Universal induced interaction between heavy polarons in superfluid

— Effective field theory approach to polaron physics —

KF, M. Hongo, & T. Enss, arXiv:2206.01048 (2022)

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on Recent Progress in Many-Body Theories XXI
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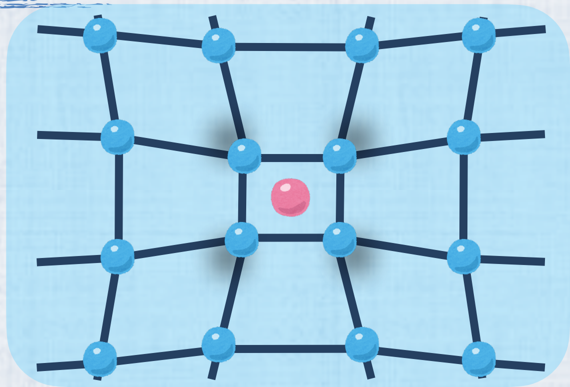
Plan of this talk

1. **Introduction of the polaron**
    - Polaron in ultracold atoms
    - Induced interaction between polarons
2. **Induced interaction between polarons**
    - Theoretical formulation of polaron physics
    - Yukawa potential
    - EFT approach : focusing on linear dispersion phonons
3. **Magnitude of the potential in the BCS-BEC crossover**
4. **Summary**

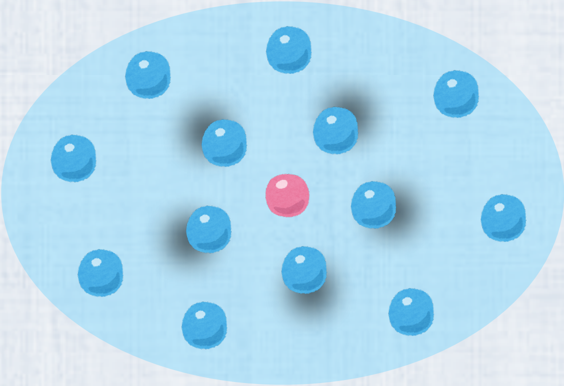
What is the polaron?



Polaron (Landau's original definition)

: an electron interacting with phonons in a crystal

lattice wave inducing **polarization**



Polaron in ultracold atoms

: an impurity interacting with quantum gas particles

▶ Ultracold atoms provide a simple and ideal research platform.

✓ **High experimental controllability**

- quantum statistics & internal degrees of freedom
- impurity-medium and medium-medium interaction

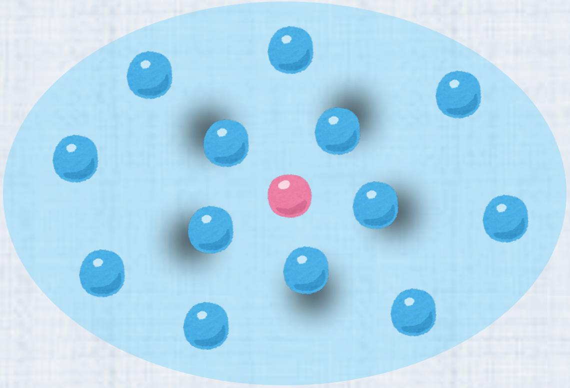
From one-body to two-body

Focusing on impurities immersed in a superfluid

In many cases, impurities in a Bose medium, called Bose polarons

One impurity problem

: effective mass, mobility, dressing cloud, etc.



Two impurity problem

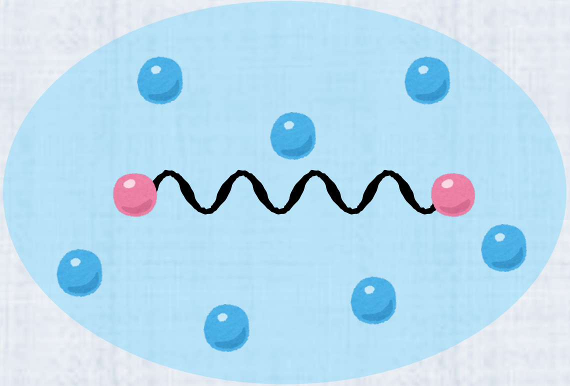
: induced interaction, bipolaron state, etc.

Interaction mediated by exchanging bosonic quanta

✓ **Crucial in modern physics**

e.g.

