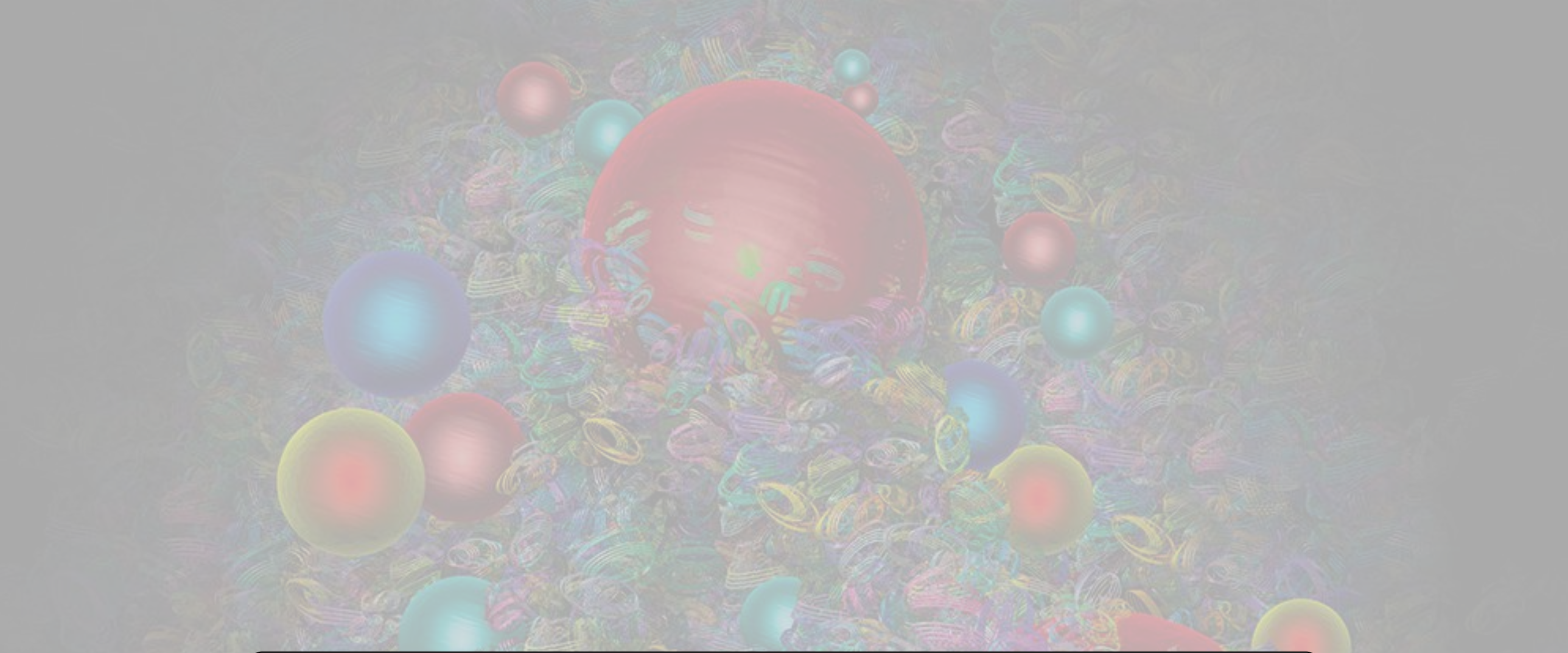
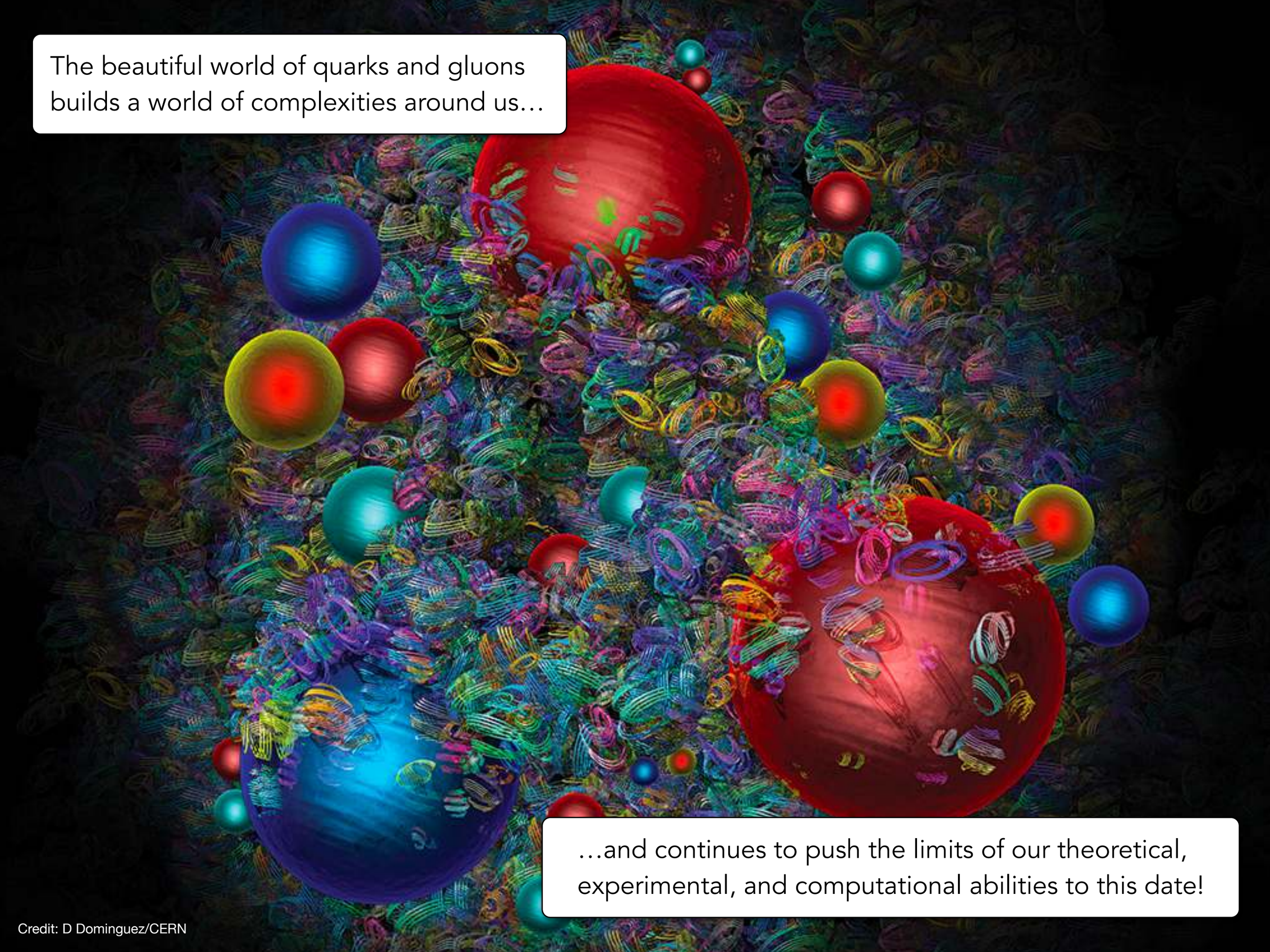


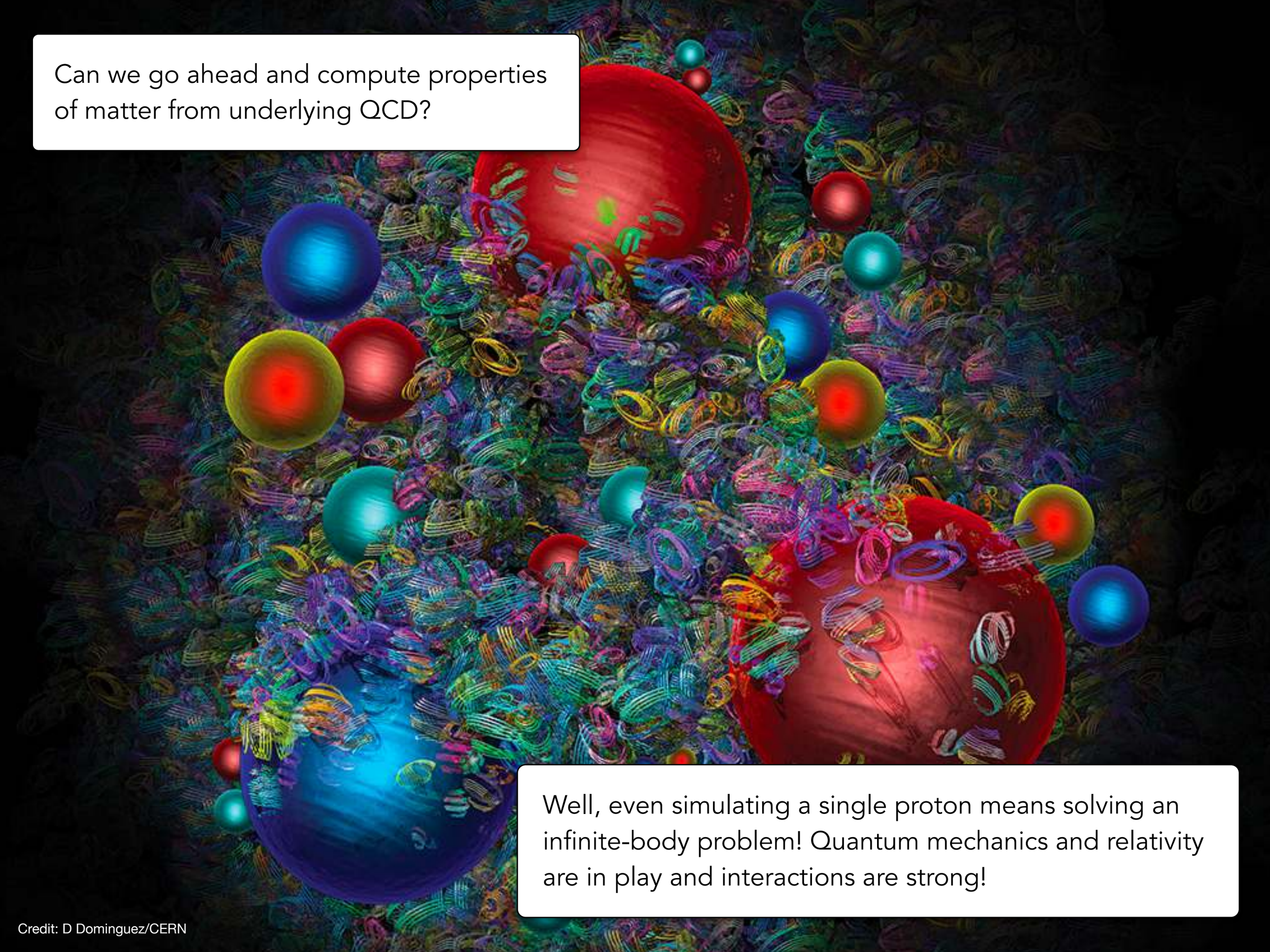
The International Conference on Recent Progress in
Many-Body Theories XXI
University of North Carolina, Chapel Hill
Sep 12-16, 2022

A Quantum Simulation Program for QCD?

Zohreh Davoudi
University of Maryland, College Park



Credit: D Dominguez/CERN

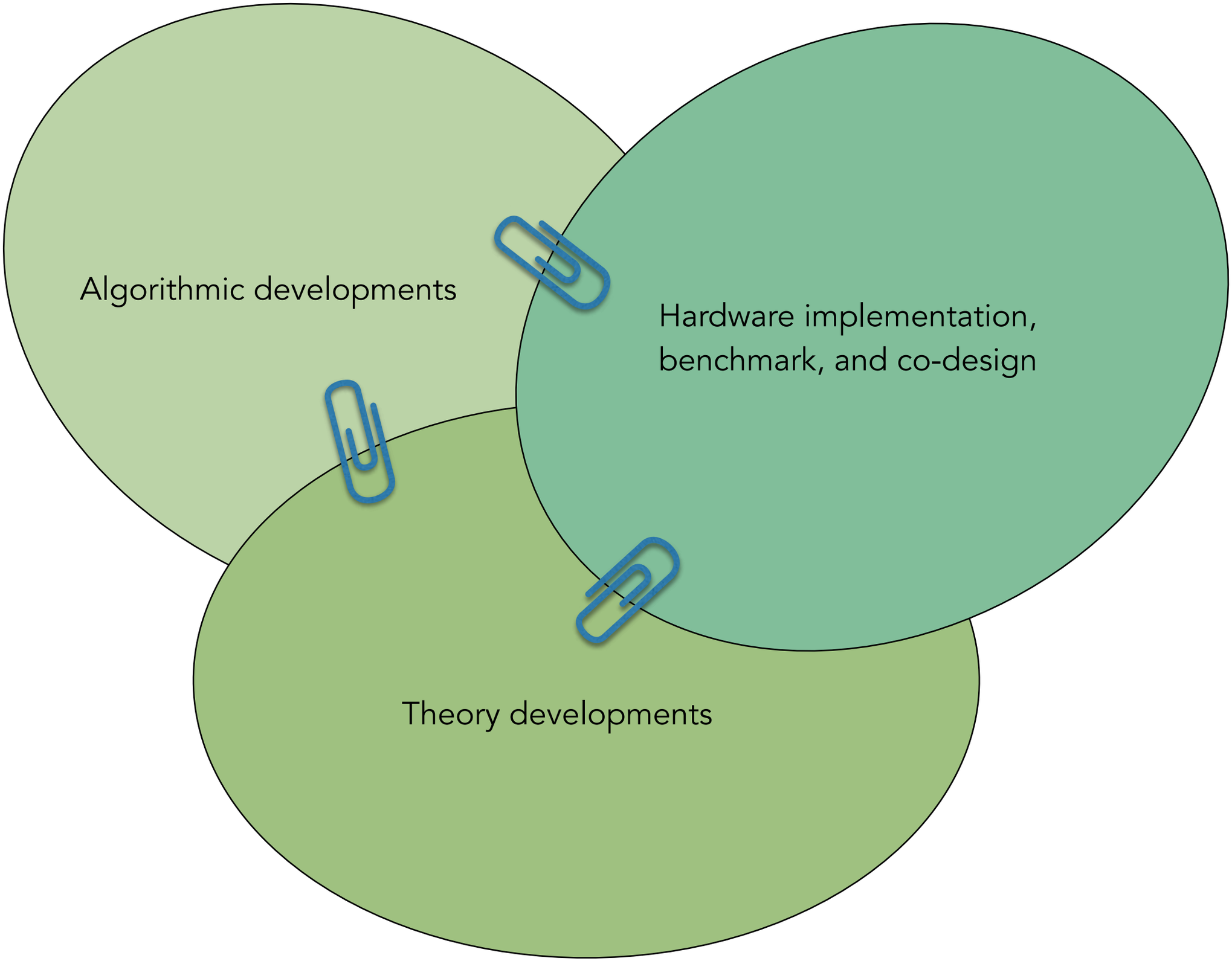


Can we go ahead and compute properties of matter from underlying QCD?

Well, even simulating a single proton means solving an infinite-body problem! Quantum mechanics and relativity are in play and interactions are strong!

Credit: D Dominguez/CERN

LATTICE QCD: A MULTI-PRONG PROGRAM THAT SIMULATES QCD NON-PERTURBATIVELY



QCD ON A CHIP!

