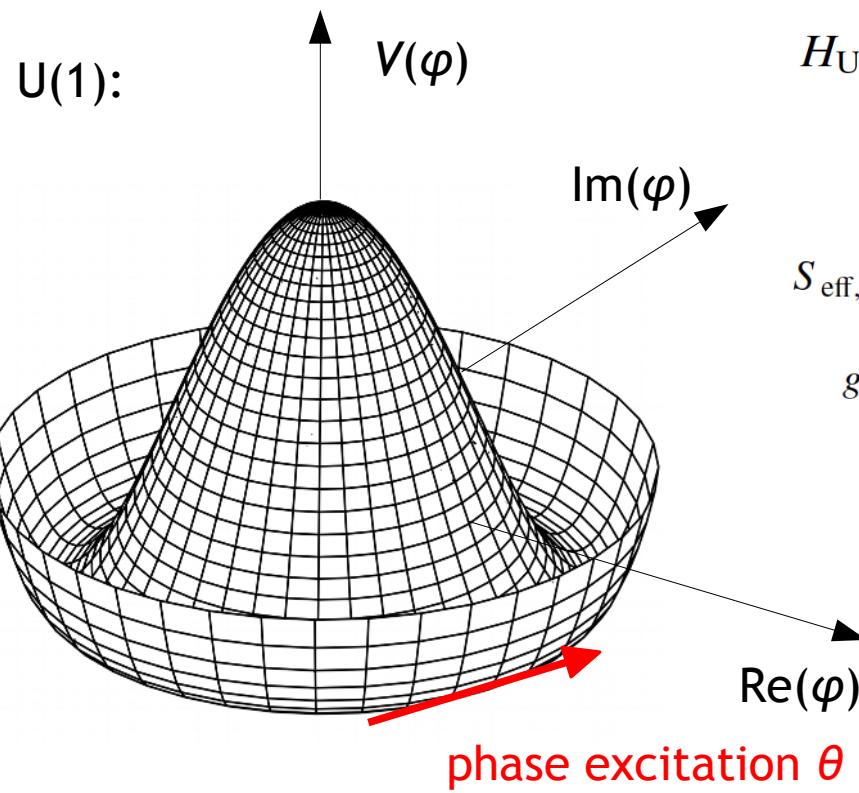
A. Piñeiro Orioli, K. Boguslavski, J. Berges, PRD 92, 025041 (15)

R. Walz, K. Boguslavski, J. Berges, PRD 97, 116011 (18)

I. Chantesana, A. Piñeiro Orioli, TG, PRA 99, 043620 (19),

A.N. Mikheev, C.M. Schmied, TG, PRA 99, 063622 (19)

# Low-energy effective theory



$$\varphi_a(\mathbf{x}, t) = \sqrt{\rho_a(\mathbf{x}, t)} \exp \{i\theta_a(\mathbf{x}, t)\}$$

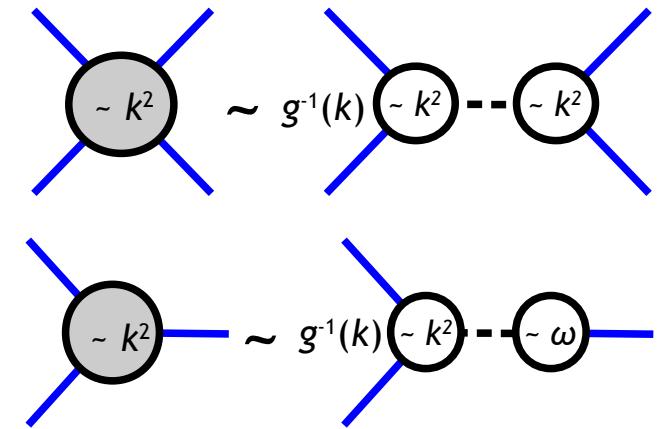
$$H_{\text{U}(N)} = \int d^d x \left[ -\Phi_a^\dagger \frac{\nabla^2}{2m} \Phi_a + \frac{g}{2} (\Phi_a^\dagger \Phi_a)^2 \right]$$

phase  
excitations

$$S_{\text{eff,G}}[\theta] = - \int_{\mathbf{k},t} \frac{1}{2g_B(\mathbf{k})} \theta_B(\mathbf{k}, t) [\partial_t^2 + \omega_B(\mathbf{k})^2] \theta_B(-\mathbf{k}, t)$$