- the fundamental interaction by gauge bosons
- the nuclear force by pions
- an attractive electron-electron interaction by lattice phonons for superconductivity



Induced Interaction between impurities

Focusing on impurities immersed in a superfluid

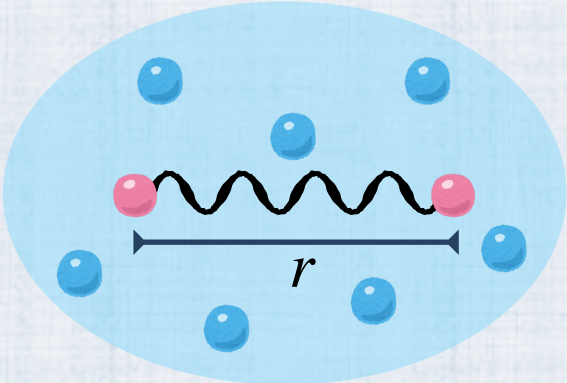
In many cases, impurities in a Bose medium, called Bose polarons

Two impurity problem

: induced interaction, bipolaron state, etc.

Interaction mediated by exchanging bosonic quanta

Superfluid phonons



✓ **The Yukawa potential** at weak impurity-medium interaction when the medium is a weakly interacting Bose gas

$$V(r) \sim -\frac{e^{-\sqrt{2}r/\xi}}{r}$$

(ξ : healing length)

See e.g. Pethick & Smith's text book "Bose-Einstein condensation in Dilute gases"

- Short-range potential mediated by a **gapped** mode at first glance
- There is a **gapless** mode (superfluid phonon) governing long-range physics

Why does a gapped mode appear?

Is there a long-range induced interaction mediated by gapless modes?

Plan of this talk

1. Introduction of the polaron

Why does a gapped mode appear?

Is there a long-range induced interaction mediated by gapless modes?

2. **Induced interaction between polarons**

    - Theoretical formulation of polaron physics
    - Yukawa potential
    - EFT approach : focusing on linear dispersion phonons

3. Magnitude of the potential in the BCS-BEC crossover

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Theoretical formulation of polaron physics

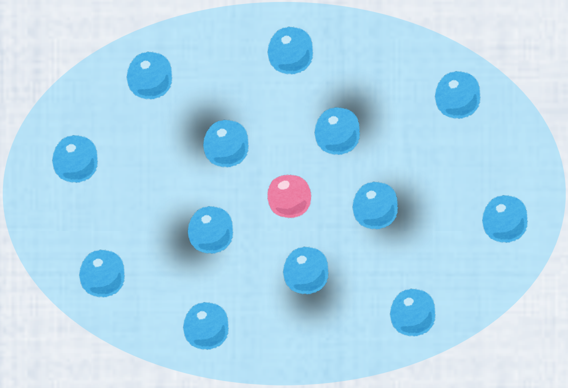
✓ **Microscopic model :**

Impurities interacting with a medium

$$\mathcal{L}_{\rm micro}(x) = \mathcal{L}_{\rm imp}(x) + \mathcal{L}_{\rm medium}(x) + \mathcal{L}_{\rm int}(x)$$

▸ Impurity-medium interaction in the contact *s*-wave channel

$$\mathcal{L}_{\rm int}(x) = -g_{IM}\underbrace{\Phi^\dagger(x)\Phi(x)}_{\text{Impurity density}}\underbrace{\psi^\dagger(x)\psi(x)}_{\text{Medium density}}$$



✓ **Our problem is to find** $S_{\rm polaron}[\Phi, \Phi^\dagger]$ by integrating out the medium

$$\exp\left[iS_{\rm polaron}[\Phi, \Phi^\dagger]\right] = \int \mathcal{D}(\psi, \psi^\dagger)\ \exp\left[i\int dt d^3x\, \mathcal{L}_{\rm micro}(x)\right]$$