534

Nuclear Instruments and Methods in Physics Research 222 (1984) 534–539
North-Holland, Amsterdam

HARDWARE MATRIX MULTIPLIER/ACCUMULATOR FOR LATTICE GAUGE THEORY CALCULATIONS *

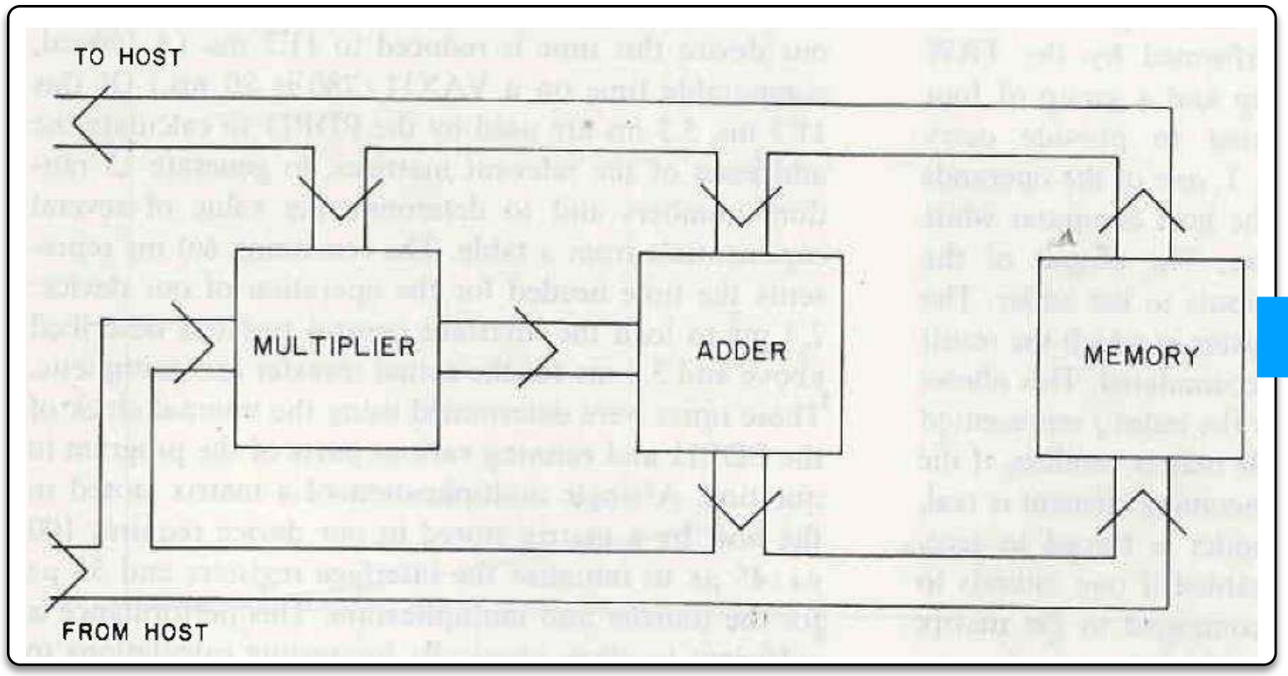
Norman H. CHRIST and Anthony E. TERRANO

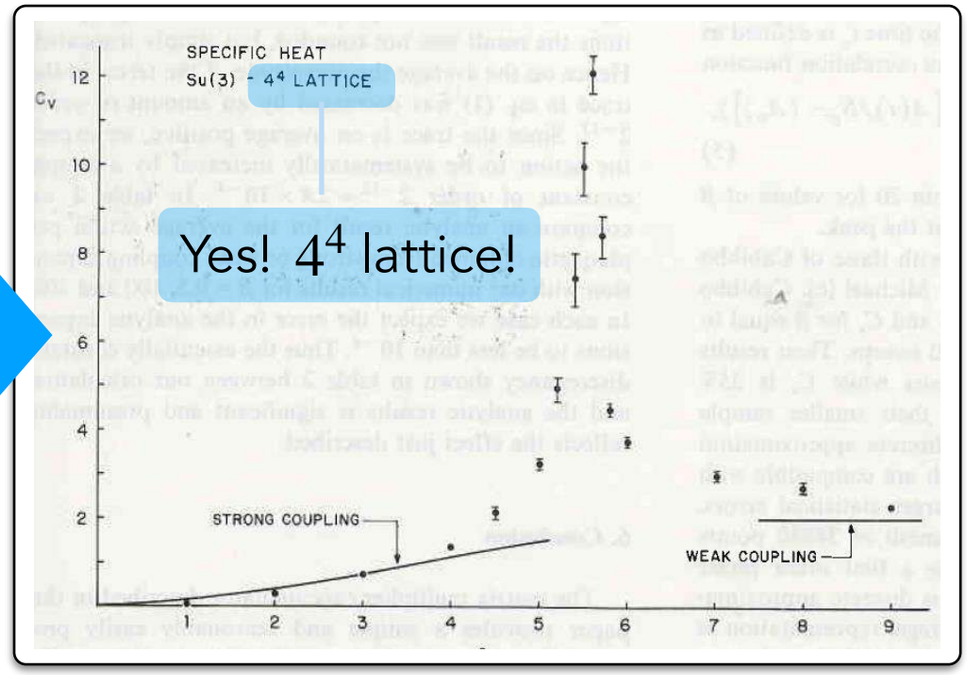
Columbia University, New York, NY 10027, USA

Received 30 September 1983

~40 years ago!

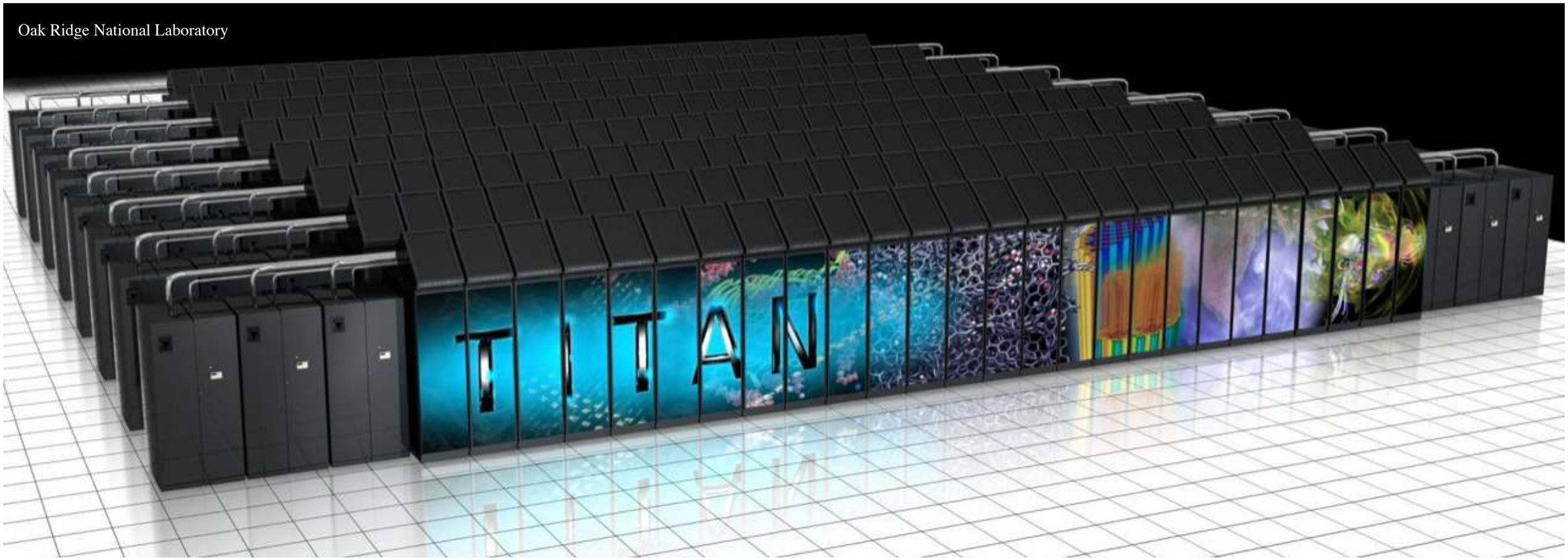
10^{10} times or more slower than current supercomputers!
Only few Kbytes of memory!





Yes! 4^4 lattice!

PUTTING ALL THESE HEROIC THEORY, ALGORITHM, AND CO-DESIGN EFFORTS TO WORK AND HAVING ACCESS TO HUNDREDS OF MILLION CPU HOURS ON THE LARGEST SUPERCOMPUTERS IN THE WORLD…

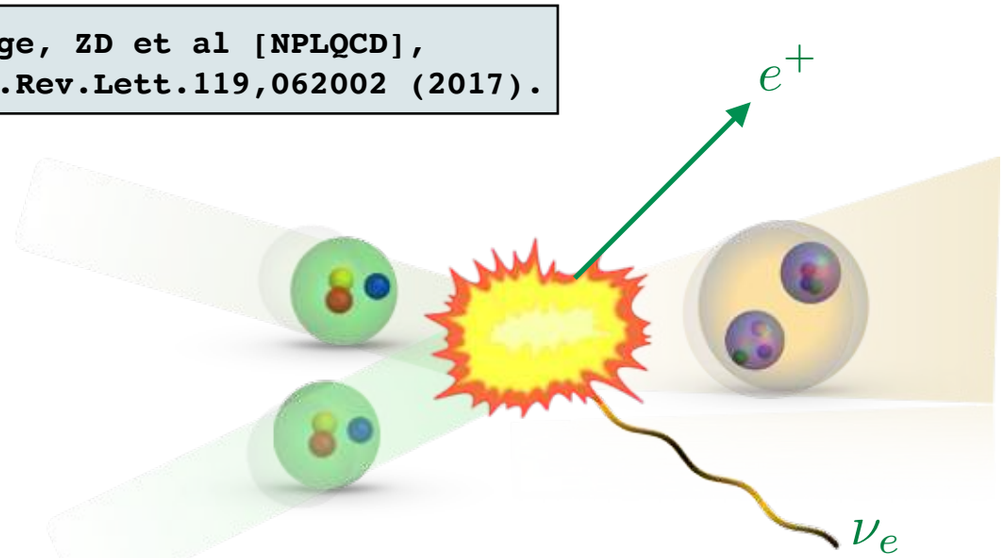


Titan supercomputer, Oak Ridge National Laboratory, USA

HAS LED TO TENS OF SUCCESS EXAMPLES
BUT LET ME TELL YOU ABOUT ONE…

pp fusion cross section

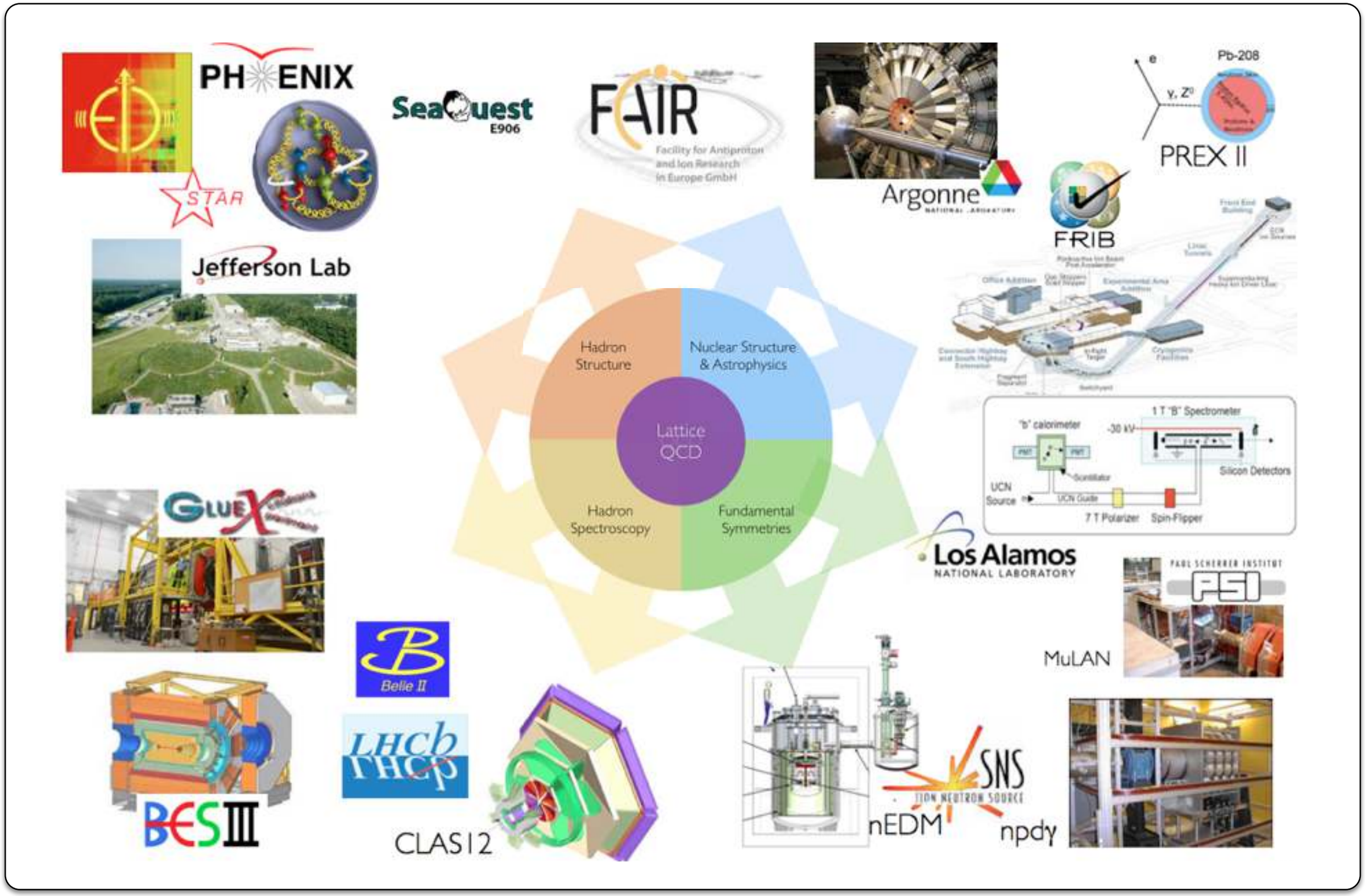
Savage, ZD et al [NPLQCD], Phys.Rev.Lett.119,062002 (2017).



$$L_{1,A} = 3.9(0.1)(1.0)(0.3)(0.9)\ \mathrm{fm}^3 \ @\ \mu = m_\pi^{\mathrm{phys.}} = 140\ \mathrm{MeV}$$

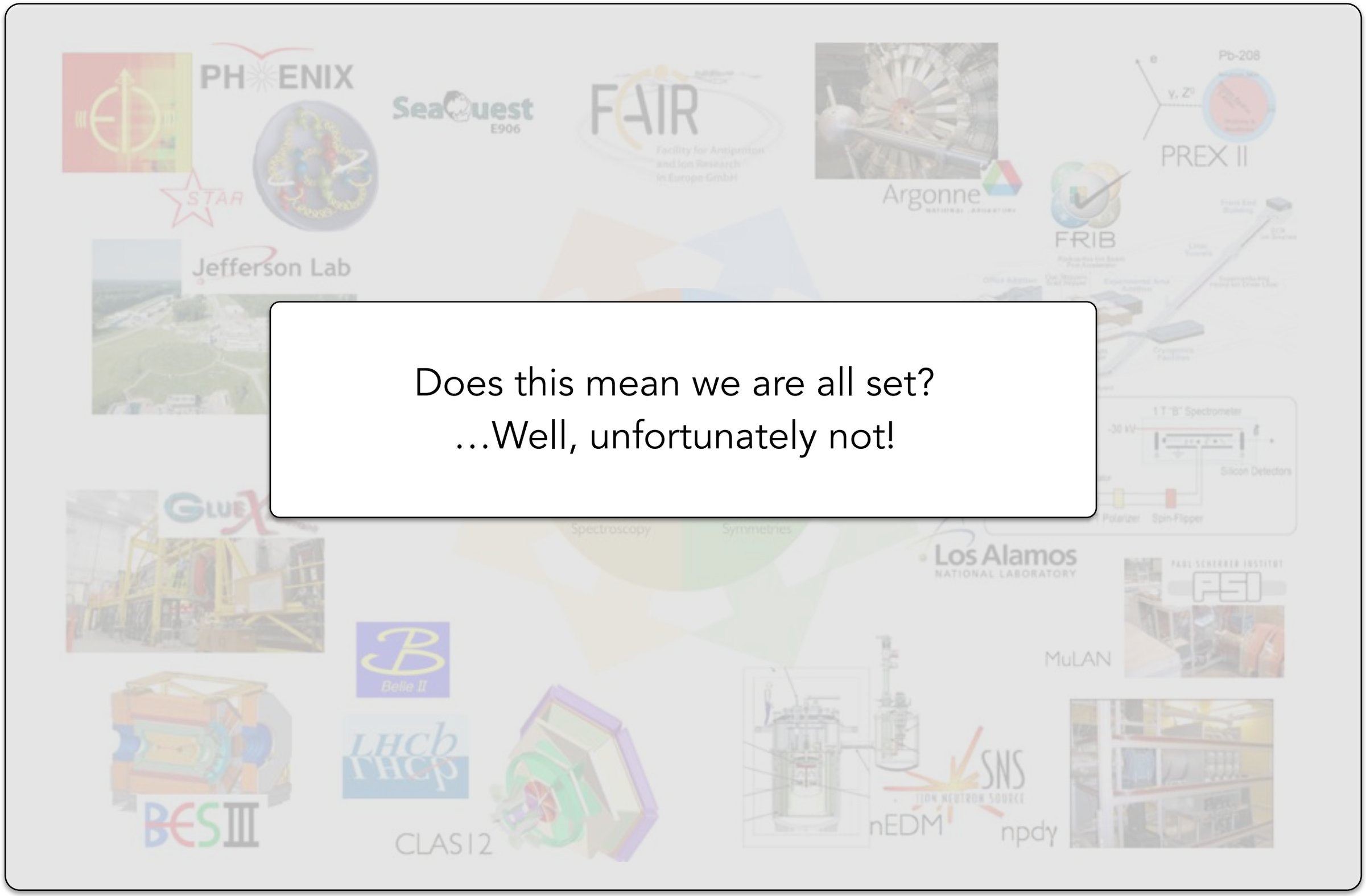
ZD, Detmold, Orginos, Parreño, Savage, Shanahan, Wagman, *Phys.Rept.*900,1-74 (2021).