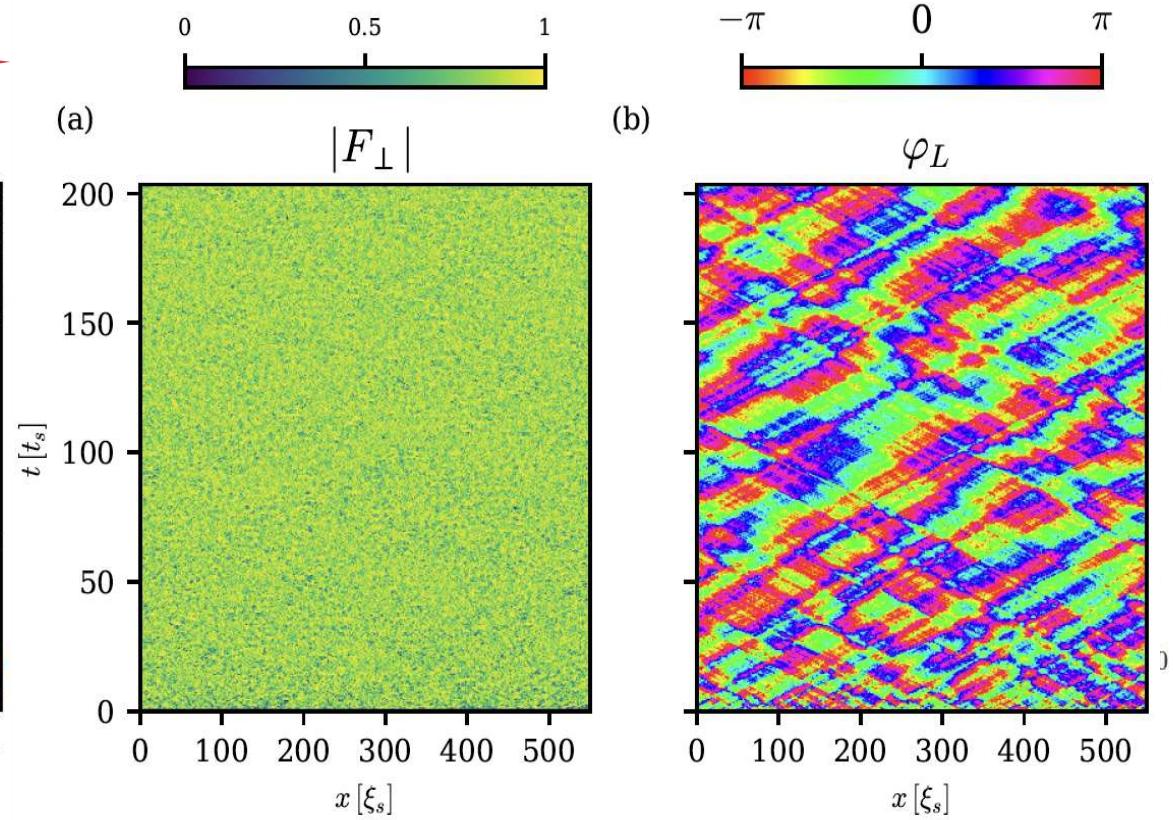
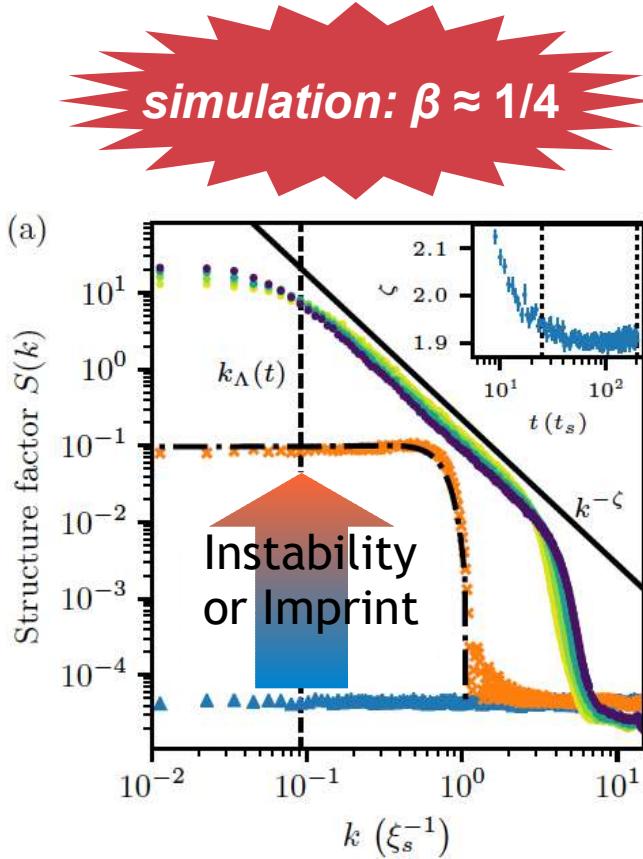
$$g_B(\mathbf{k}) = gN(1 + \mathbf{k}^2/2k_\Xi^2)$$

= Luttinger, plus non-linear interactions:



[M. Mikheev, C.M. Schmied, TG, PRA 99, 063622 (2019)]