▸ Formally simple, but difficult to perform the integration

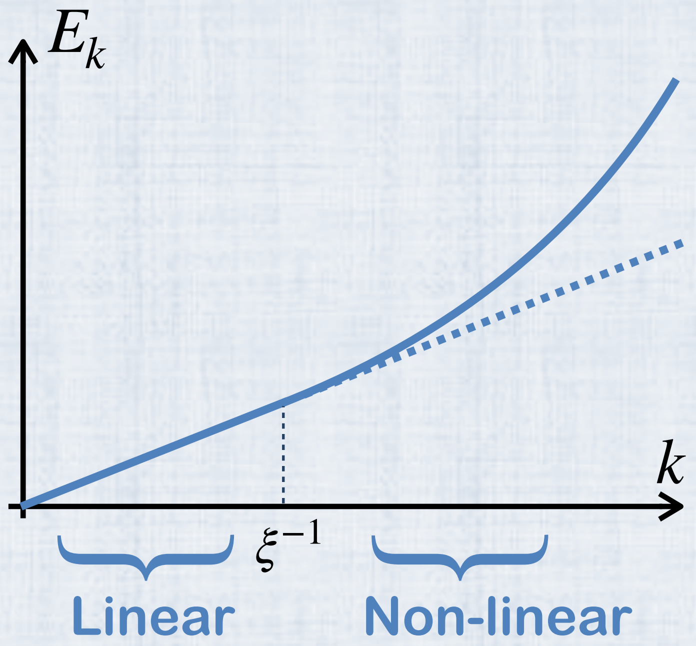
Usual method: Bogoliubov approximation

✓ Bogoliubov approximation for the medium

Medium = weakly interacting Bose gas

$$\int d^3x\, \mathcal{L}_{\mathrm{medium}}(x) \simeq \sum_{\boldsymbol{k}} \left[i b^\dagger_{\boldsymbol{k}}(t)\partial_t b_{\boldsymbol{k}}(t) - E_{\boldsymbol{k}} b^\dagger_{\boldsymbol{k}}(t) b_{\boldsymbol{k}}(t) \right]$$

Bogoliubov dispersion $E_{\boldsymbol{k}} = \sqrt{\epsilon_{\boldsymbol{k}}(\epsilon_{\boldsymbol{k}} + 2\mu)}$ $\quad \epsilon_{\boldsymbol{k}} = \dfrac{\boldsymbol{k}^2}{2m}$



- The interaction term is rewritten in terms of the Bogoliubov basis

- **The Yukawa potential is derived from the nonlinear part**

$$V(r) = \int \frac{d^3k}{(2\pi)^3} \tilde{V}(k) e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \sim -\frac{e^{-\sqrt{2}r/\xi}}{r} \quad \text{with} \quad \tilde{V}(k) \sim -g_{IM}^2 \frac{n}{V} \frac{1}{\epsilon_{\boldsymbol{k}} + 2\mu}$$

  - like a **gapped-mode** propagator
  - Only the non-linear part survives

$$E_{\boldsymbol{k}} = \sqrt{\epsilon_{\boldsymbol{k}}(\epsilon_{\boldsymbol{k}} + 2\mu)}$$

Plan of this talk

Why does a gapped mode appear?

→ The non-linear dispersion part only survives
and behaves like a gapped propagator

Is there a long-range induced interaction mediated by gapless modes?

2. Induced interaction between polarons

- Theoretical formulation of polaron physics
- Yukawa potential
- EFT approach : focusing on linear dispersion phonons

3. Magnitude of the potential in the BCS-BEC crossover

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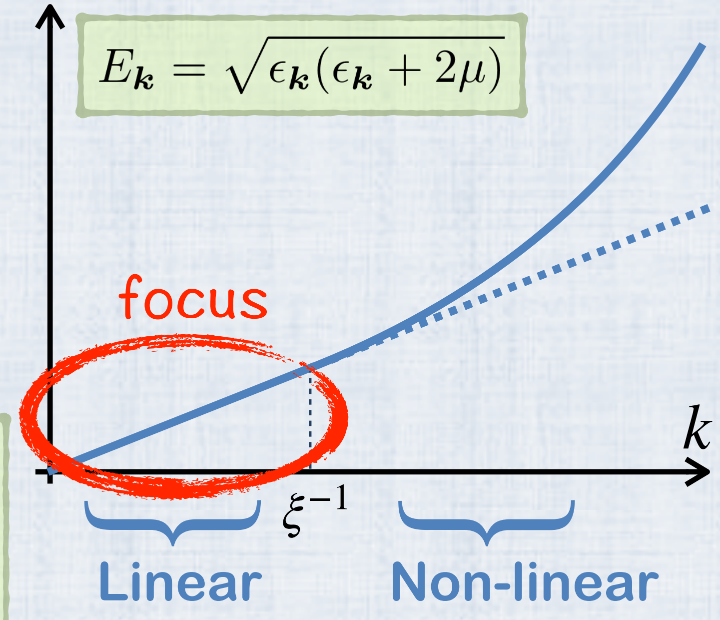
Effective field theory method: Superfluid EFT

Focus only on the linear dispersion regime

DoF: phonon field $\phi(x)$, showing a linear dispersion



Due to **the Galilean invariance** of the medium,
the Lagrangian is **generally** given in

✓ Galilean superfluid EFT for the medium

$$\mathcal{L}_{\mathrm{medium}}(x) = \mathcal{P}(\theta(x))$$

$\mathcal{P}(\mu)$: Pressure as a function of μ

Galilean invariant combination $\theta(x) = \mu - \partial_t\phi(x) - \dfrac{(\nabla\phi(x))^2}{2m}$

M. Greiter, F. Wilczek, & E. Witten, Mod. Phys. Lett. B **3**, 903 (1989);
D. T. Son & M. Wingate, Ann. Phys. **321**, 197 (2006).