LATTICE QCD IS SUPPORTING A MULTI-BILLION DOLLAR EXPERIMENTAL PROGRAM!



Slide content courtesy of Martin Savage.

LATTICE QCD IS SUPPORTING A MULTI-BILLION DOLLAR EXPERIMENTAL PROGRAM!



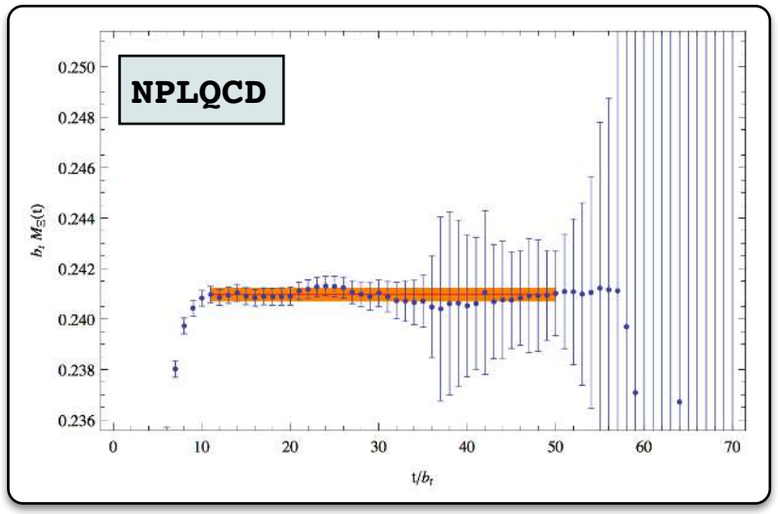
Does this mean we are all set?

…Well, unfortunately not!

Slide content courtesy of Martin Savage.

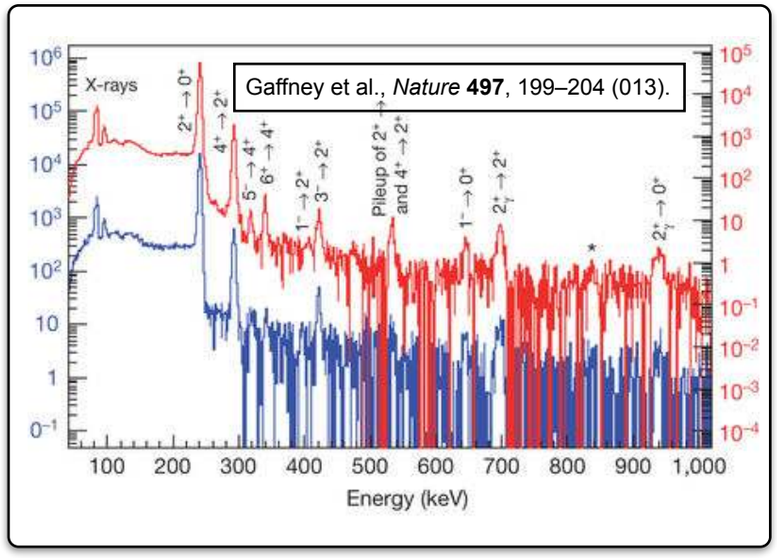
THREE FEATURES MAKE LATTICE QCD CALCULATIONS OF NUCLEI HARD:

i) The complexity of systems grows factorially with the number of quarks.



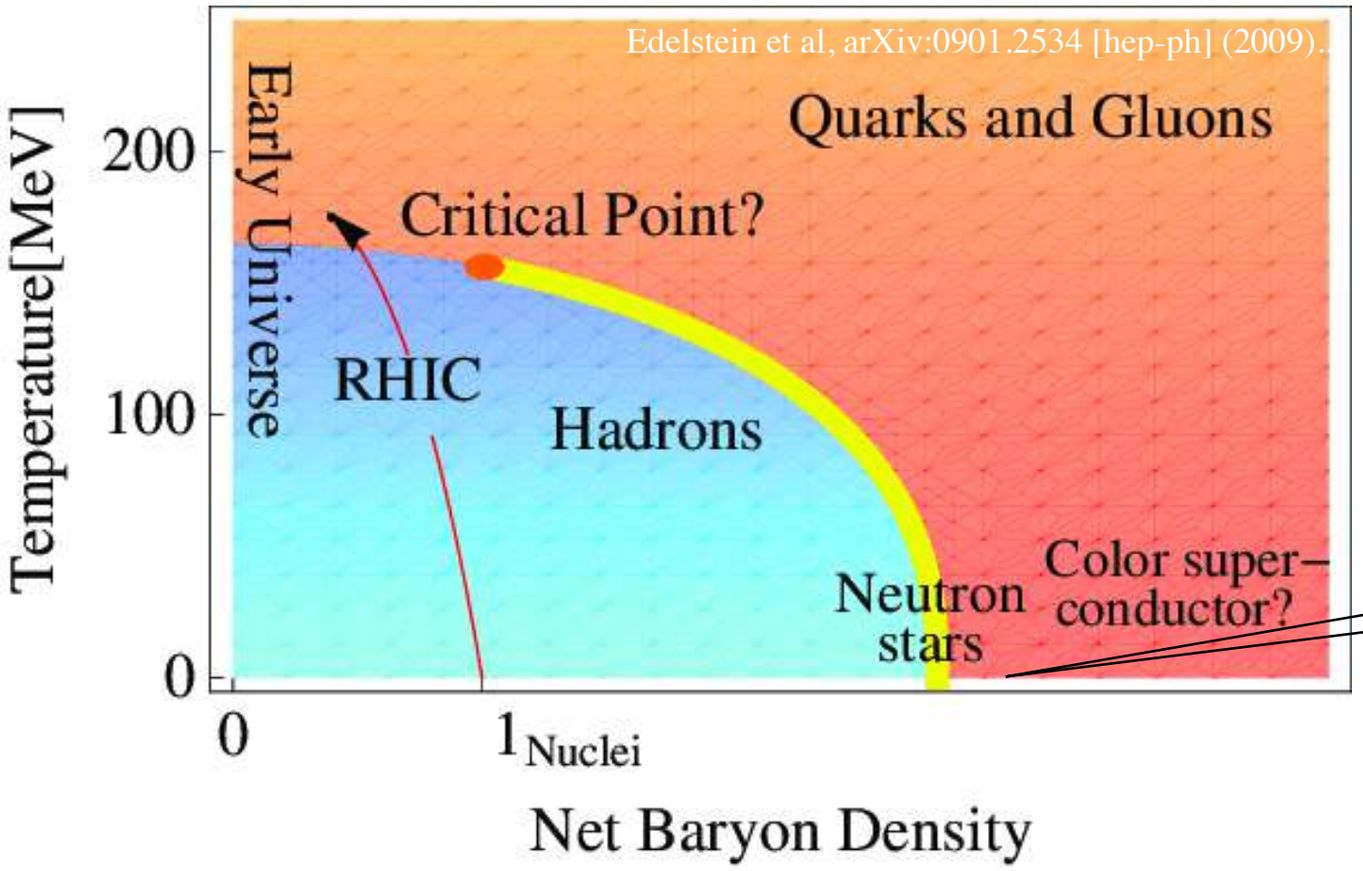
ii) There is a severe signal-to-noise degradation.

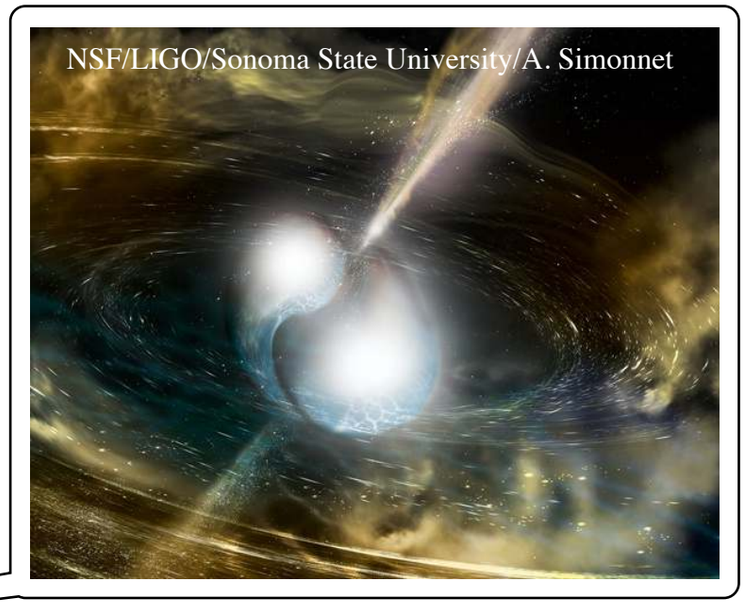
iii) Excitation energies of nuclei are much smaller than the QCD scale.



ADDITIONALLY THE SIGN PROBLEM FORBIDS:

i) Studies dense matter such as interior of neutron stars and phase diagram of QCD





Path integral formulation:

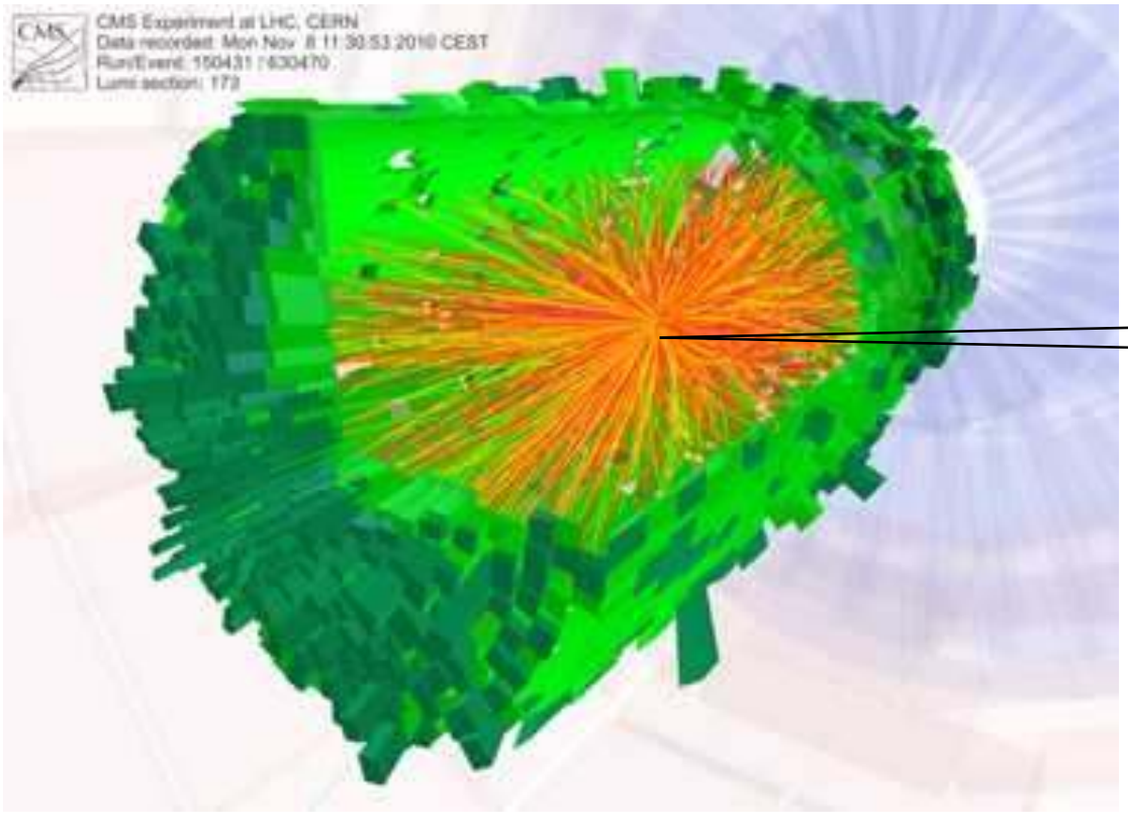
$$e^{-S[U,q,\bar{q}]}$$

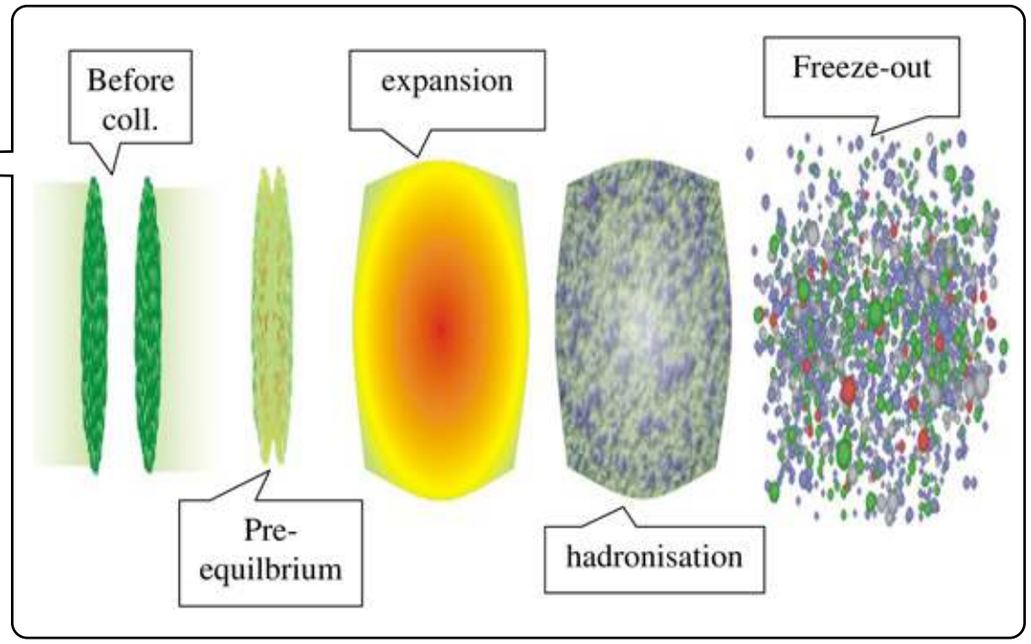
with a complex action:

$$\mathcal{L}_{\mathrm{QCD}} \to \mathcal{L}_{\mathrm{QCD}} - i\mu \sum_f \bar{q}_f \gamma^0 q_f$$

ADDITIONALLY THE SIGN PROBLEM FORBIDS:

ii) Real-time dynamics of matter in heavy-ion collisions or after Big Bang…





…and a wealth of dynamical response functions, transport properties, parton distribution functions, and non-equilibrium physics of QCD.

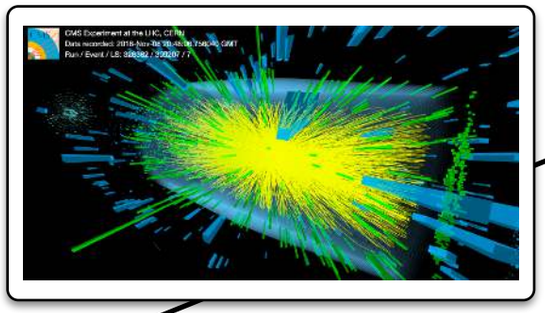
Path integral formulation:

$$e^{iS[U,q\bar{q}]}$$

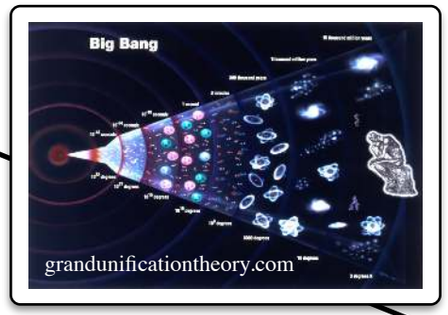
Hamiltonian evolution:

$$U(t) = e^{-iHt}$$

PLUS MANY INTRACTABLE QUESTIONS IN HIGH ENERGY PHYSICS AS WELL…



Collider phenomenology

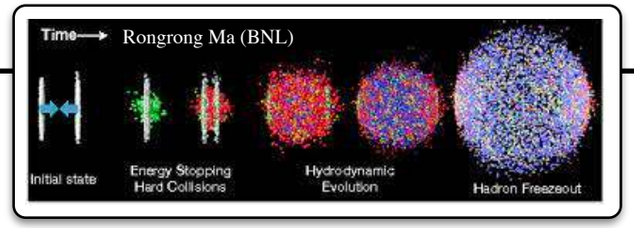


Cosmology and early universe

Neutrino (astro)physics

Quantum gravity

Matter in and out of equilibrium



Bauer, ZD et al, arXiv:2204.03381 [quant-ph].