# Back to: Pattern formation & Coarsening

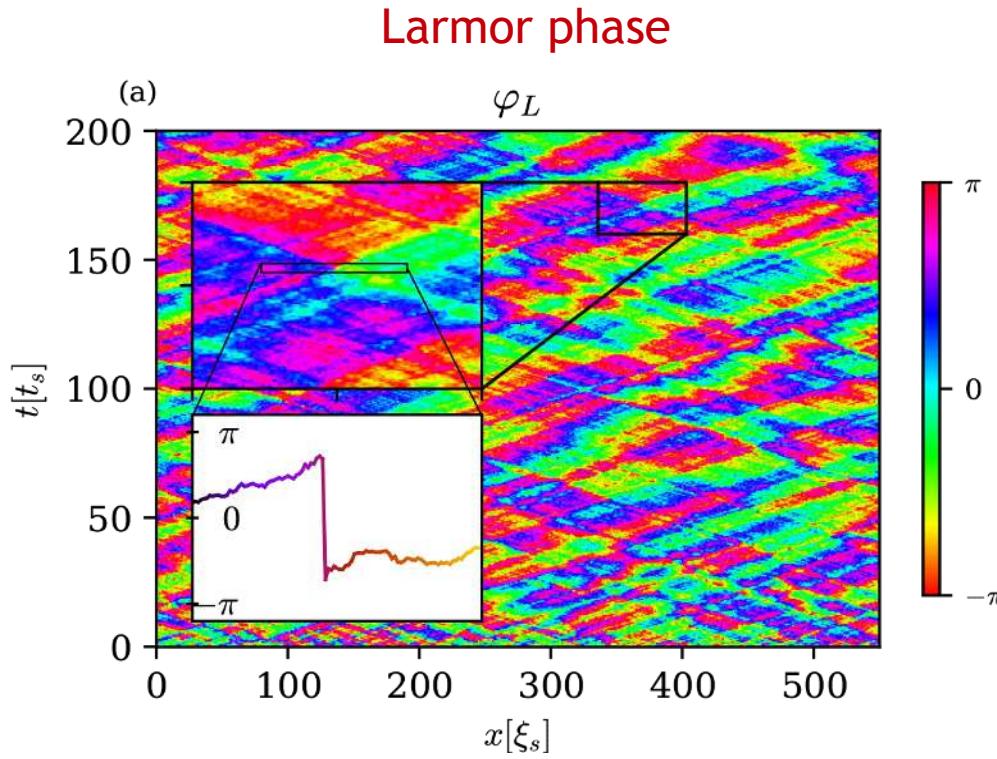


$$S(k, t) = \langle |F_{\perp}(k, t)|^2 \rangle$$

$$F_{\perp} = F_x + iF_y = |F_{\perp}|e^{i\varphi_L}$$

C.M. Schmied, M. Prüfer, M.K. Oberthaler, TG, PRA 99, 033611 (2019); arXiv:[1812.08571 \[cond-mat.quant-gas\]](https://arxiv.org/abs/1812.08571)  
I. Siovitz, C.M. Schmied, S. Lannig, H. Strobel, M.K. Oberthaler, TG et al., unpublished

# Non-linear excitations?

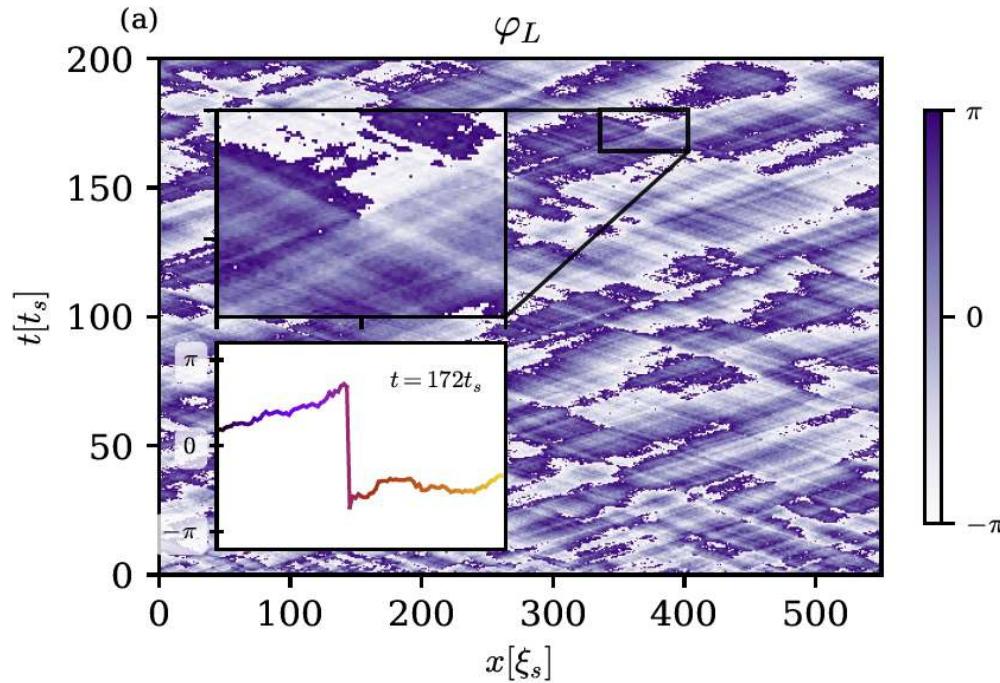


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I. Siovitz, C.M. Schmied, S. Lannig, H. Strobel, M.K. Oberthaler, TG et al., unpublished

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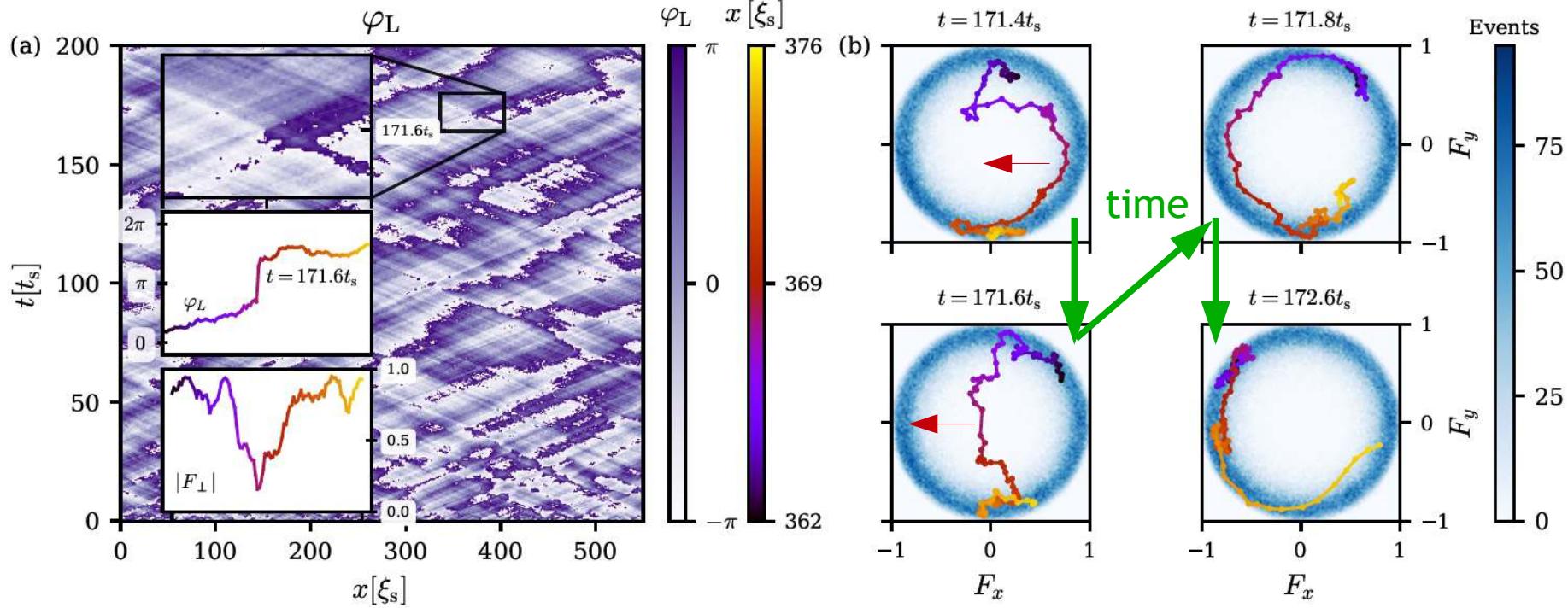
Larmor phase