✓ Interaction term

$$\mathcal{L}_{\mathrm{int}}(x) = -g_{IM}\Phi^\dagger(x)\Phi(x)n(\theta(x))$$

with $n(\mu) = \mathcal{P}'(\mu)$

cf.

$$\mathcal{L}_{\mathrm{int}}(x) = -g_{IM}\Phi^\dagger(x)\Phi(x)\psi^\dagger(x)\psi(x)$$

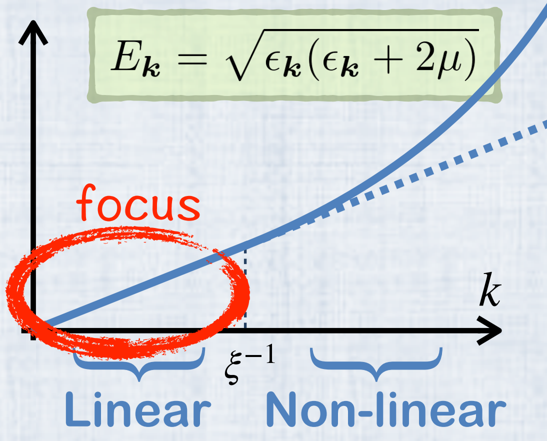
Medium density

Effective theory for impurities in a superfluid

✓ Our effective theory

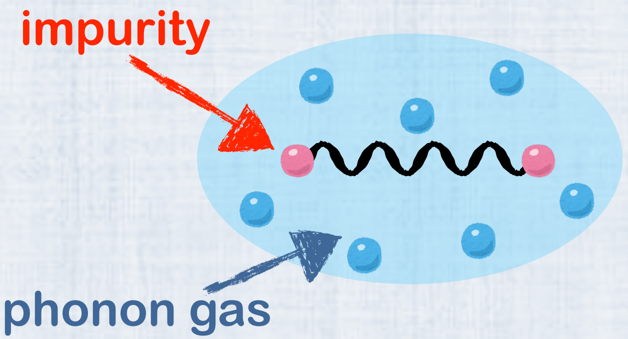
$$\mathcal{L}_{\rm eff}(x) = \mathcal{L}_{\rm imp}(x) + \mathcal{P}(\theta(x)) - g_{IM}\Phi^\dagger(x)\Phi(x)n(\theta(x))$$

▸ **Galilean invariant combination** $\theta(x) = \mu - \partial_t\phi(x) - \dfrac{(\nabla\phi(x))^2}{2m}$



▸ Our assumptions are only two:

- **Galilean invariant medium**
- **Contact s-wave impurity-medium coupling**

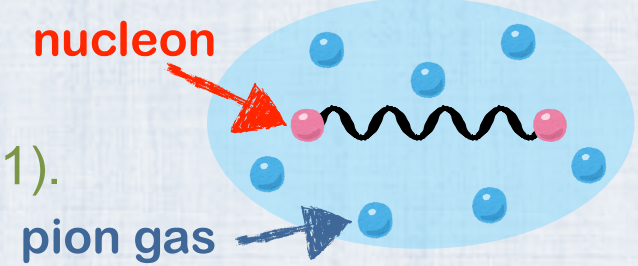


➡ Universal !! : **Independent of the details of the medium**

Our remaining task is **to calculate induced interactions from our effective theory**

cf. nuclear forces are computed from chiral effective field theory

See e.g., R. Machleidt & D. R. Entem,
"Chiral effective field theory and nuclear forces," Phys. Rept. **503**, 1 (2011).



Induced interaction mediated by phonons

Expanding $\mathcal{P}(\theta)$ & $n(\theta)$ and keeping the leading terms with rescaling $\varphi = \sqrt{\chi}\phi$

$$\mathcal{L}(x) = \mathcal{L}_{\mathrm{imp}}(x) - g_{IM} n \Phi^\dagger \Phi + \underbrace{\frac{1}{2}(\partial_t \varphi)^2 - \frac{1}{2} c_s^2 (\nabla \varphi)^2} + \underbrace{g_{IM}\left[\sqrt{\chi}\partial_t \varphi + \frac{(\nabla \varphi)^2}{2m}\right]\Phi^\dagger \Phi} + \cdots$$