Quantum Simulation for High-energy Physics

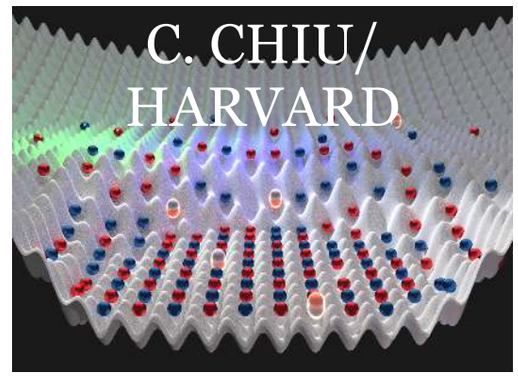
Christian Bauer,[1,a] Zohreh Davoudi,[2,b] A. Baha Balantekin,[3] Tanmoy Bhattacharya,[4] Marcela Carena,[5,6,7] Wibe A. de Jong,[1] Nate Gemelke,[8] Dmitri Kharzeev,[9] Henry Lamm,[5] Ying-Ying Li,[5] Yannick Meurice,[10] Christopher Monroe,[11,12,13,14] Benjamin Nachman,[1] Guido Pagano,[15] John Preskill,[16] Alessandro Roggero,[17,18] David I. Santiago,[19,20] Martin J. Savage,[21] Irfan Siddiqi,[19,20,22] George Siopsis,[23] Yukari Yamauchi,[2] and Kübra Yeter-Aydeniz[24]

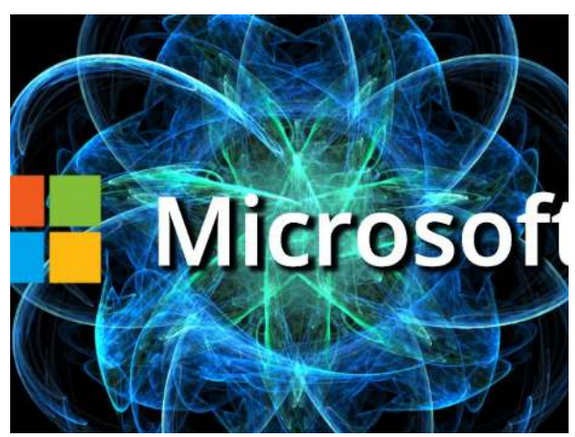
An opportunity to explore new paradigms and new technologies:
Turning to quantum simulation

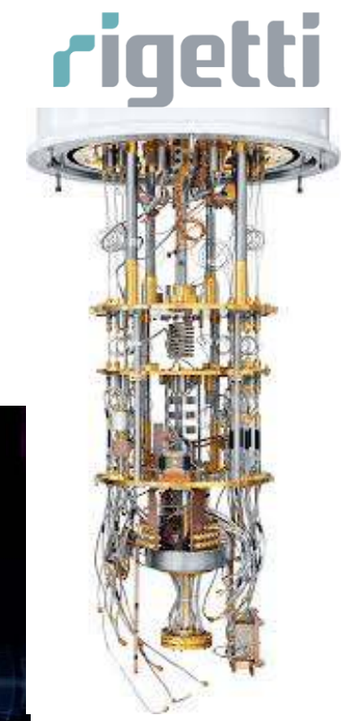
https://www.pechakucha.com/

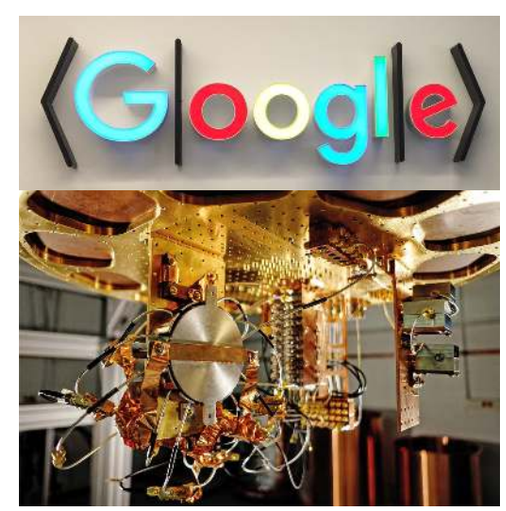
A RANGE OF QUANTUM SIMULATORS WITH VARING CAPACITY AND CAPABILITY IS AVAILABLE!

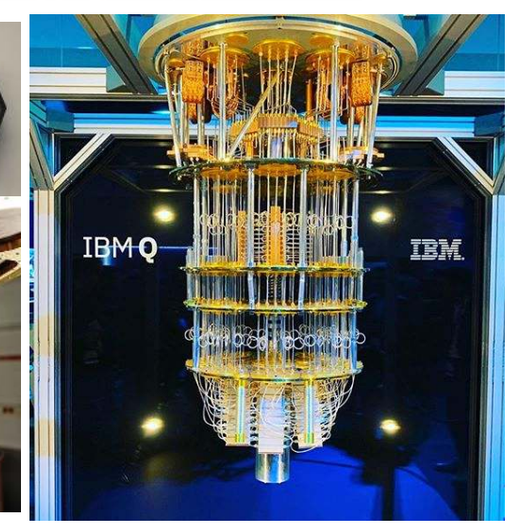
- Atomic systems (trapped ions, cold atoms, Rydbergs)
- Condensed matter systems (superconducting circuits, dopants in semiconductors such as in Silicon, NV centers in diamond)
- Laser-cooled polar molecules
- Optical systems (cavity quantum electrodynamics)





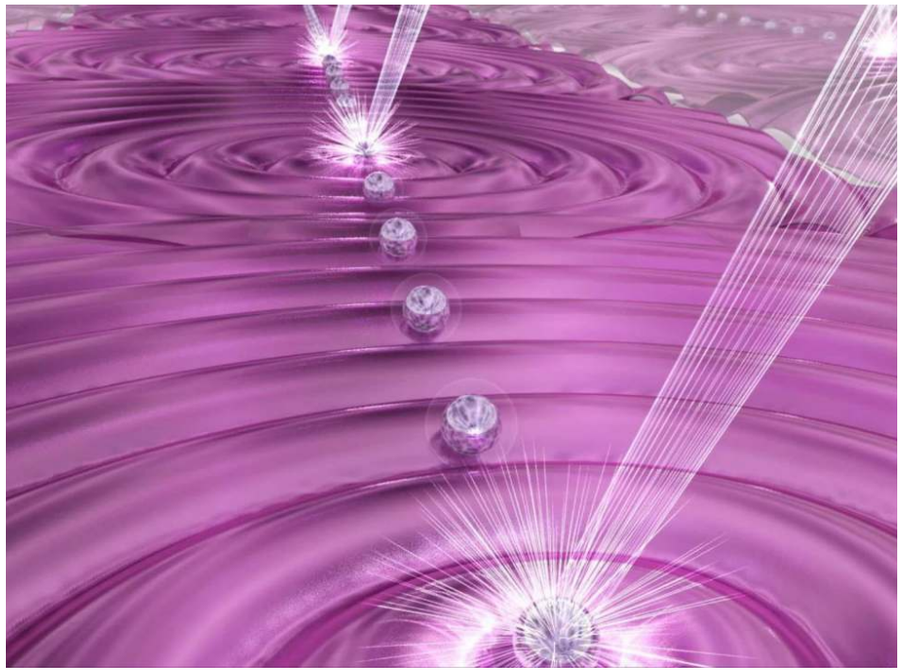




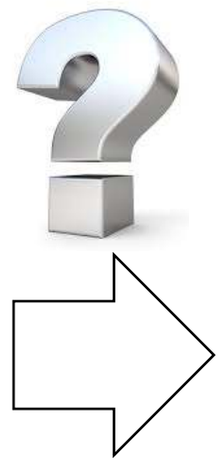


QUANTUM SIMULATION OF QCD?

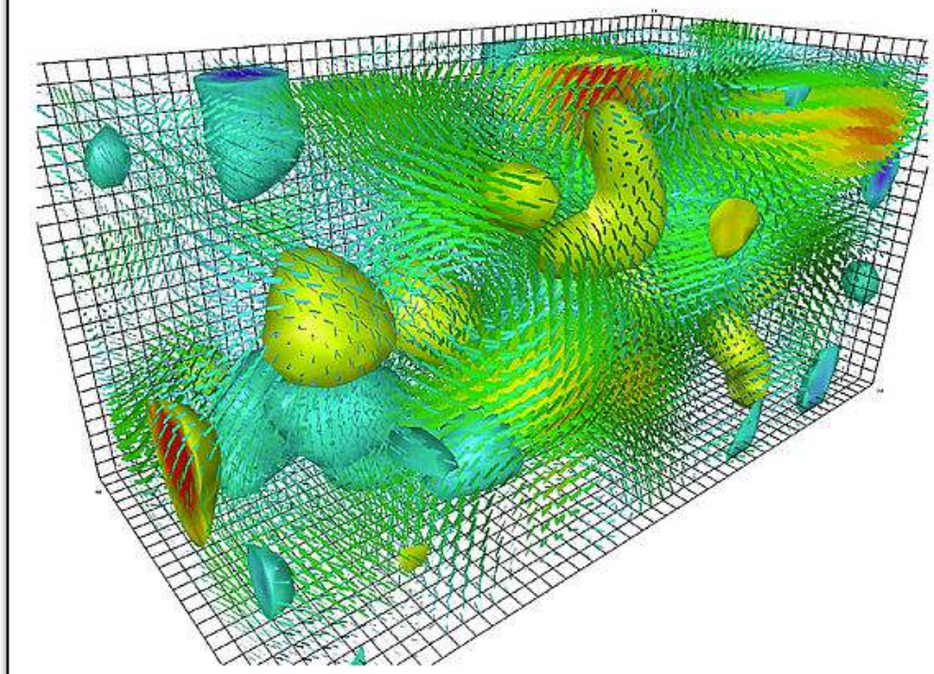
A controlled quantum system



CREDIT: EMILY EDWARDS, UNIVERSITY OF MARYLAND

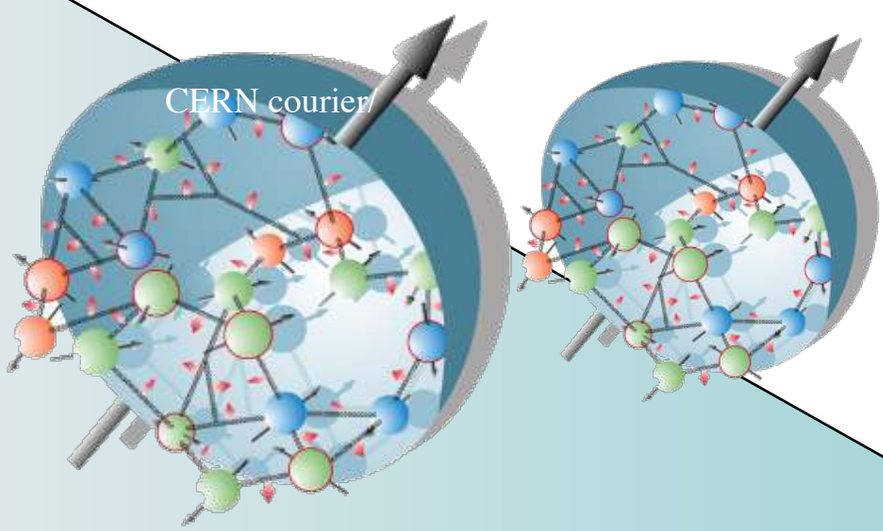


Strong-interaction physics



COPY RIGHT: UNIVERSITY OF ADELAIDE

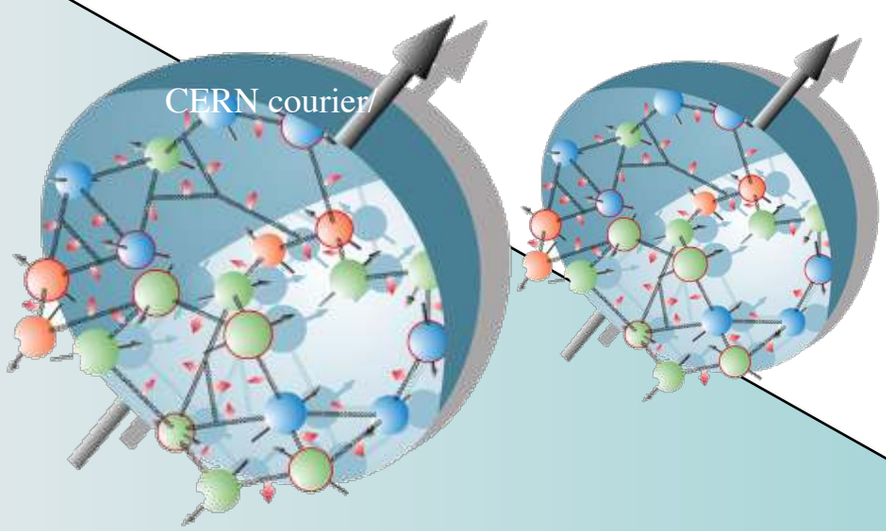
DIFFERENT FROM QUANTUM-CHEMISTRY SIMULATIONS



Starting from the Standard Model

Both bosonic and fermionic DOF are dynamical and coupled, exhibit both global and local (gauge) symmetries, relativistic hence particle number not conserved, vacuum state nontrivial in strongly interacting theories.

DIFFERENT FROM QUANTUM-CHEMISTRY SIMULATIONS



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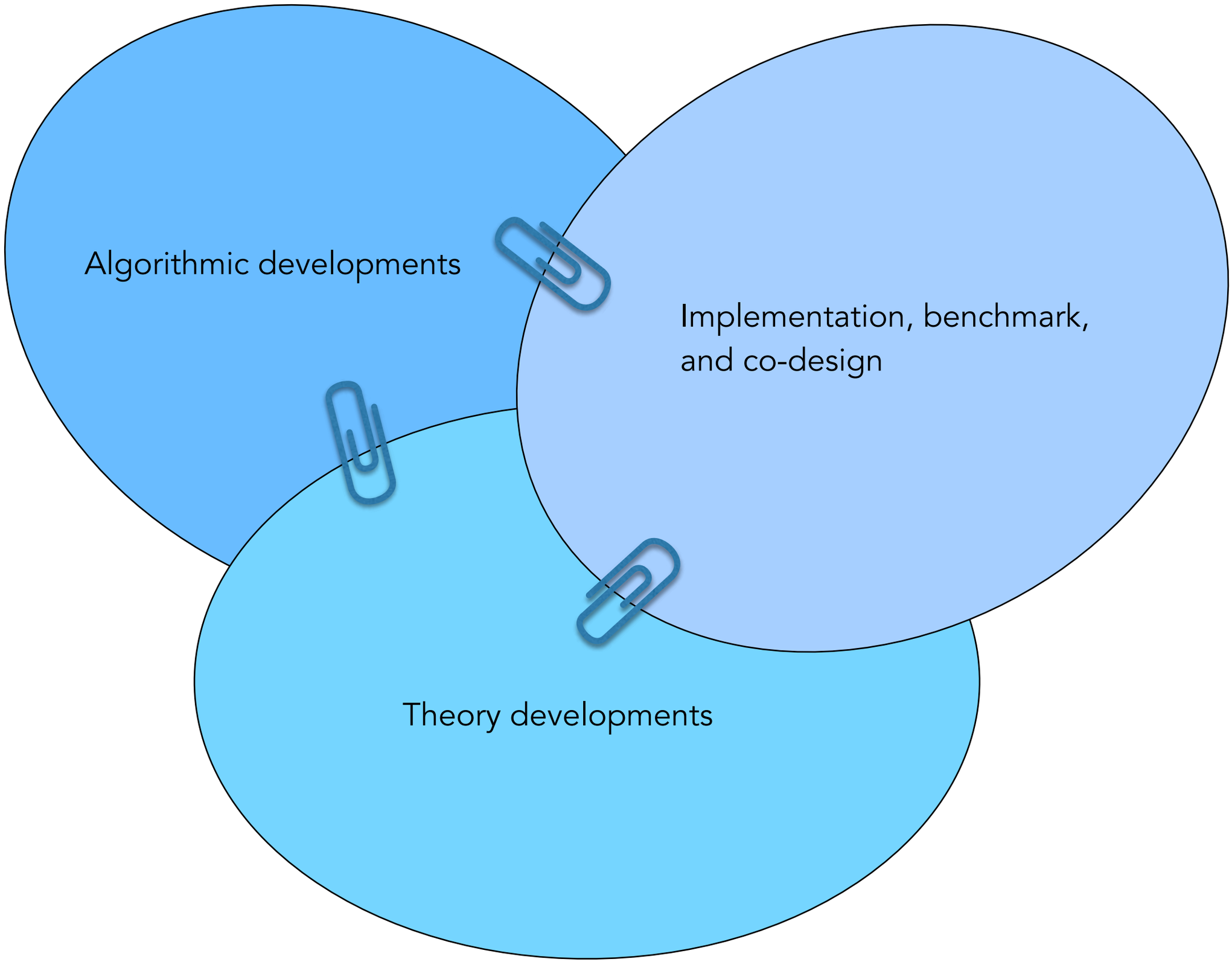
Both bosonic and fermionic DOF are dynamical and coupled, exhibit both global and local (gauge) symmetries, relativistic hence particle number not conserved, vacuum state nontrivial in strongly interacting theories.