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I. Siovitz, C.M. Schmied, S. Lannig, H. Strobel, M.K. Oberthaler, TG et al., unpublished

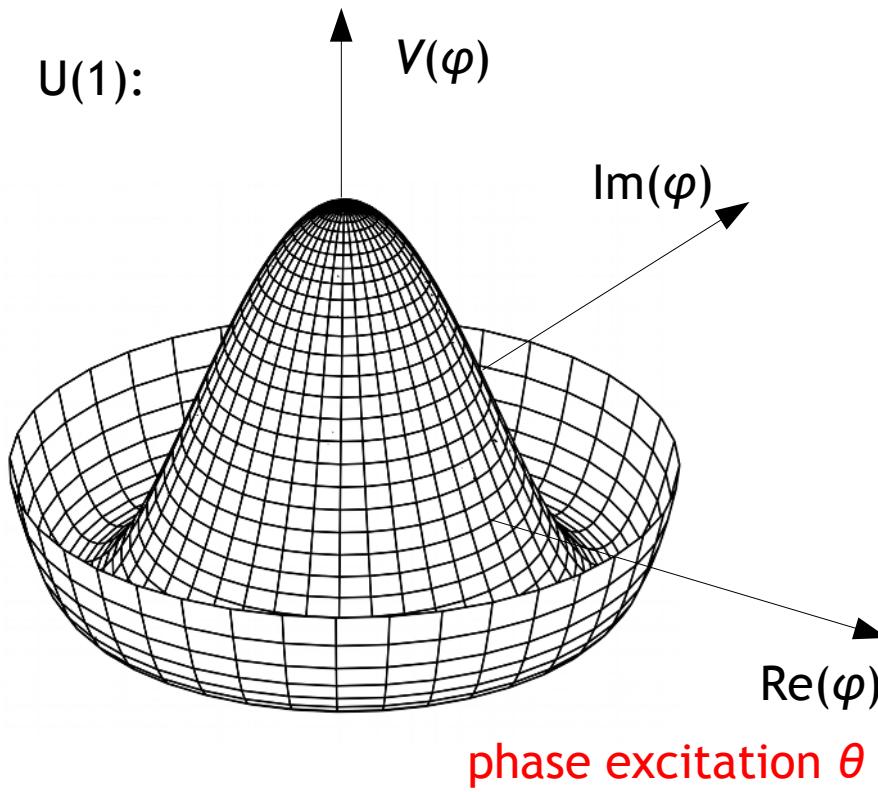
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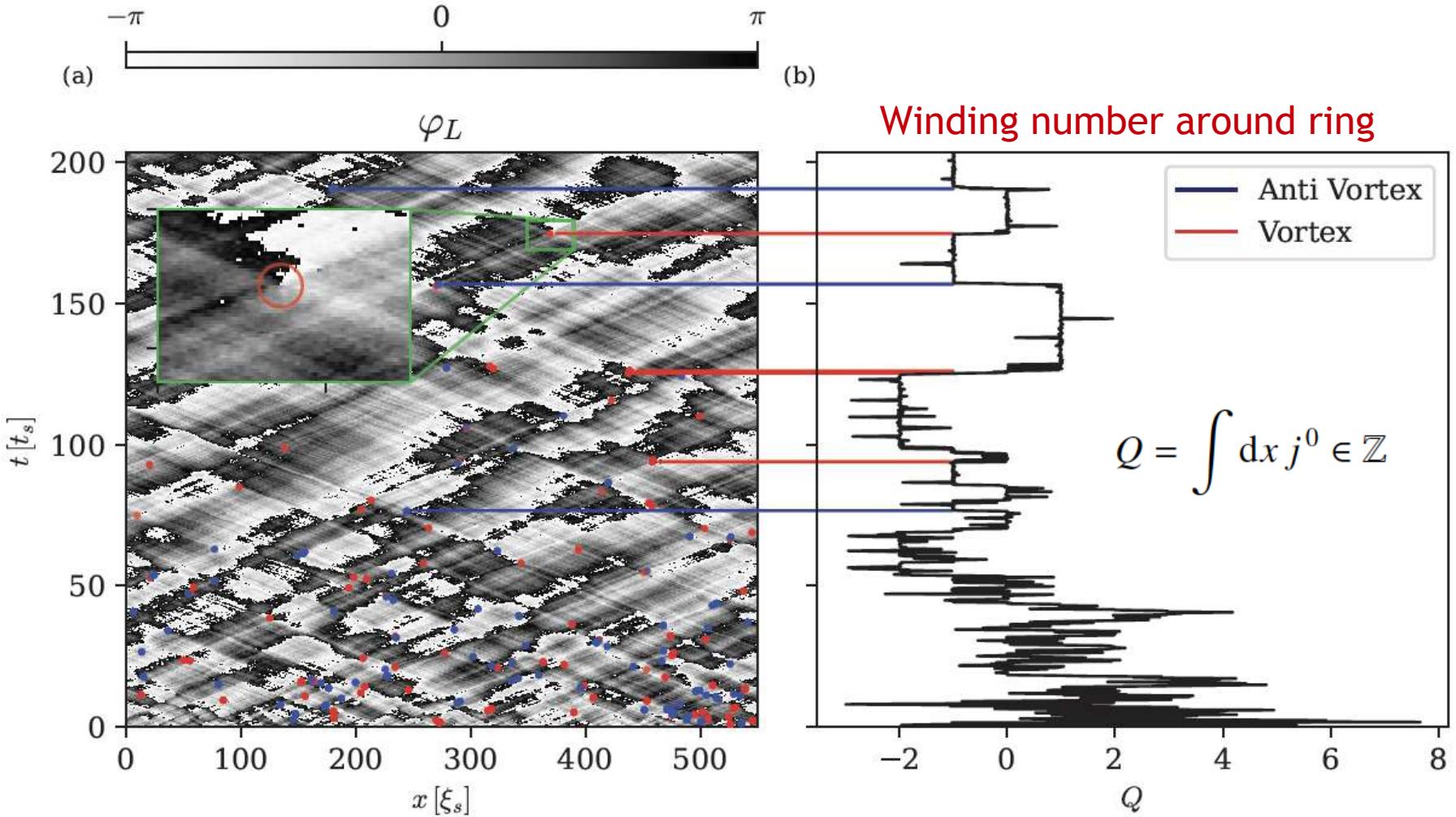
# Instantons