$\chi = n'(\mu)$: compressibility

$c_s = \sqrt{n/(m\chi)}$: speed of sound

Kinetic term for phonons showing the linear dispersion

✓ Interaction terms between impurities and phonons

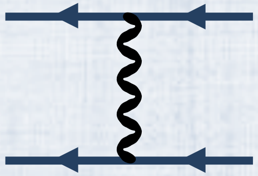
$g_{IM}\sqrt{\chi}\partial_t \varphi \Phi^\dagger \Phi$: one-body coupling

$g_{IM}\dfrac{(\nabla \varphi)^2}{2m}\Phi^\dagger \Phi$: two-body coupling

- **The coefficients are constrained by the Galilean invariance**

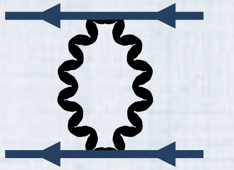
  ⟷ The Bogoliubov approx. breaks the Galilean invariance.

- One-body coupling ➡ one-phonon exchange $\tilde{V}(k) \sim$



$$V(r) = \int \frac{d^3k}{(2\pi)^3} \tilde{V}(k) e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$

- Two-body coupling ➡ two-phonon exchange $\tilde{V}(k) \sim$



One-phonon exchange potential

Expanding $\mathcal{P}(\theta)$ & $n(\theta)$ and keeping the leading terms with rescaling $\varphi = \sqrt{\chi}\phi$

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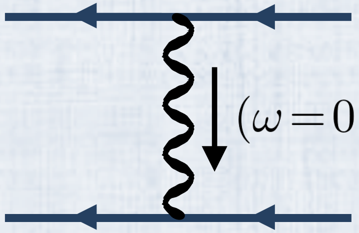
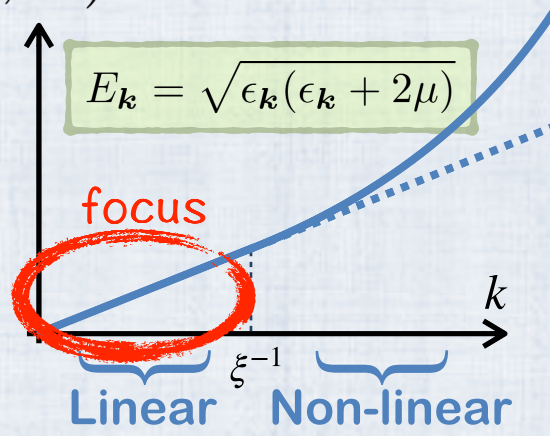
Kinetic term for phonons showing the linear dispersion

✔ **Static potential = exchanging purely spatial modes with** $(\omega = 0,\ \boldsymbol{k})$

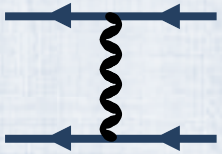
$\tilde{V}(k) \sim$  $(\omega = 0,\ \boldsymbol{k})$ $\sim \left(g_{IM}\sqrt{\chi}\,\omega\right)^2 \Delta(\omega, \boldsymbol{k}) \ \underline{= 0}$ at $\omega = 0$



▸ **Consistent with the previous result**

: The Yukawa potential effectively vanishes at $\underline{r \gg \xi}$

cf. One-Bogoliubov mode exchange has NO contribution from the linear dispersion part

$\tilde{V}(k) \sim$  $\sim -g_{IM}^2 \dfrac{n}{V} \dfrac{1}{\epsilon_{\boldsymbol{k}} + 2\mu}$

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Expanding $\mathcal{P}(\theta)$ & $n(\theta)$ and keeping the leading terms with rescaling $\varphi = \sqrt{\chi}\phi$

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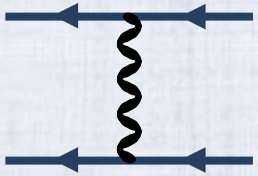
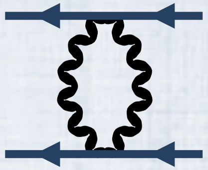
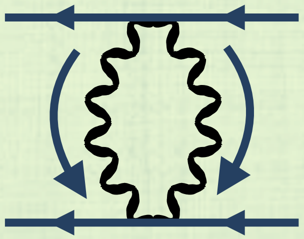
- One-body coupling ➡ one-phonon exchange $\tilde{V}(k) \sim$  = 0
- Two-body coupling ➡ two-phonon exchange $\tilde{V}(k) \sim$  = finite

Van der Waals force from two-phonon exchange

✓ Two-phonon exchange potential from $g_{IM}\dfrac{(\nabla\varphi)^2}{2m}\Phi^\dagger\Phi$