Attempts to cast QFT problems in a language closer to quantum chemistry and NR simulations: Kreshchuk, Kirby, Goldstein, Beauchemin, Love, arXiv:2002.04016 [quant-ph], Kreshchuk, Jia, Kirby, Goldstein, Vary, Love, Entropy 2021, 23, 597, Liu, Xin, arXiv:2004.13234 [hep-th], Barata , Mueller, Tarasov, Venugopalan (2020)

QUANTUM SIMULATION OF QUANTUM FIELD THEORIES: A MULTI-PRONG EFFORT



How to formulate QCD in the Hamiltonian language?

What are the efficient formulations? Which bases will be most optimal toward the continuum limit?

How to preserve the symmetries? How much should we care to retain gauge invariance?

How to quantify systematics such as finite volume, discretization, boson truncation, time digitization, etc?

Theory developments

QUANTUM SIMULATION OF GAUGE FIELD THEORIES: THEORY DEVELOPMENTS

Hamiltonian formalism maybe more natural than the path integral formalism for quantum simulation/computation:

Kogut and Susskind formulation:

$$H = -t\sum s\ \left(\psi\ U\ \psi + \psi\ U\ \psi\right) + m\sum s\ \psi\ \psi + \frac{g}{2}\sum\left(L + R\right) - \frac{1}{4g}\sum \mathrm{Tr}\left(U + U\right).$$

Fermion hopping term

Fermion mass

Energy of color electric field

Energy of color magnetic field

QUANTUM SIMULATION OF GAUGE FIELD THEORIES: THEORY DEVELOPMENTS

Hamiltonian formalism maybe more natural than the path integral formalism for quantum simulation/computation:

An infinite-dimensional Hilbert space!

Kogut and Susskind formulation:

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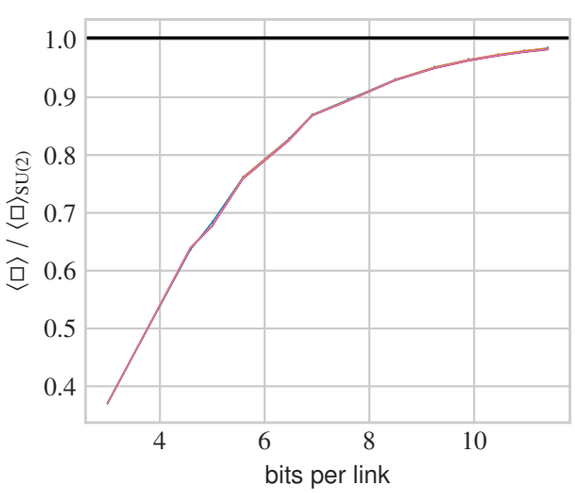
Fermion hopping term | Fermion mass | Energy of color electric field | Energy of color magnetic field

Gauge-field truncation

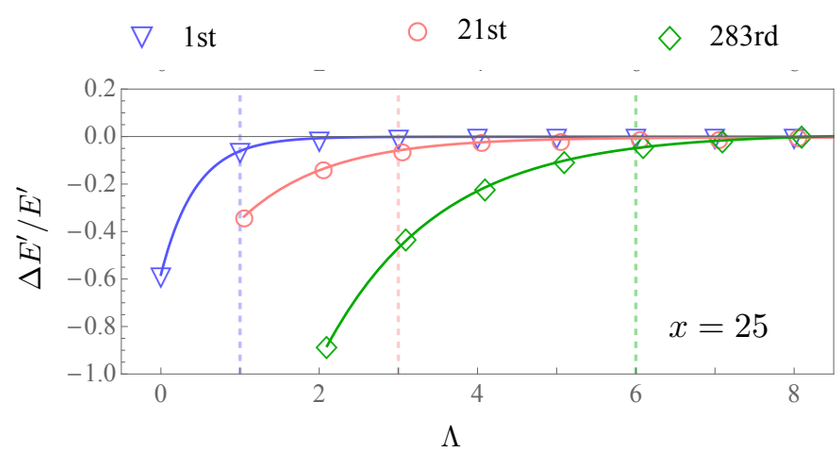
Tong, Albert, McClean, Preskill, and Su (2021).

SU(2) pure gauge in 3+1 D in group element basis



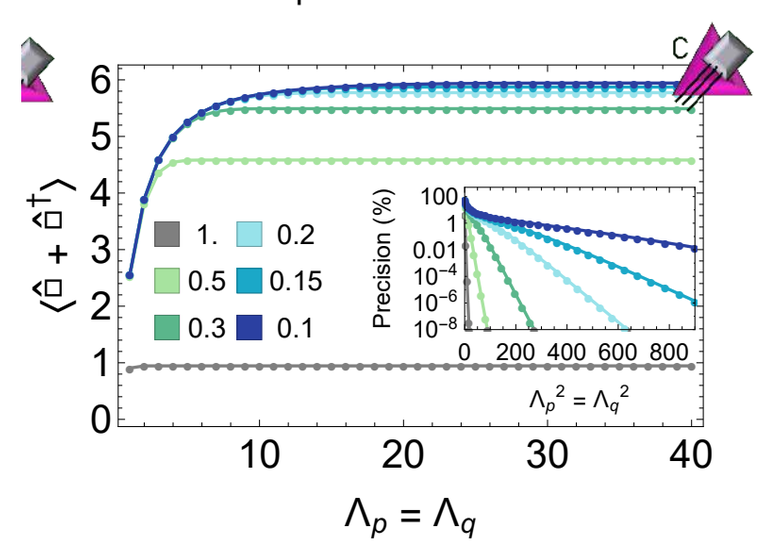
Hackett et al, Phys. Rev. A 99, 062341 (2019)

SU(2) with matter in 1+1 D in electric-field basis



ZD, Raychowdhury, and Shaw, arXiv:2009.11802 [hep-lat]

SU(3) pure gauge in 2+1 D in local-irreps basis



Ciavarella, Klco, and Savage, arXiv:2101.10227 [quant-ph]

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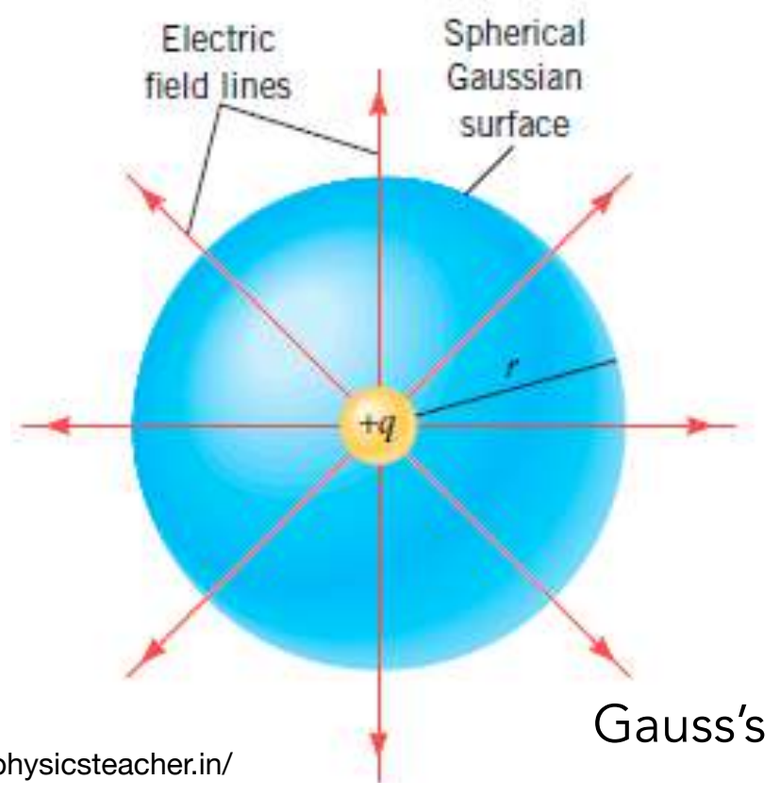
Fermion hopping term

Fermion mass

Energy of color electric field

Energy of color magnetic field

Generator of infinitesimal gauge transformation $G = \psi\ \lambda\ \psi + \sum\left(L + R\right) \Rightarrow G_x^i|\psi(\{q_x^{(i)}\})\rangle = q_x^{(i)}|\psi(\{q_x^{(i)}\})\rangle$



Gauss's law

https://physicsteacher.in/

QUANTUM SIMULATION OF GAUGE FIELD THEORIES: THEORY DEVELOPMENTS

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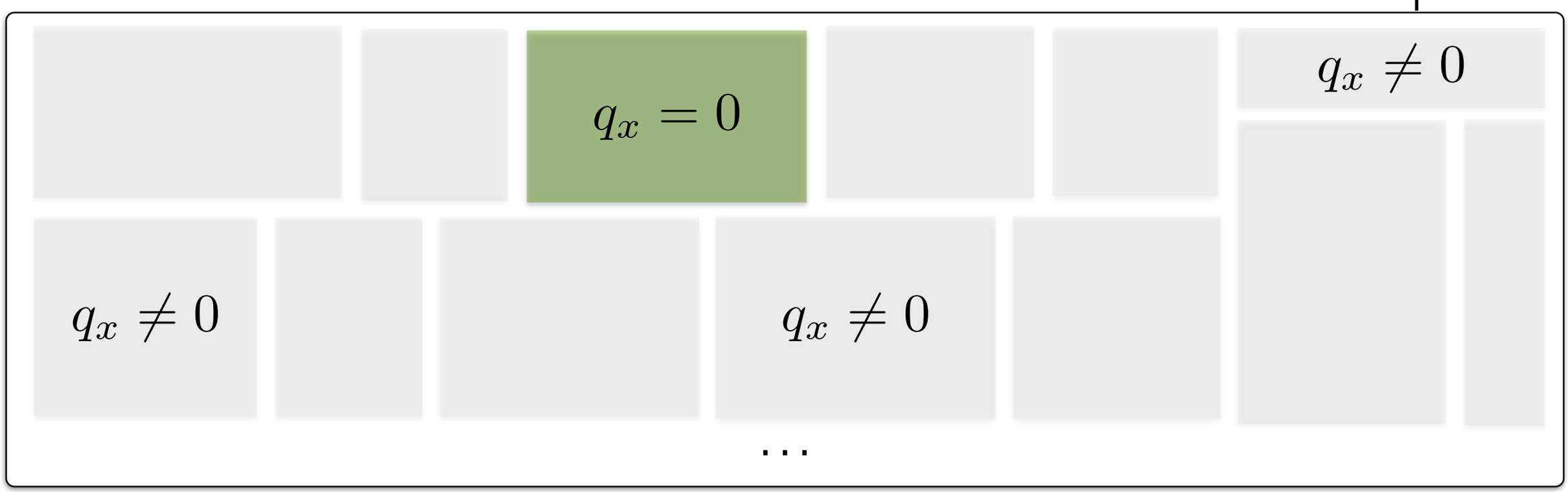
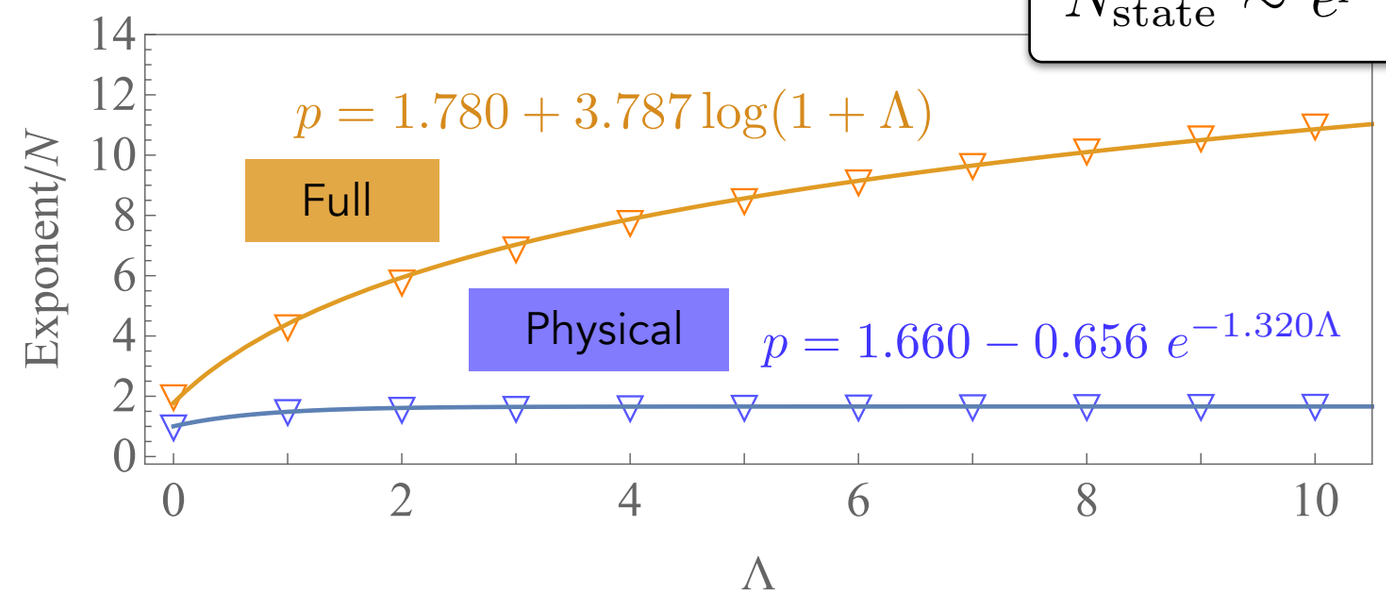


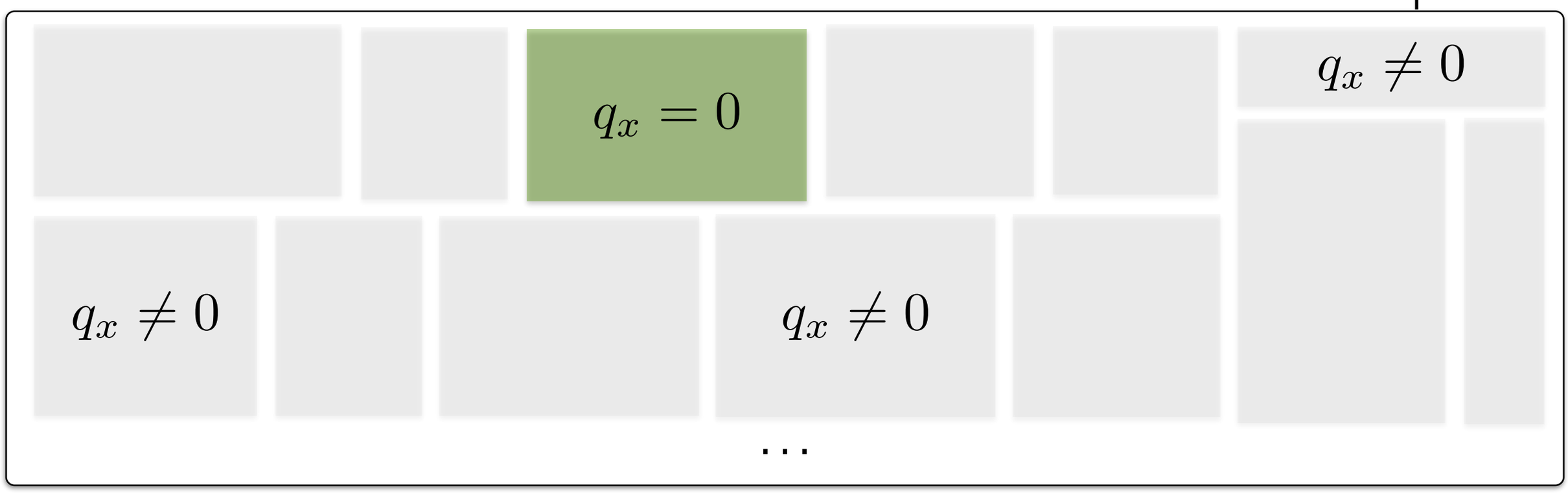
QUANTUM SIMULATION OF GAUGE FIELD THEORIES: THEORY DEVELOPMENTS

SU(2) gauge theory with matter in 1+1D

$$N_{\text{state}} \sim e^{pN}$$

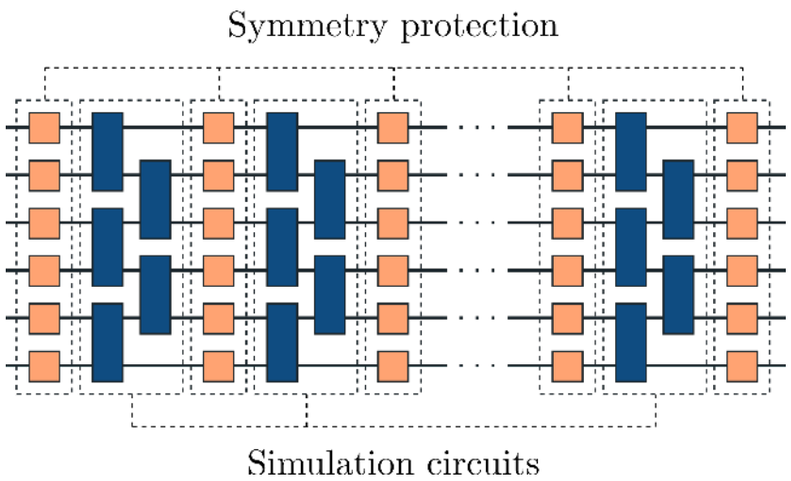


ZD, Raychowdhury, and Shaw, Phys. Rev. D 104, 074505, arXiv:2009.11802 [hep-lat]