$$\varphi_a(\mathbf{x}, t) = \sqrt{\rho_a(\mathbf{x}, t)} \exp \{i\theta_a(\mathbf{x}, t)\}$$



# Instanton distribution & dilution



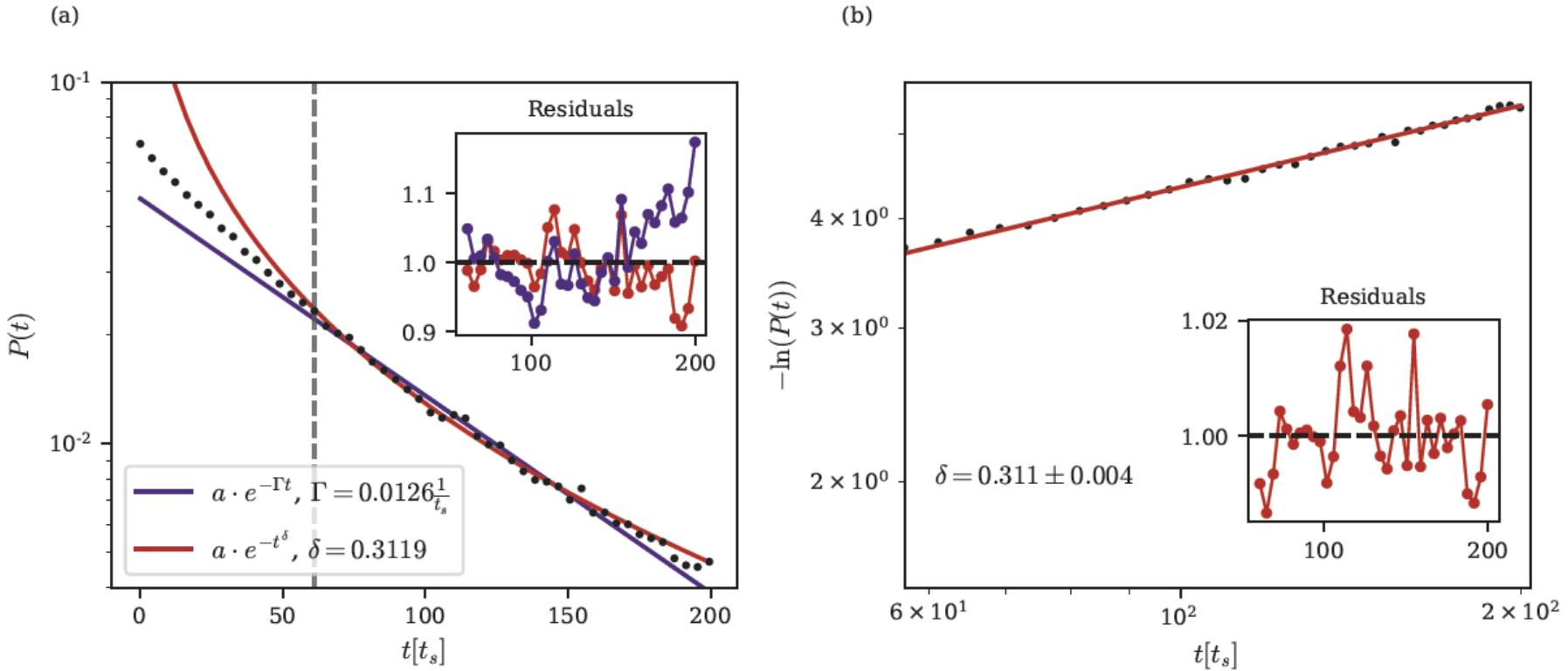
$$F_{\perp} = F_x + iF_y = |F_{\perp}|e^{i\varphi_L}$$