At zero temperature

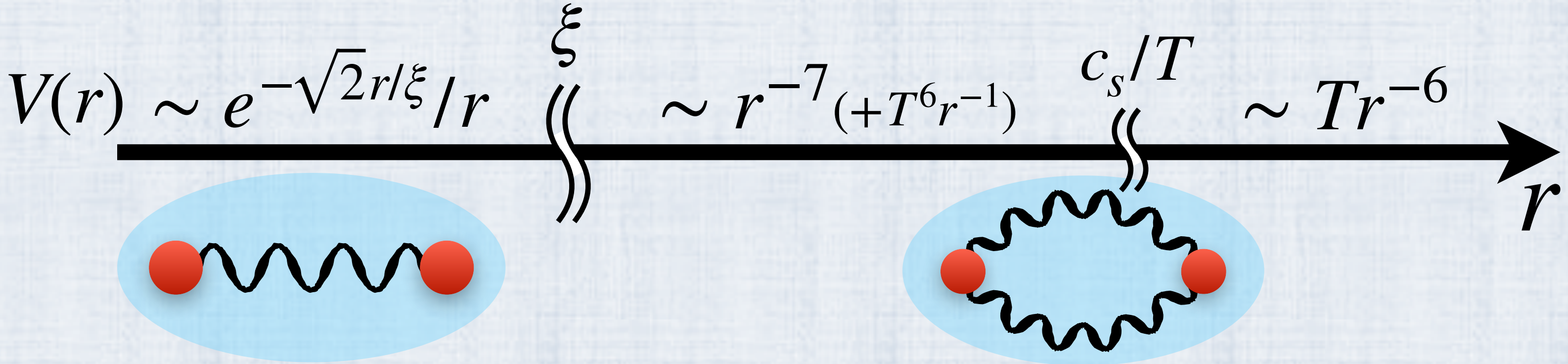
$$V_{T=0}(r) = -g_{IM}^2\frac{43}{128\pi^3 m^2 c_s^3}\frac{1}{r^7}$$

relativistic van der Waals

At finite temperatures (c_s/T : temperature length scale)

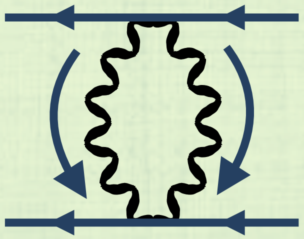
$$V(r) = \begin{cases} V_{T=0}(r) - g_{IM}^2\dfrac{\pi^3 T^6}{135 m^2 c_s^9}\dfrac{1}{r} & (r \ll c_s/T) \\ -g_{IM}^2\dfrac{3T}{16\pi^2 m^2 c_s^4}\dfrac{1}{r^6} & (r \gg c_s/T) \end{cases}$$

non-relativistic van der Waals



Van der Waals force from two-phonon exchange

✓ **Two-phonon exchange potential** from $g_{IM}\frac{(\boldsymbol{\nabla}\varphi)^2}{2m}\Phi^\dagger\Phi$



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non-relativistic van der Waals

- proportional to g_{IM}^2, as in the Yukawa potential
- power-low behavior: stronger than the Yukawa potential at large distance
- the sound velocity controls the magnitude of the potentials
- **the phonon-induced Casimir interaction**, as an analogy to the usual Casimir effect

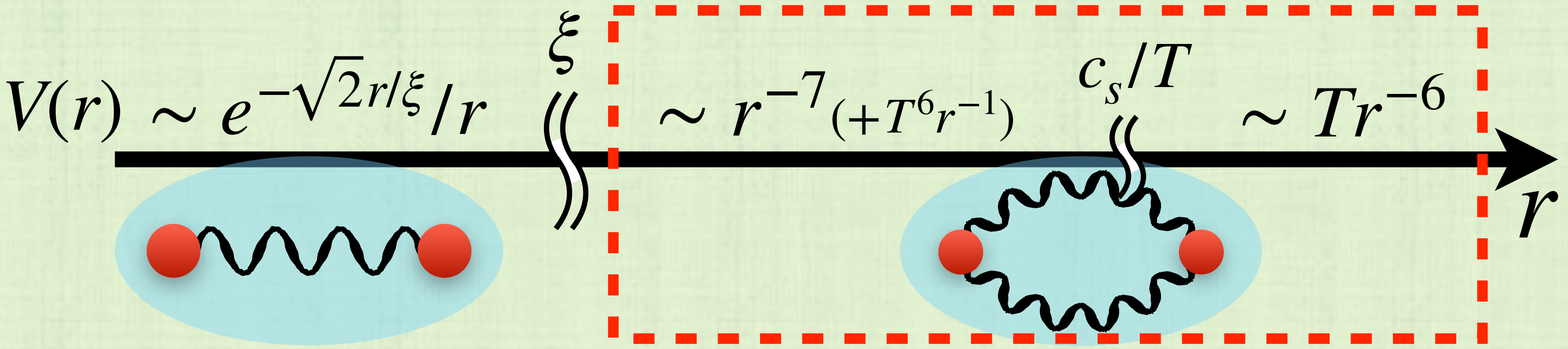
Plan of this talk

Why does a gapped mode appear?