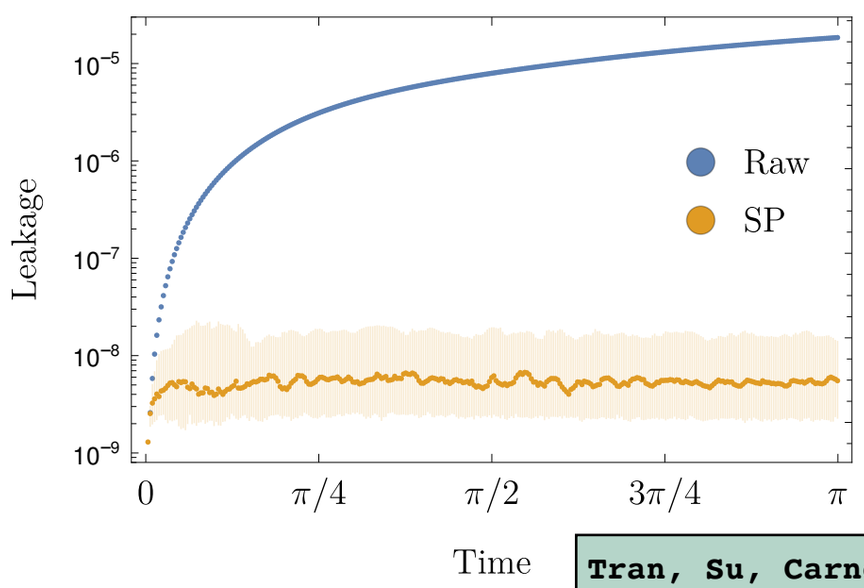
IDEAS TO SUPPRESS LEAKAGE TO UNPHYSICAL SECTOR IN THE SIMULATION

Can restore symmetries lost by Trotter expansion of the evolution…



$$U_t \approx \prod_{k=1}^{r} C_k^\dagger S_{\delta t} C_k. \qquad \text{with} \qquad [C, H] = 0 \quad \forall\, C \in \mathcal{S}.$$

Example: a 4-site Schwinger model

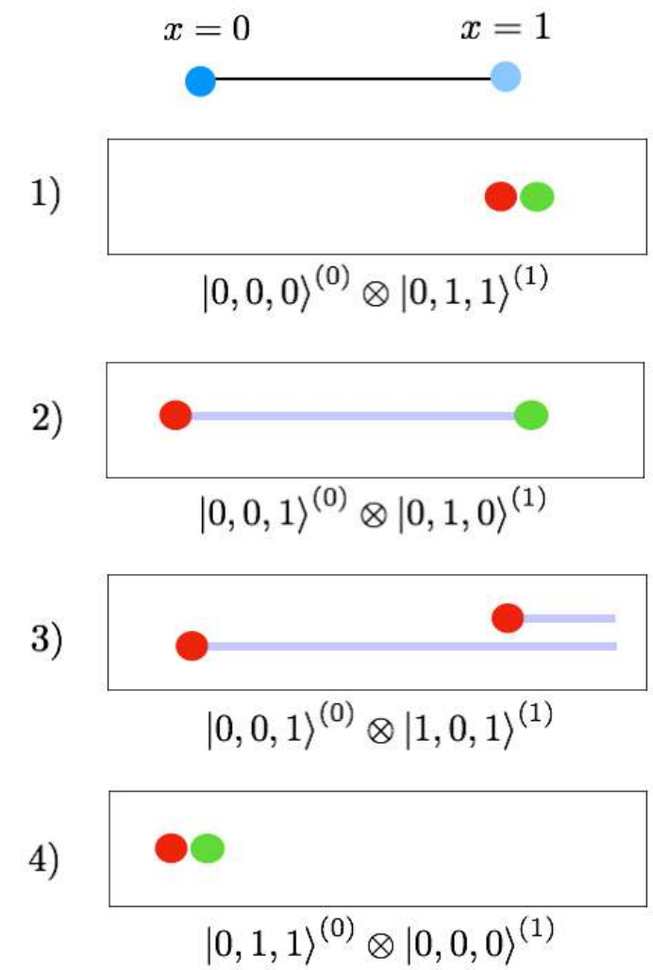


Tran, Su, Carney, Taylor, PRX Quantum 2, 010323 (2021).

Experimental implementation in: Nguyen, Tran, Zhu, Green, Huerta Alderete, ZD, Linke, *PRX Quantum* 3 (2022) 2, 020324.

See also Stannigel, et al Phys. Rev. Lett. 112, 120406, Halimeh, Lang, Mildenberger, Jiang, Hauke, arXiv:2007.00668 [quant-ph], Lamm, Lawrence, Yamauchi, arXiv:2005.12688 [quant-ph], and Kasper et al, arXiv:2012.08620 [quant-ph.

IDEAS TO SUPPRESS LEAKAGE TO UNPHYSICAL SECTOR IN THE SIMULATION



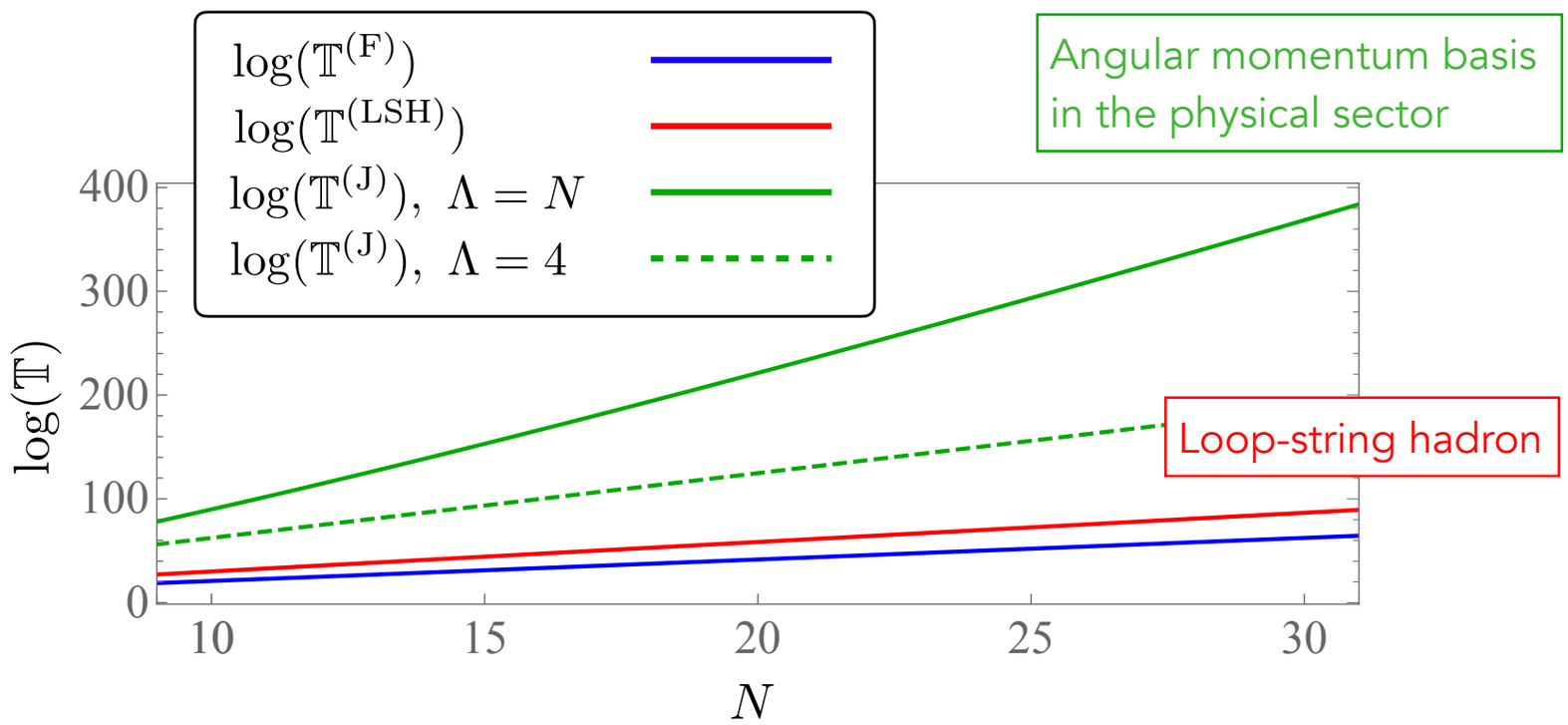
Loop-string-hadron formulation: Building blocks (quantum numbers) are already local and gauge-invariant.

Raychowdhury, Stryker, Phys. Rev. D 101, 114502 (2020).

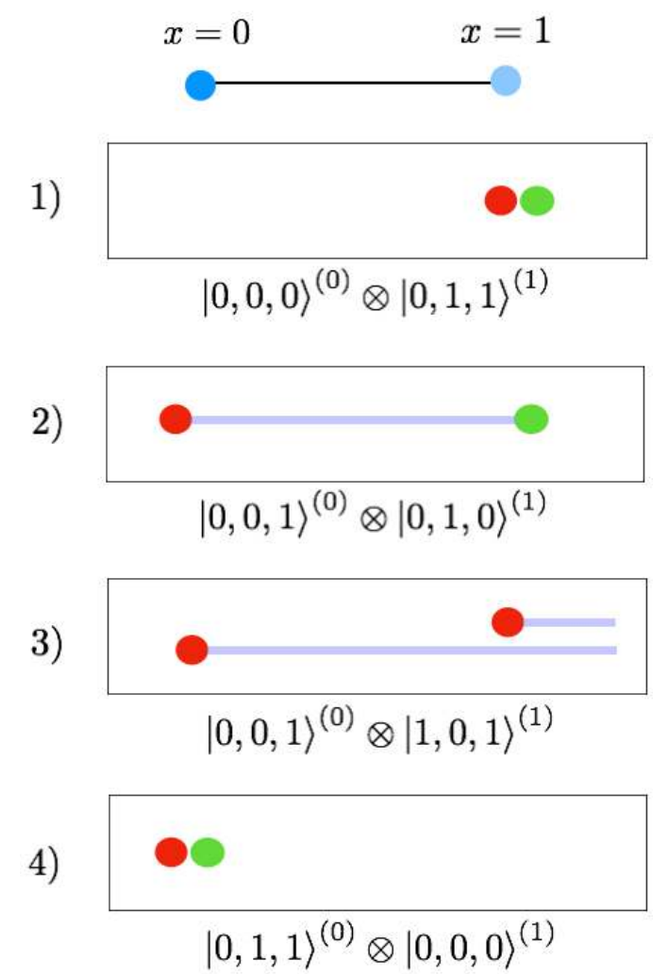
LSH for SU(3) under development by: Kadam, Raychowdhury, Stryker (2022).

IDEAS TO SUPPRESS LEAKAGE TO UNPHYSICAL SECTOR IN THE SIMULATION

The time complexity of classical Hamiltonian-simulation algorithms for each formulation.



ZD, Raychowdhury, and Shaw, Phys. Rev. D 104, 074505 (2020).



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MANY HAMILTONIAN FORMULATIONS OF GAUGE THEORIES EXIST, BUT WHICH ONE TO PICK?

Gauge-field theories (Abelian and non-Abelian):

Group-element representation
Zohar et al; Lamm et al

Prepotential formulation
Mathur, Raychowdhury et al

Loop-String-Hadron basis
Raychowdhury and Stryker

Link models, qubitization
Chandrasekharan, Wiese et al, Alexandru, Bedaque, et al.

Fermionic basis
Hamer et al; Martinez et al; Banuls et al

Bosonic basis
Cirac and Zohar

Light-front quantization
Kreshchuk, Love, Goldstien, Vary et al.; Ortega at al

Local irreducible representations
Byrnes and Yamamoto; Ciavarella, Klco, and Savage

Manifold lattices
Buser et al

Dual plaquette (magnetic) basis
Bender, Zohar et al; Kaplan and Styker; Unmuth-Yockey; Hasse et al; Bauer and Grabowska

Spin-dual representation
Mathur et al

Scalar field theory

Field basis
Jordan, Lee, and Preskill

Continuous-variable basis
Pooser, Siopsis et al

Harmonic-oscillator basis
Klco and Savage

Single-particle basis
Barata , Mueller, Tarasov, and Venugopalan.

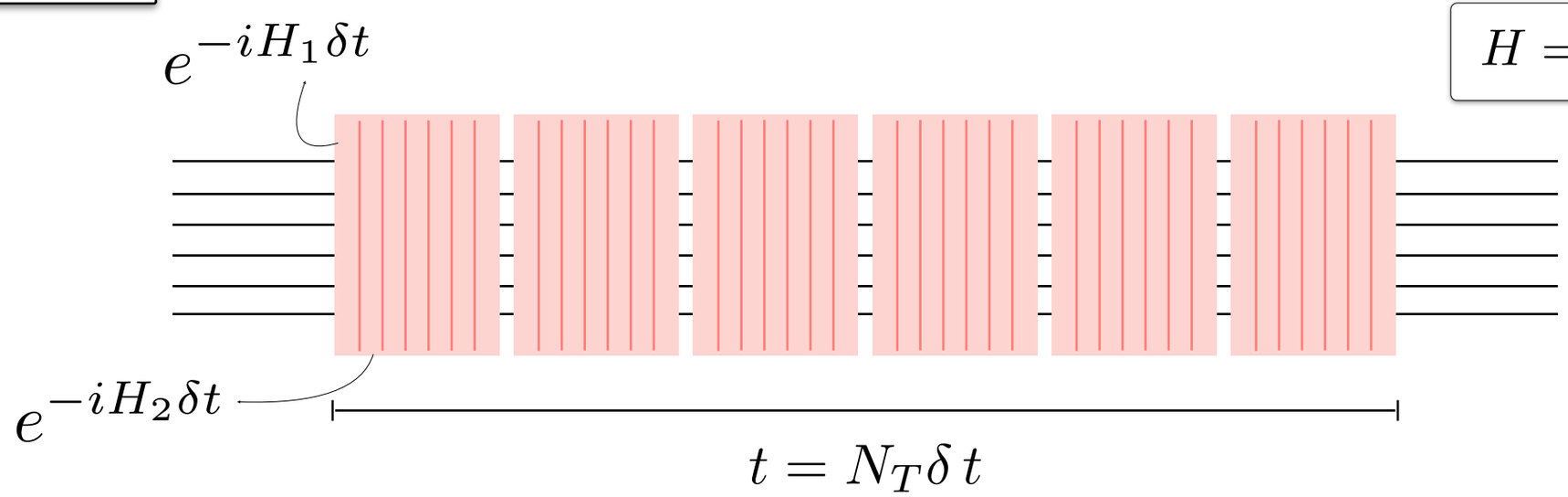
Lots of interesting ideas using **analog simulators** will not be covered here.

