

A Pairing-Field Approach to Ultracold Fermi Gases on the Lattice

Florian Ehmann, TU Darmstadt
RPMBT 21 at UNC Chapel Hill, September 2022

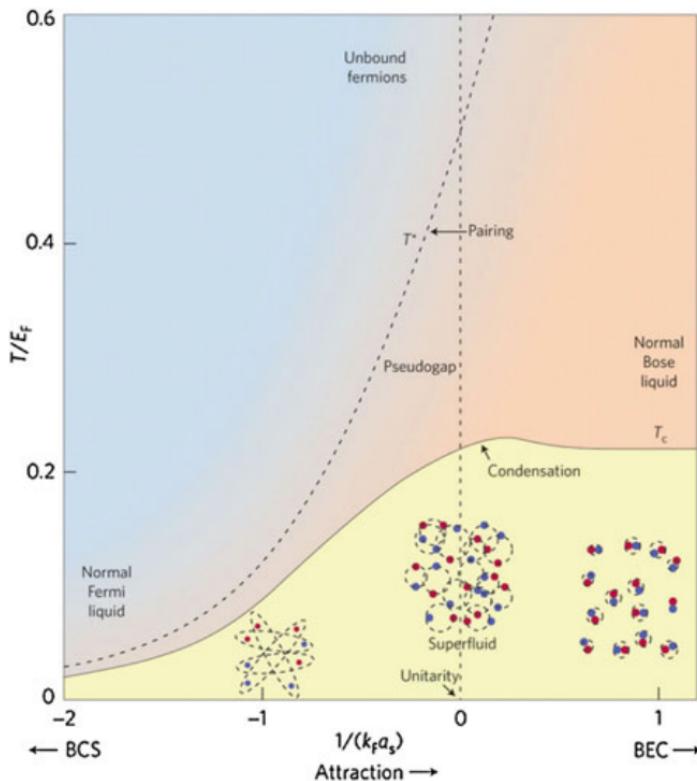


System / Motivation

$$\hat{H} = \int d^d r \left(-\hat{\psi}_\sigma^\dagger(\mathbf{r}) \frac{\nabla^2}{2m_\sigma} \hat{\psi}_\sigma(\mathbf{r}) - g \hat{\psi}_\uparrow^\dagger \hat{\psi}_\uparrow \hat{\psi}_\downarrow^\dagger \hat{\psi}_\downarrow(\mathbf{r}) \right)$$

- two-component Fermi gas $\sigma \in \{\uparrow, \downarrow\}$
- attractive contact interaction, i.e. $g > 0$
- potentially interesting phase structure, e.g. in the unitary limit

System / Motivation



[C. Sade Melo, M. Randeria, J. Engelbrecht, '93]

[W. Zwerger, The BCS-BEC Crossover and the Unitary Fermi Gas, '12]

Auxiliary-Field Formalism

Hubbard-Stratonovich Transformation

$$Z = \int \mathcal{D}(\psi^\dagger, \psi) e^{-S_F} \quad \rightarrow \quad Z = \int \mathcal{D}(\phi^*, \phi) e^{-S_B}$$

exchange fermionic fields for auxiliary bosonic fields

use bosonized theory to calculate observables

Pairing Field vs. Density Field

pairing

[review: W. Zwerger, The BCS-BEC Crossover and the Unitary Fermi Gas,
'12]

[on the lattice: FE, J. Drut, J. Braun, '22, in preparation]

$$\langle \phi \rangle \sim \langle \psi_{\uparrow} \psi_{\downarrow} \rangle$$

easy access to:

- pair condensate
- pair-correlation functions
- phase structure

density

[e.g. in review: J. Drut, A. Nicholson, '13]

$$\langle \phi \rangle \sim \left\langle \psi_{\uparrow}^* \psi_{\uparrow} + \psi_{\downarrow}^* \psi_{\downarrow} \right\rangle$$

easy access to:

- density
- density-density correlation functions

To the Lattice

$$\rightarrow \quad \vec{\psi} = \begin{pmatrix} \psi_{\tau_1, \mathbf{r}_1} \\ \psi_{\tau_1, \mathbf{r}_2} \\ \psi_{\tau_1, \mathbf{r}_3} \\ \psi_{\tau_2, \mathbf{r}_1} \\ \psi_{\tau_2, \mathbf{r}_2} \\ \psi_{\tau_2, \mathbf{r}_3} \\ \psi_{\tau_3, \mathbf{r}_1} \\ \psi_{\tau_3, \mathbf{r}_2} \\ \psi_{\tau_3, \mathbf{r}_3} \end{pmatrix}$$

Our Formalism: Lattice Formulation

$$S_F = \psi_\sigma^\dagger \left(D_\tau^{(\text{bw})}(\mu_\sigma) - \frac{R_- D_\Delta}{2m_\sigma} \right) \psi_\sigma \\ - g (A_-(\psi_\uparrow^* \circ \psi_\downarrow^*))^\top (\psi_\uparrow \circ \psi_\downarrow)$$

- field configuration vectors: $\psi = (\psi_{(\tau,\mathbf{r})_1}, \dots, \psi_{(\tau,\mathbf{r})_N})^\top$
- time-shifting matrices A_- and R_- , such that

$$(A_- \psi^*)^\top \psi = \sum_\tau \sum_{\mathbf{r}} \psi_{(\tau+a_\tau, \mathbf{r})}^* \psi_{(\tau, \mathbf{r})}$$

Our Formalism: Matrix Notation

- reduces visual noise
- makes potentially daunting tasks trivial, e.g.

$$\psi_{\downarrow}^{\dagger} D_{\tau}^{(\text{bw})} \psi_{\downarrow} = \psi_{\downarrow}^{\top} (?) \psi_{\downarrow}^*$$

$$\begin{aligned}\psi_{\downarrow}^{\dagger} D_{\tau}^{(\text{bw})} \psi_{\downarrow} &= \psi_{\downarrow}^{\top} \left(D_{\tau}^{(\text{bw})} \right)^{\top} \psi_{\downarrow}^* \\&= \psi_{\downarrow}^{\top} (1 - R_-)^{\top} \psi_{\downarrow}^* \\&= \psi_{\downarrow}^{\top} (1 - A_-) \psi_{\downarrow}^* \\&= \psi_{\downarrow}^{\top} \left(-D_{\tau}^{(\text{fw})} \right) \psi_{\downarrow}^*\end{aligned}$$

Our Formalism: Bosonization

$$\begin{aligned} S'_F = & \psi_\sigma^\dagger \left(D_\tau^{(\text{bw})} - \frac{R_- D_\Delta}{2m_\sigma} \right) \psi_\sigma \\ & - g \left(A_- (\psi_\uparrow^* \circ \psi_\downarrow^*) \right)^\top (\psi_\uparrow \circ \psi_\downarrow) \\ & + g \phi^\dagger \phi \end{aligned}$$

need to shift ϕ integration in a way that eliminates four-fermion interaction

Problem: time-shift between starred and unstarred fields

(with $1 = \int \mathcal{D}(\phi^*, \phi) e^{-g\phi^\dagger \phi}$)

Our Formalism: Bosonization

Solution:

$$\begin{aligned}\phi^* &\rightarrow \phi^* - A_- (\psi_\uparrow^* \circ \psi_\downarrow^*) \\ \phi &\rightarrow \phi + (\psi_\uparrow \circ \psi_\downarrow)\end{aligned}$$

include time-shift in the HS-shift terms

Our Formalism: Bosonized Theory

fermions can be integrated out to obtain a fully bosonized theory

$$Z(\beta, \mu_\uparrow, \mu_\downarrow) = \int \mathcal{D}(\phi^*, \phi) e^{-S_B}$$

with

$$S_B = g \phi^\dagger \phi - \log \det \mathcal{M}(\phi^*, \phi)$$

this theory can now be solved numerically

Our Pairing-Field Formalism

[FE, J. Drut, J. Braun '22, in preparation]

- on the lattice / ab initio
- supports finite temperature
- supports imbalance in mass and chemical potential
- calculated using the Complex Langevin approach, due to sign-problem