$$j^0 = \partial^x \varphi_L, \quad j^1 = \partial^t \varphi_L$$

I. Siovitz, C.M. Schmied, S. Lannig, H. Strobel, M.K. Oberthaler, TG et al., unpublished

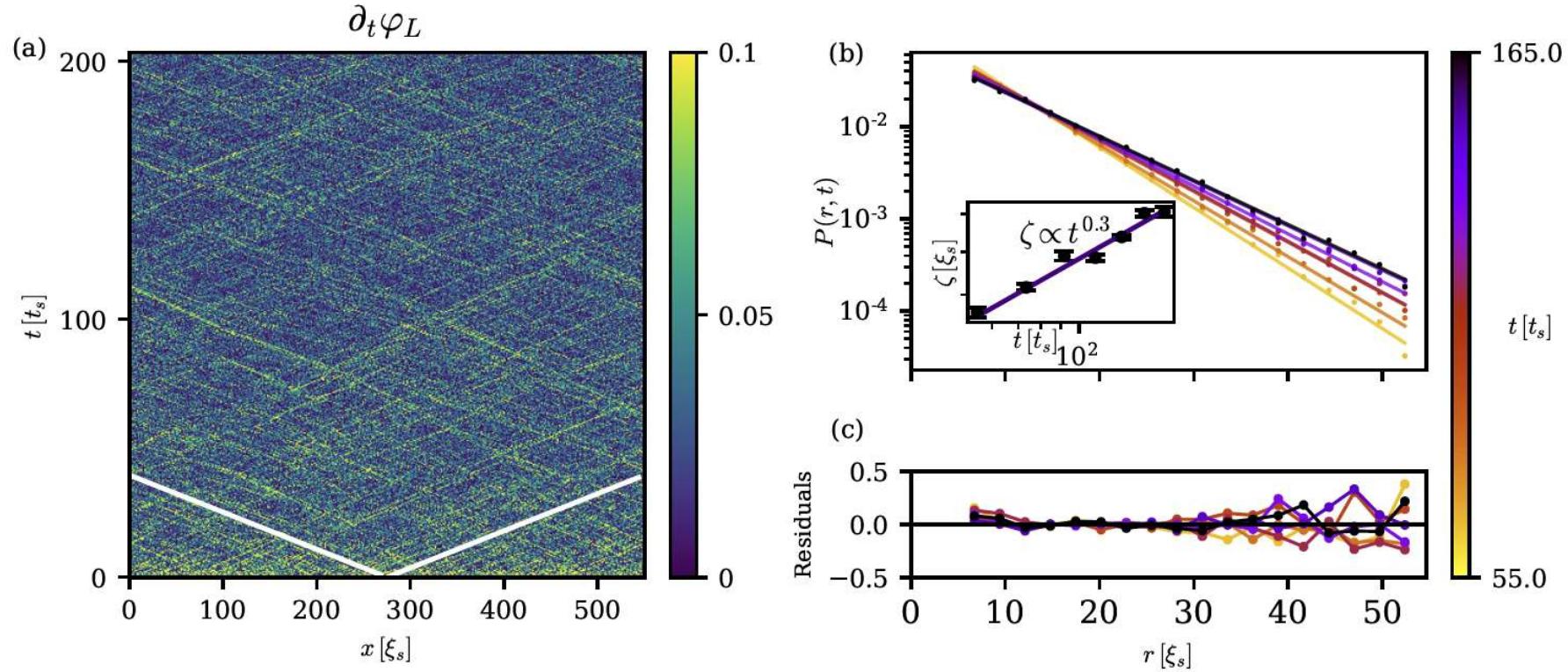
# Universal dynamics in a 1D Spin-1 Bose gas ( $\delta \approx 0.3$ )

Temporal distribution  $P(t) \sim \exp[-(t/t_{\text{ref}})^{\delta}]$  of *instanton* events



# Universal dynamics in a 1D Spin-1 Bose gas ( $\delta \approx 0.3$ )

Spatio-temporal distribution  $P(r, t) \sim \exp[-r/\zeta(t)]$  of *instantons*;  $\zeta(t) = \zeta_0 \cdot (t/t_{\text{ref}})^{\delta}$



# Non-thermal fixed points in other systems

Video: Approach of a non-thermal fixed point in a 1-component 2D gas

<https://www.kip.uni-heidelberg.de/gasenzer/projects/anomalousntp>

## Reviews, Lecture notes, Summary articles, & Recent progress:

**Vortex motion quantifies strong dissipation in a holographic superfluid**

P. Wittmer, C.-M. Schmied, T. Gasenzer, C. Ewerz,

Phys. Rev. Lett. 127, 101601 (2021); arXiv:[2011.12968 \[hep-th\]](https://arxiv.org/abs/2011.12968)

**Violation of single-length scaling dynamics via spin vortices in an isolated spin-1 Bose gas**

C.-M. Schmied, T. Gasenzer, P. B. Blakie

Phys. Rev. A 100, 033603 (2019); arXiv:[1904.13222 \[cond-mat.quant-gas\]](https://arxiv.org/abs/1904.13222)

**Non-thermal fixed points: Universal dynamics far from equilibrium**

C.-M. Schmied, A. N. Mikheev, TG,

in Proc. Julian Schwinger Centennial Conf. and Workshop, Singapore, 7-12 Feb 2018.