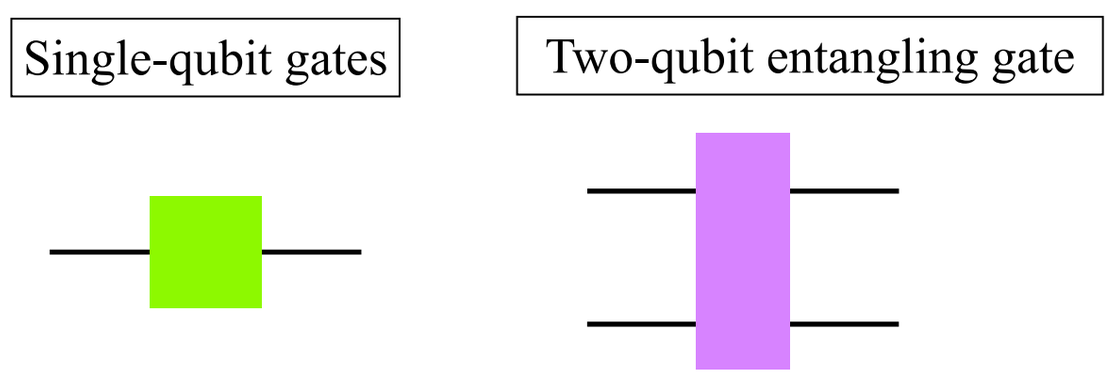
Algorithmic developments [Digital]

- Near- and far-term algorithms with bounded errors and resource requirement for gauge theories?
- Can given formulation/encoding reduce qubit and gate resources?
- Should we develop gauge-invariant simulation algorithms?
- How do we do state preparation and compute observables like scattering amplitudes?

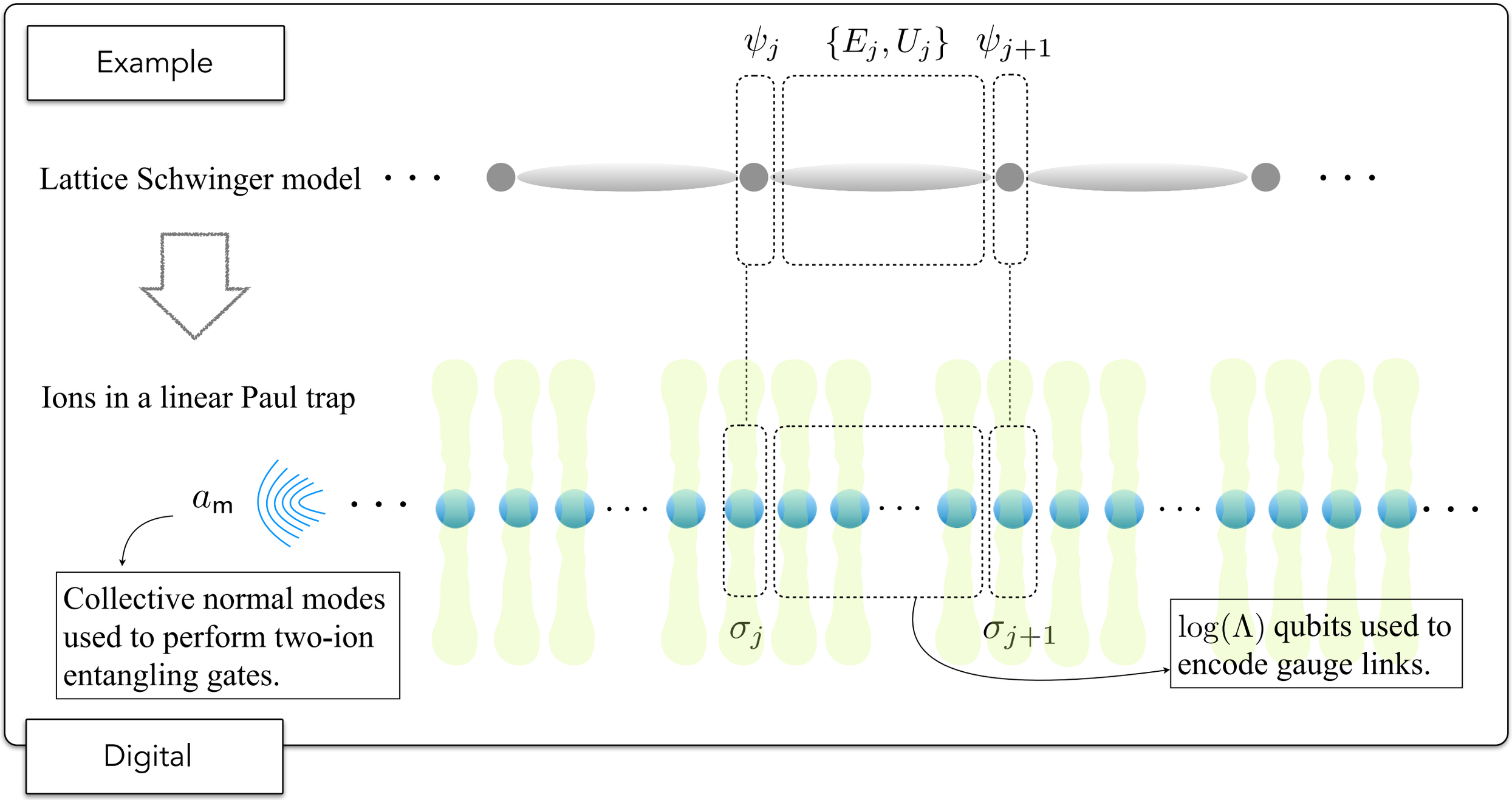
Digital

$$H = H_1 + H_2 + \cdots$$





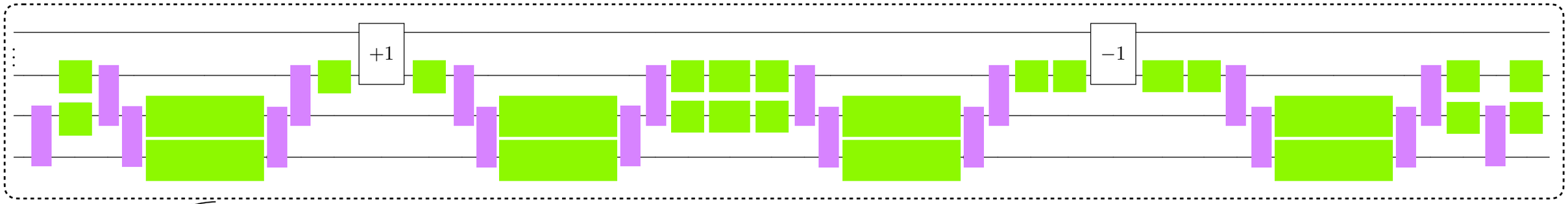
How many qubits and gates are required to achieve accuracy ϵ in a given observables? Are there algorithms that scale optimally?



$$H = -ix \sum_{n=1}^{N-1} \left[\psi_n^\dagger U_n \psi_{n+1} - \text{h.c.}\right] + \sum_{n=1}^{N-1} E_n^2 + \mu \sum_{n=1}^{N} (-1)^n \psi_n^\dagger \psi_n$$

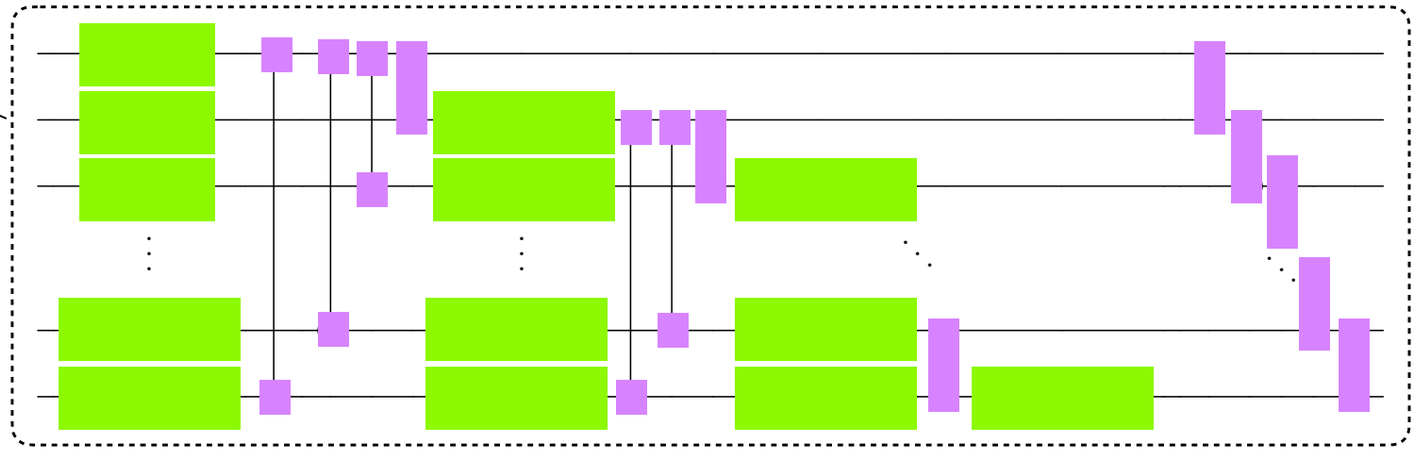
Circuit and recourse analysis

Shaw, Lougovski, Stryker, Wiebe, Quantum 4, 306 (2020).



Sample gauge-fermion interaction block

Part of electric field interactions acting on gauge DOF registers



Near term cost

	$\delta_g = 10^{-3}$		$\delta_g = 10^{-4}$		$\delta_g = 10^{-5}$		$\delta_g = 10^{-6}$		$\delta_g = 10^{-7}$	
	$\tilde{\epsilon}^2$	CNOT	$\tilde{\epsilon}^2$	CNOT	$\tilde{\epsilon}^2$	CNOT	$\tilde{\epsilon}^2$	CNOT	$\tilde{\epsilon}^2$	CNOT
$x = 10^{-2}$	—	7.3e4	—	1.6e5	—	3.4e5	—	7.3e5	5.6e-2	1.6e6
$x = 10^{-1}$	—	1.6e4	—	3.5e4	—	7.5e4	5.9e-2	1.6e5	2.7e-3	3.5e5
$x = 1$	—	4.6e3	—	9.9e3	1.0e-1	2.1e4	4.7e-3	4.6e4	2.2e-4	9.9e4
$x = 10^2$	—	2.8e3	8.3e-1	6.1e3	3.8e-2	1.3e4	1.8e-3	2.8e4	8.2e-5	6.0e4

$$H = -ix \sum_{n=1}^{N-1} \left[\psi_n^\dagger U_n \psi_{n+1} - \text{h.c.}\right] + \sum_{n=1}^{N-1} E_n^2 + \mu \sum_{n=1}^{N} (-1)^n \psi_n^\dagger \psi_n$$

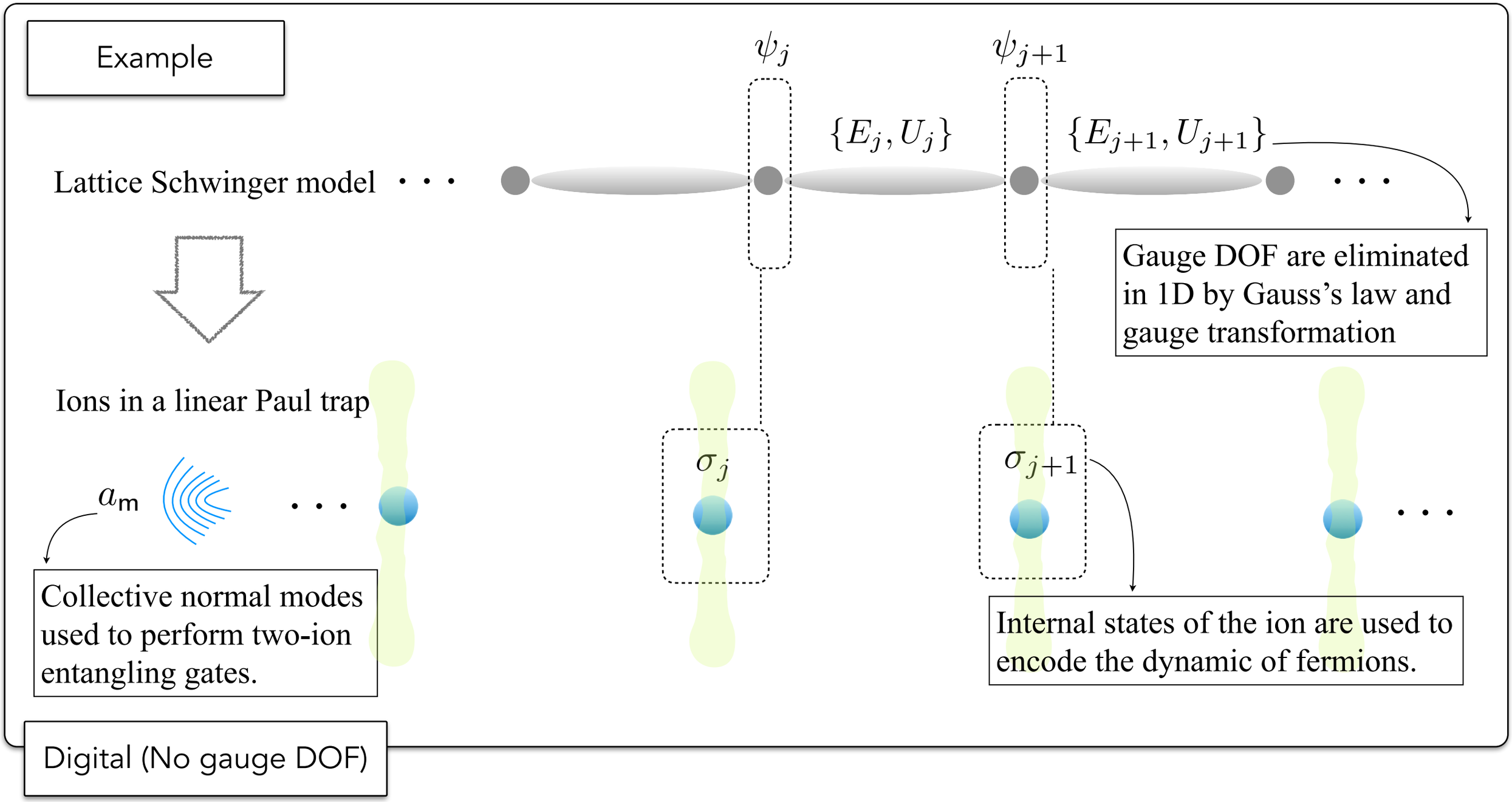
Implementation, benchmark, and co-design

What is the capability limit of the hardware for gauge-theory simulations so far?

What is the nature of noise in hardware and how can it best be mitigated?

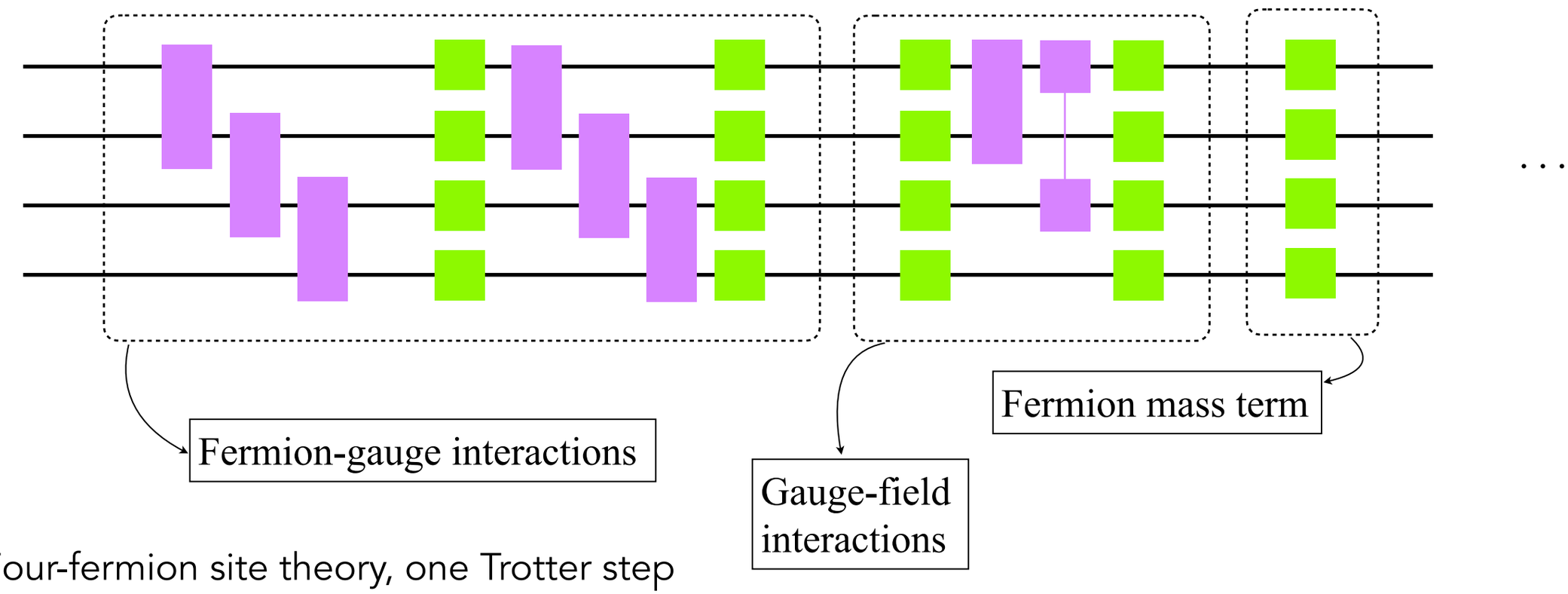
Can we co-design dedicated systems for gauge-theory simulations?