First Results

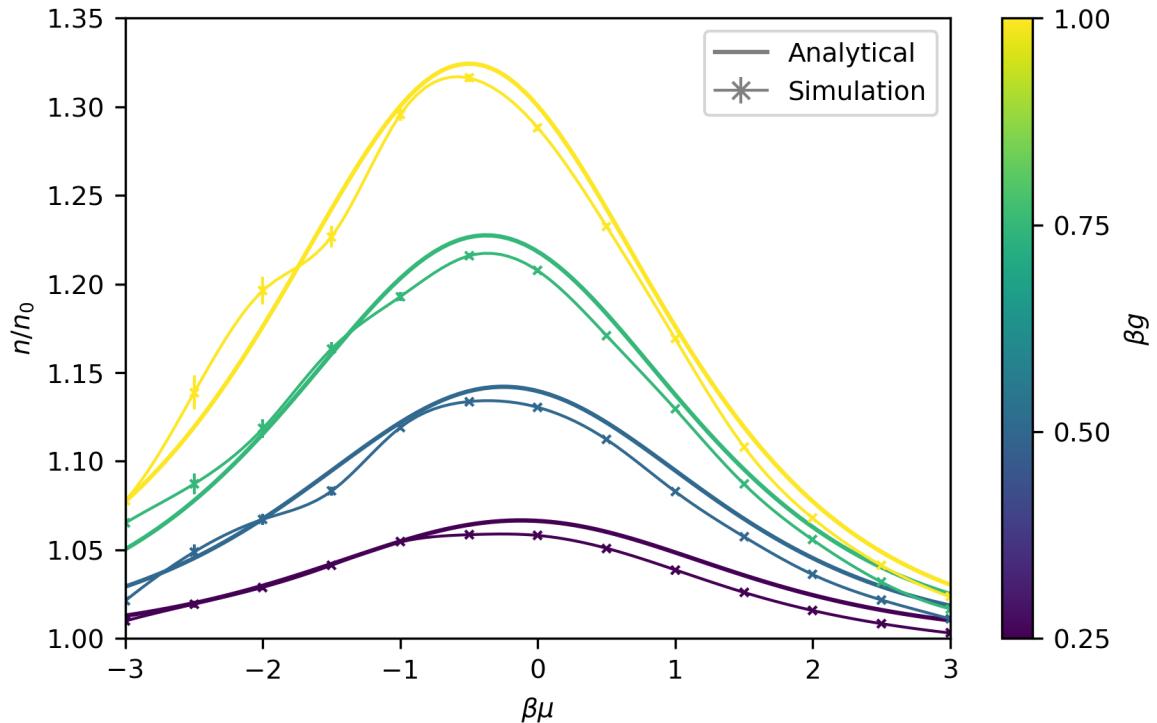
- proof of concept
- 0+1 dimensional systems

$$\hat{H} = -g \hat{\psi}_\uparrow^\dagger \hat{\psi}_\uparrow \hat{\psi}_\downarrow^\dagger \hat{\psi}_\downarrow$$

analytical solution exists!