IJMPA 34, 1941006 (2019); arXiv:[1810.08143 \[cond-mat.quant-gas\]](https://arxiv.org/abs/1810.08143)

**Low-energy effective theory of non-thermal fixed points in a multicomponent Bose gas**

A. N. Mikheev, C.-M. Schmied, TG,

Phys. Rev. A 99, 063622 (2019); arXiv:[1807.10228 \[cond-mat.quant-gas\]](https://arxiv.org/abs/1807.10228)

**Prescaling in a far-from-equilibrium Bose gas**

C.-M. Schmied, A. N. Mikheev, TG,

Phys. Rev. Lett. 122: 170404 (2019); arXiv:[1807.07514 \[cond-mat.quant-gas\]](https://arxiv.org/abs/1807.07514)

**Kinetic theory of non-thermal fixed points in a Bose gas**

I. Chantesana, A. Piñeiro Orioli, TG,

Phys. Rev. A 99, 043620 (2019); arXiv:[1801.09490 \[cond-mat.quant-gas\]](https://arxiv.org/abs/1801.09490)

**Prethermalization and universal dynamics in near-integrable quantum systems**

T. Langen, TG, J. Schmiedmayer,

JSTAT 064009, 2016; arXiv:[1603.09385 \[cond-mat.quant-gas\]](https://arxiv.org/abs/1603.09385)

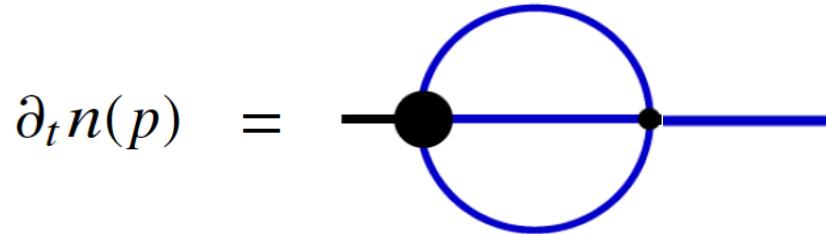
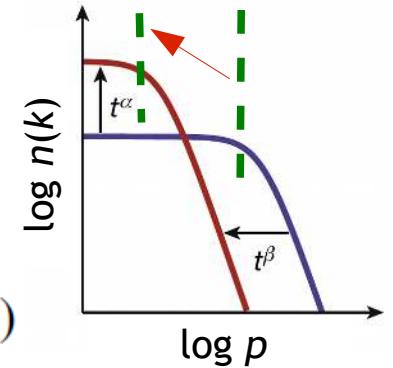
*The End*

# Scaling analysis of Boltzmann transport

Radial transport equation as fixed-point equation:

$$\partial_t n_Q(\mathbf{p}, t) = I[n_Q](\mathbf{p}, t)$$

$$I[n_Q](\mathbf{p}, t) = \int_{\mathbf{kqr}} |T_{\mathbf{pkqr}}|^2 \delta(\mathbf{p} + \mathbf{k} - \mathbf{q} - \mathbf{r}) \delta(\omega_{\mathbf{p}} + \omega_{\mathbf{k}} - \omega_{\mathbf{q}} - \omega_{\mathbf{r}}) \\ \times [(n_{\mathbf{p}} + n_{\mathbf{k}})n_{\mathbf{q}}n_{\mathbf{r}} - n_{\mathbf{p}}n_{\mathbf{k}}(n_{\mathbf{q}} + n_{\mathbf{r}})]$$



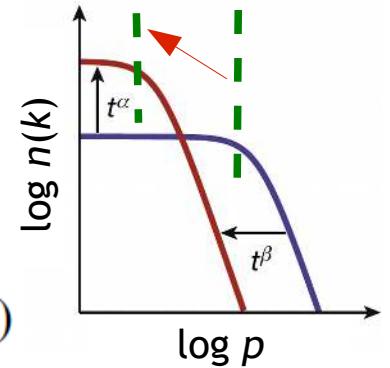
A. Piñeiro Orioli, K. Boguslavski, J. Berges, PRD 92, 025041 (2015)  
I. Chantesana, A. Piñeiro Orioli, TG, PRA 99, 043620 (2019)  
A.N. Mikheev, C.M. Schmied, TG, PRA 99, 063622 (2019)

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$$n_{\mathbf{p}} \equiv n_Q(\mathbf{p}, t) = (t/t_0)^{\alpha} f_s([t/t_0]^{\beta} p) \quad f_s(p) \sim p^{-\zeta}$$