➡ The non-linear dispersion part only survives and behaves like a gapped propagator

Is there a long-range induced interaction mediated by gapless modes?

➡ Yes, **Two-phonon exchange potential**



3. Magnitude of the potential in the BCS-BEC crossover

4. Summary

Induced potential in BCS-BEC crossover

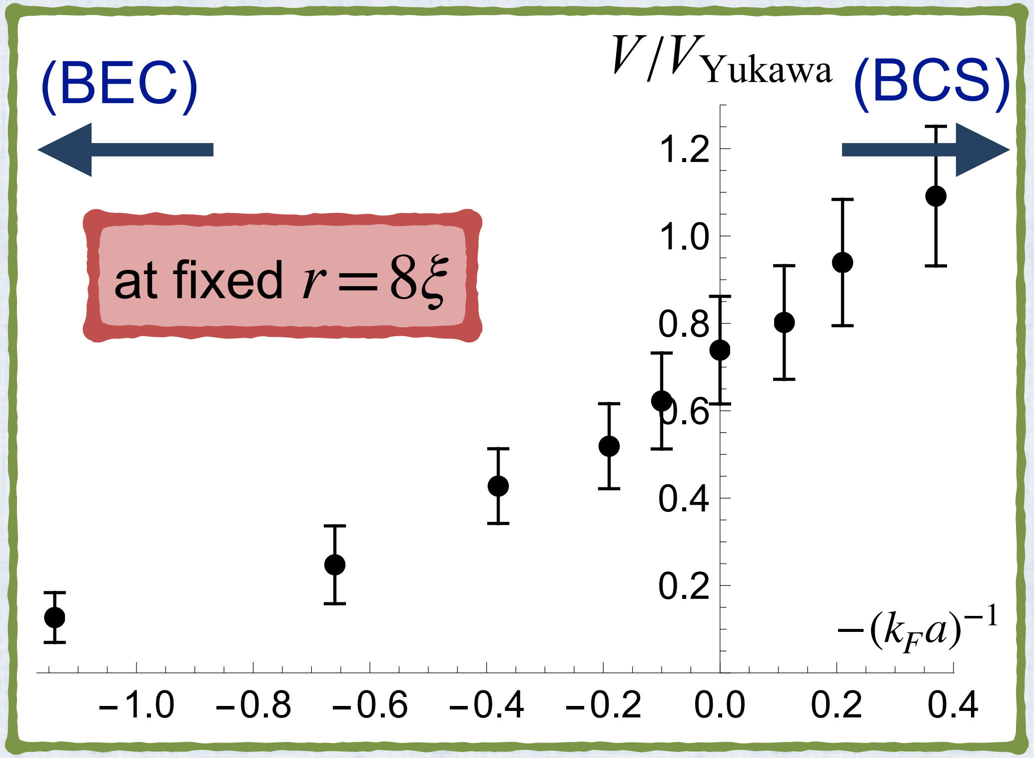
Our results are based on only two assumptions

- **Galilean invariant medium**
- **Contact s-wave impurity-medium coupling**

➡ Our results are valid in the entire BCS-BEC crossover

✓ Plotting the ratio
of the potential $V_{T=0}(r)$ to the Yukawa potential
with the use of the experimental data