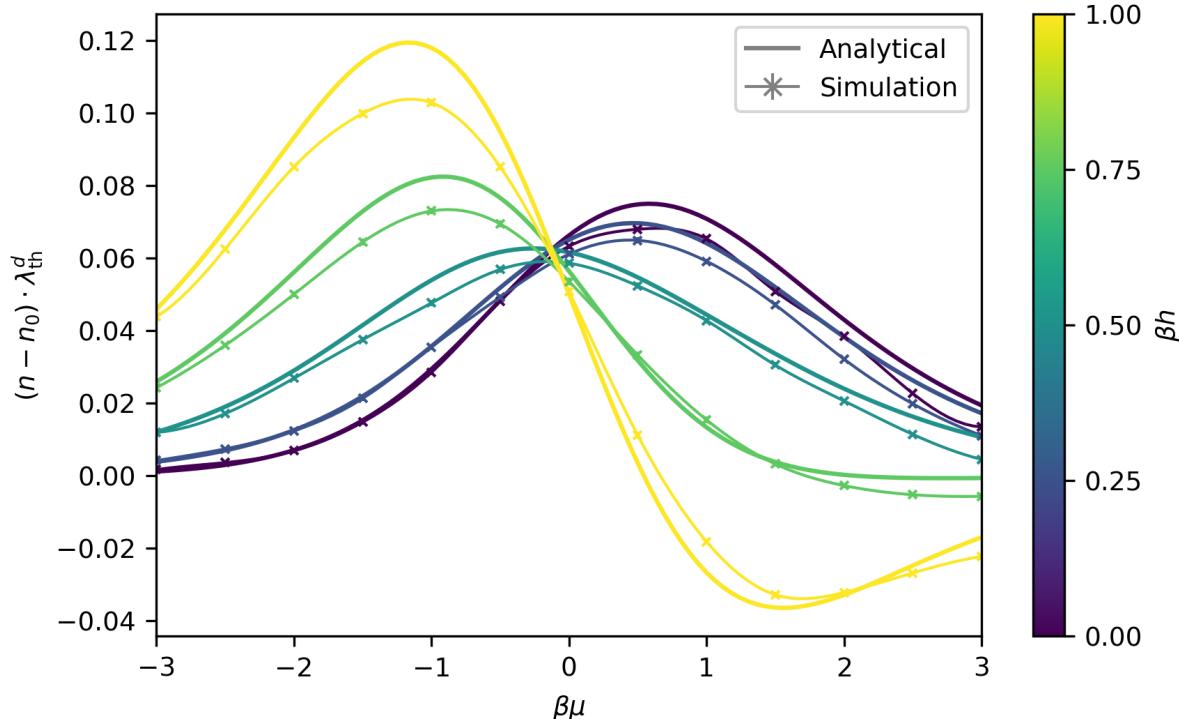
- density equations of state

Interacting Density



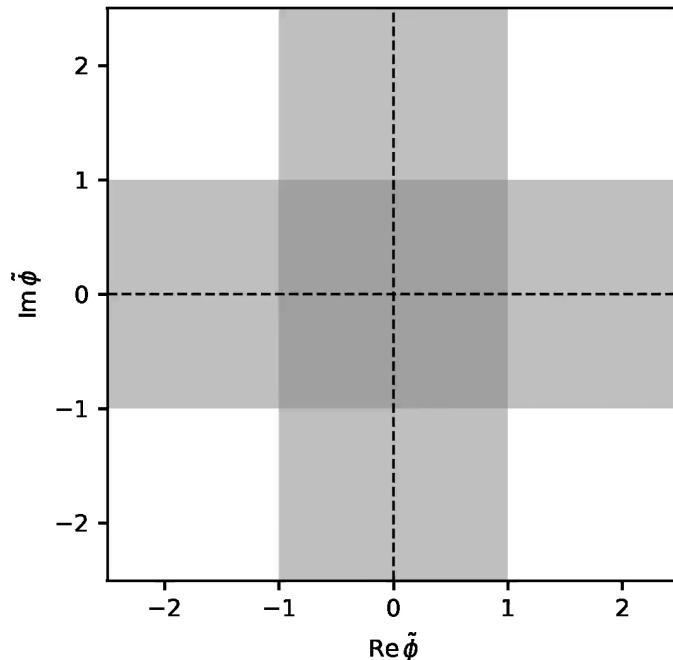
Interacting Imbalanced Density

$$\beta\mu = \frac{\beta\mu_{\uparrow} + \beta\mu_{\downarrow}}{2} \quad \text{and} \quad \beta h = \frac{\beta\mu_{\uparrow} - \beta\mu_{\downarrow}}{2}$$



Pairing Field Sampling

$$\tilde{\phi} = \lambda_{\text{th}}^{d/2} \langle \phi \rangle_{\tau, \mathbf{r}}$$



Outlook

- pairing condensate
 - $U(1)$ phase transition
 - pairing correlation functions
 - 1+1, 2+1 and 3+1 dimensional systems
- stay tuned