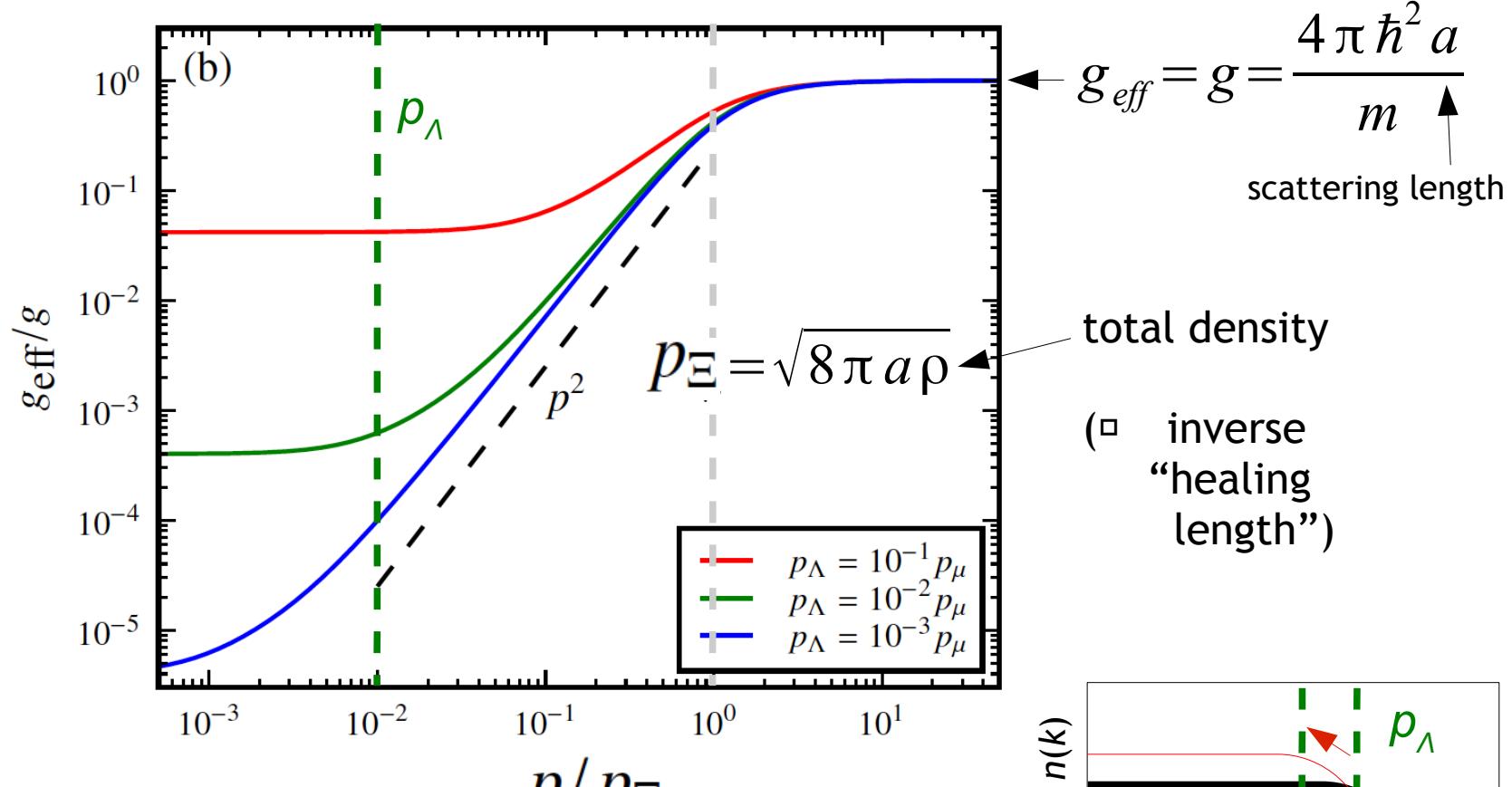
$$t^{\alpha-1} \sim t^{-\beta(3d+2m-d-z)+3\alpha}$$

- A. Piñeiro Orioli, K. Boguslavski, J. Berges, PRD 92, 025041 (2015)  
I. Chantesana, A. Piñeiro Orioli, TG, PRA 99, 043620 (2019)  
A.N. Mikheev, C.M. Schmied, TG, PRA 99, 063622 (2019)

# Universal coupling function

$$|T_{\mathbf{p}\mathbf{k}\mathbf{q}\mathbf{r}}|^2 = (2\pi)^4 g_{\text{eff}}^2(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{r}}, \mathbf{p} - \mathbf{r})$$

$$g_{\text{eff}}(p) = \frac{g}{|1 + g \Pi^R(p)|}$$



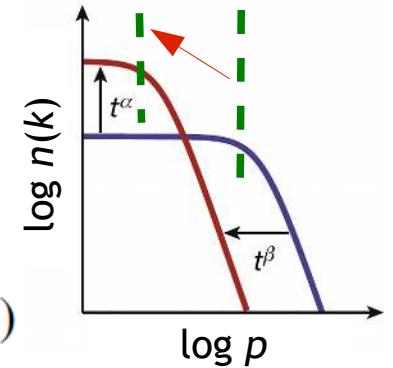
I. Chantesana, A. Piñeiro Orioli, TG, PRA 99, 043620 (2019)

# Scaling analysis of Boltzmann transport

Radial transport equation as fixed-point equation:

$$\partial_t n_Q(\mathbf{p}, t) = I[n_Q](\mathbf{p}, t)$$

$$I[n_Q](\mathbf{p}, t) = \int_{\mathbf{kqr}} |T_{\mathbf{pkqr}}|^2 \delta(\mathbf{p} + \mathbf{k} - \mathbf{q} - \mathbf{r}) \delta(\omega_{\mathbf{p}} + \omega_{\mathbf{k}} - \omega_{\mathbf{q}} - \omega_{\mathbf{r}}) \\ \times [(n_{\mathbf{p}} + n_{\mathbf{k}})n_{\mathbf{q}}n_{\mathbf{r}} - n_{\mathbf{p}}n_{\mathbf{k}}(n_{\mathbf{q}} + n_{\mathbf{r}})]$$



$$n_{\mathbf{p}} \equiv n_Q(\mathbf{p}, t) = (t/t_0)^{\alpha} f_s([t/t_0]^{\beta} p) \quad f_s(p) \sim p^{-\zeta}$$

$$t^{\alpha-1} \sim t^{-\beta(3d+2m-d-z)+3\alpha}$$

$$\alpha = \beta[d+m-z/2] - 1/2 \quad \alpha = d\beta \quad \zeta_S = d+z/2$$

fixes, together with relation from conservation laws,  $\alpha$ ,  $\beta$ , &  $\zeta$

- A. Piñeiro Orioli, K. Boguslavski, J. Berges, PRD 92, 025041 (2015)
- I. Chantesana, A. Piñeiro Orioli, TG, PRA 99, 043620 (2019)
- A.N. Mikheev, C.M. Schmied, TG, PRA 99, 063622 (2019)