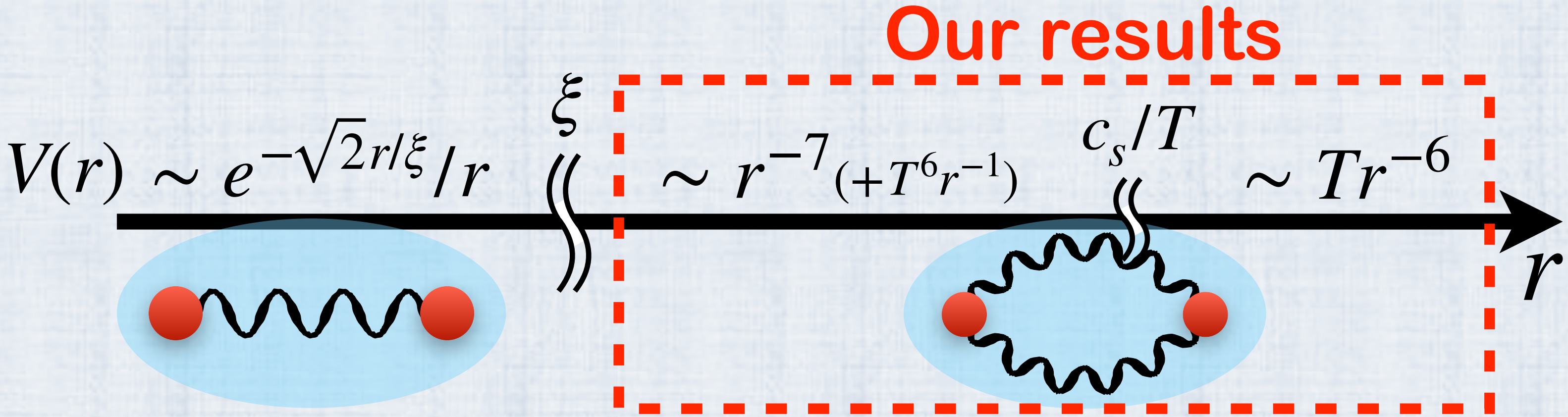
S. Hoinka, et al., Nature Physics **13**, 943 (2017)

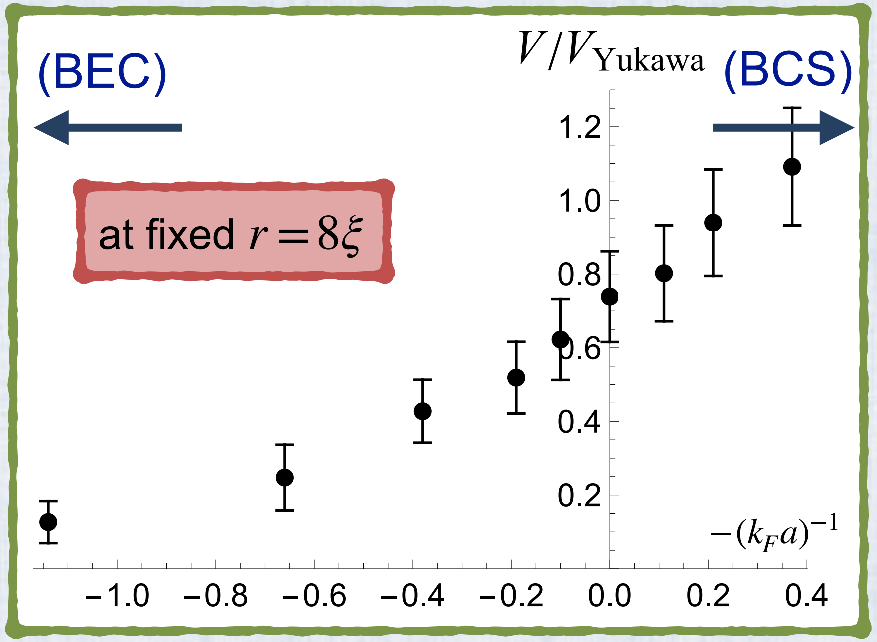


- The van der Waals potential is small in the BEC side because of the gas parameter, $(n\xi^3)^{-1}$.
- The van der Waals potential becomes relatively larger when $-(k_F a)^{-1}$ increases
- At unitarity, the van der Waals potential is dominant in $r \gtrsim 8\xi$.

Summary

✓ Induced interaction between impurities in a superfluid

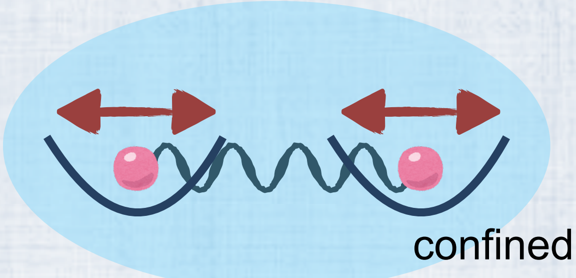




- ▸ based on only two assumptions:
  - **Galilean invariant medium**
  - **Contact s-wave impurity-medium coupling**
- ▸ The van der Waals potential becomes relatively larger when $-(k_F a)^{-1}$ increases

Experimentally measurable ?

- Ramsey interferometry
- Frequency shift of the out-of-phase mode



confined to separate micro-traps

Thank you!!