Can digital and analog ideas be combined to facilitate simulations of field theories?



$$H = x \sum_{n=1}^{N-1} \left[\sigma_+^{(n)} \sigma_-^{(n+1)} + \sigma_+^{(n+1)} \sigma_-^{(n)} \right] + \sum_{n=1}^{N-1} \left[\epsilon_0 + \frac{1}{2} \sum_{m=1}^{n} \left(\sigma_z^{(m)} + (-1)^m \right) \right]^2 + \frac{\mu}{2} \sum_{n=1}^{N} (-1)^n \sigma_z^{(n)}$$

Associated quantum circuit for Trotterized evolution:



Four-fermion site theory, one Trotter step

$$H = x \sum_{n=1}^{N-1} \left[\sigma_+^{(n)} \sigma_-^{(n+1)} + \sigma_+^{(n+1)} \sigma_-^{(n)} \right] + \sum_{n=1}^{N-1} \left[\epsilon_0 + \frac{1}{2} \sum_{m=1}^{n} \left(\sigma_z^{(m)} + (-1)^m \right) \right]^2 + \frac{\mu}{2} \sum_{n=1}^{N} (-1)^n \sigma_z^{(n)}$$

Klco, Savage, et al, Phys. Rev. A 98, 032331 (2018).

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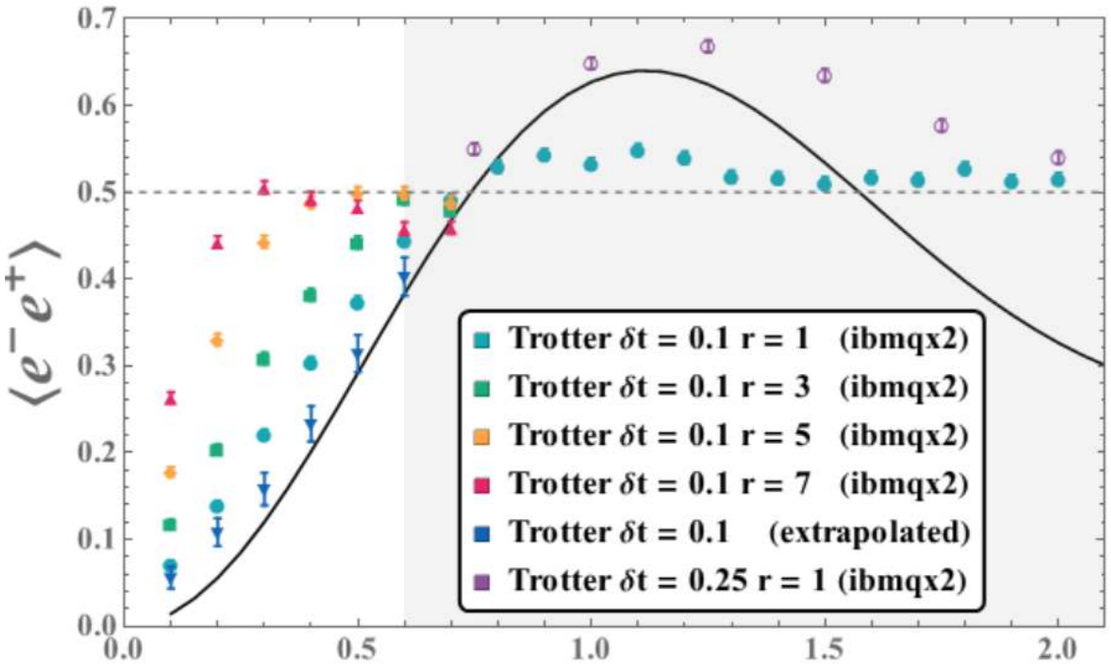

Not the spin formulation: a 2-qubit reduction of 4-qubit simulation.

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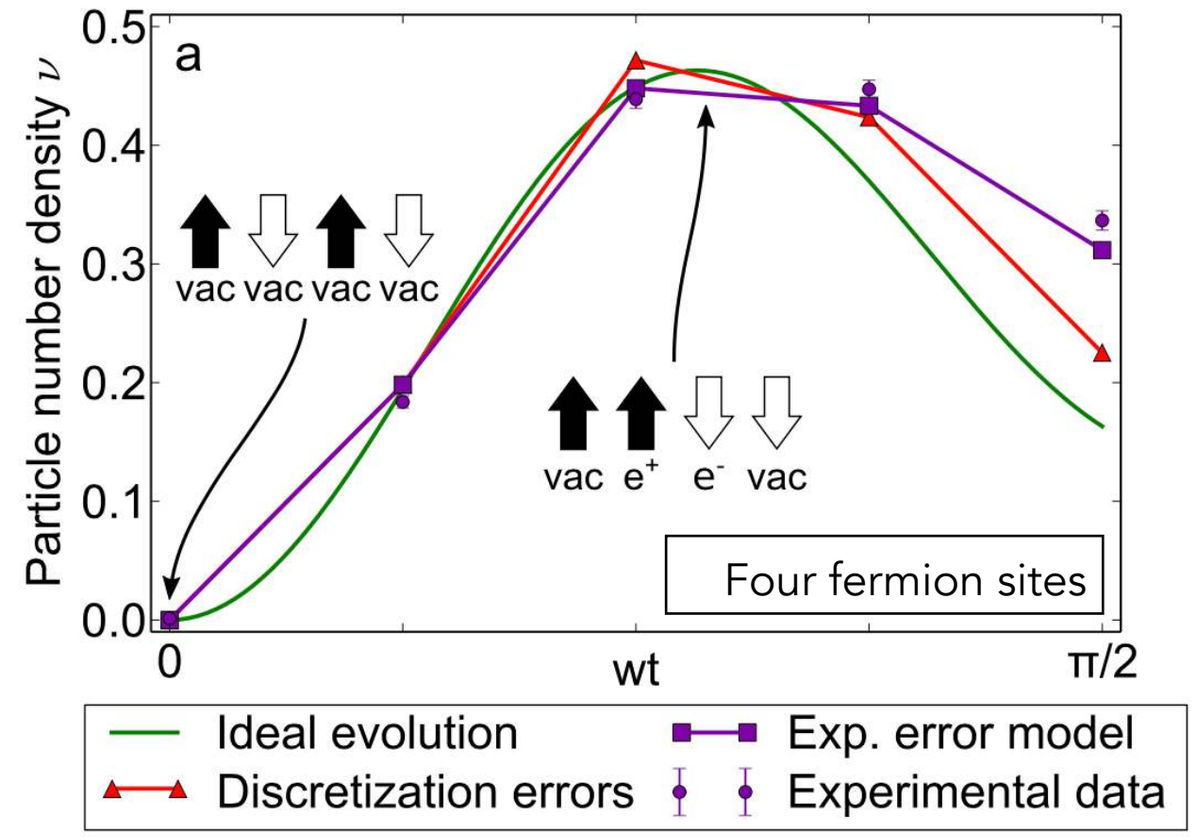
Martinez et al, Nature 534, 516 EP (2016).



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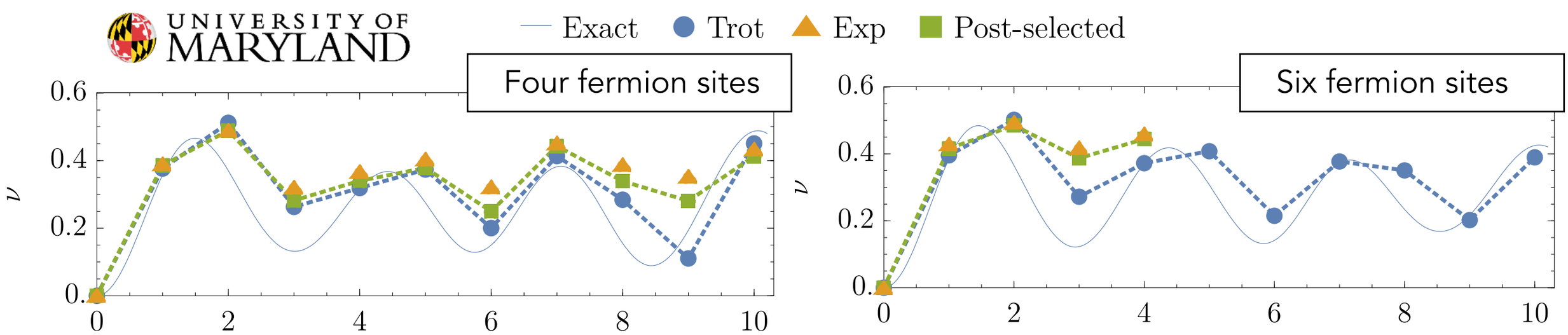

80 entangling gates!

90 entangling gates!

Nguyen, Tran, Zhu, Green, Huerta Alderete, ZD, Linke, *PRX Quantum* 3 (2022) 2, 020324.

Klco, Savage, et al, Phys. Rev. A 98, 032331 (2018).

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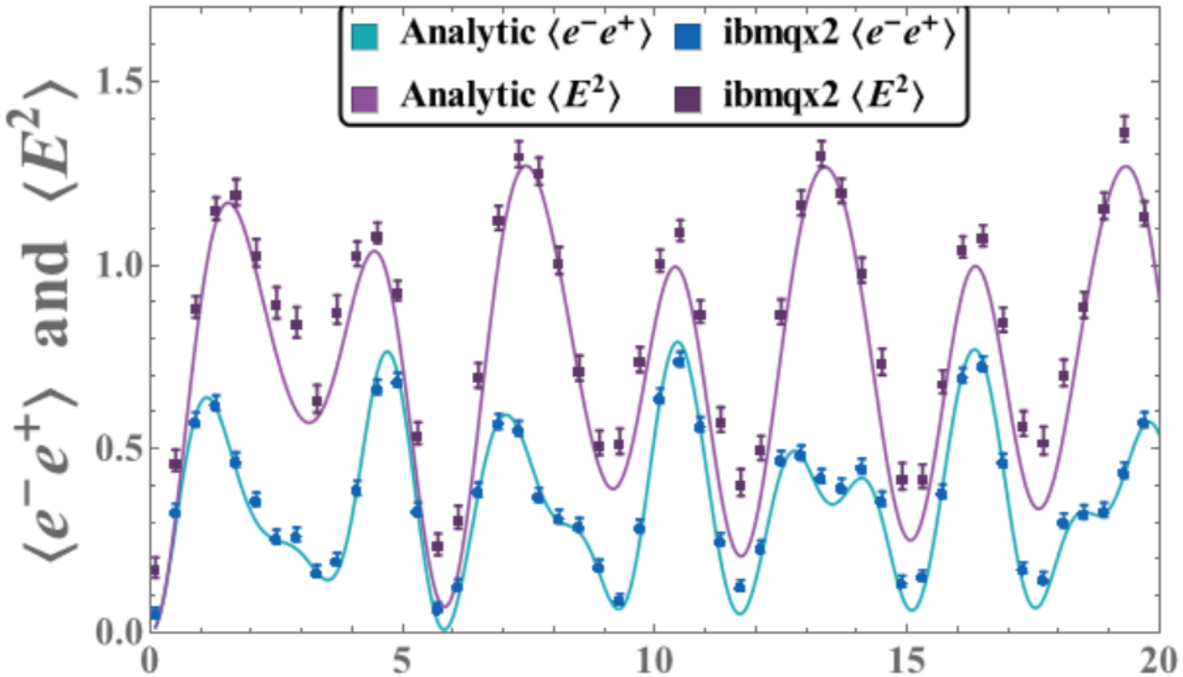

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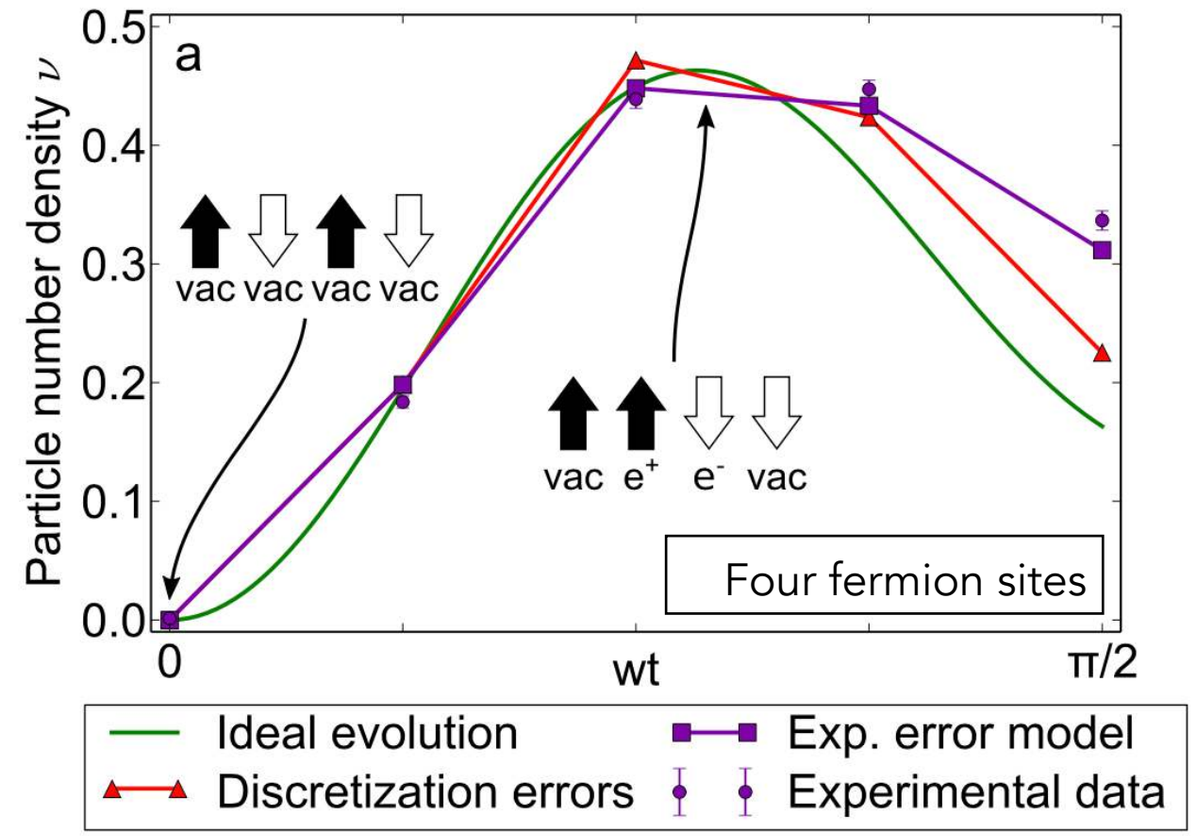

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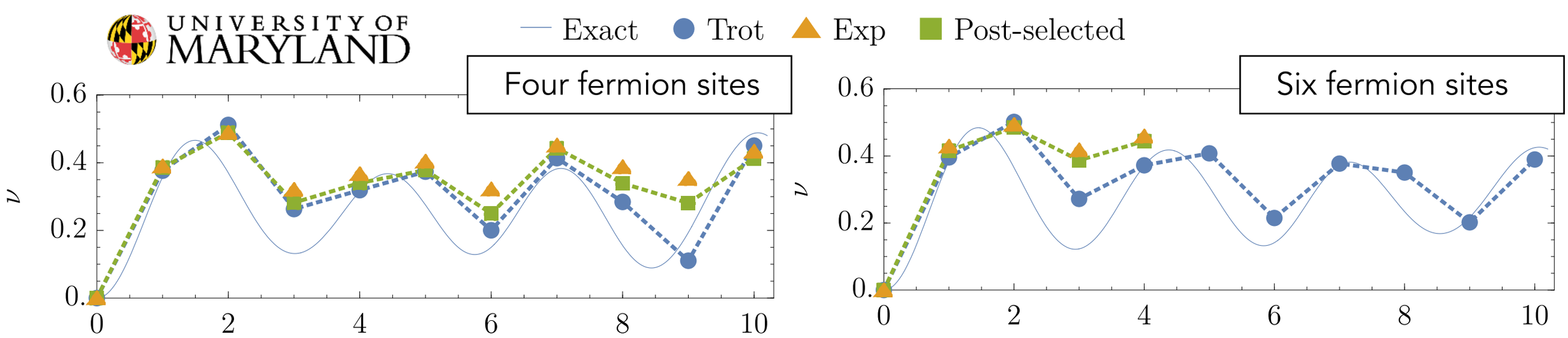

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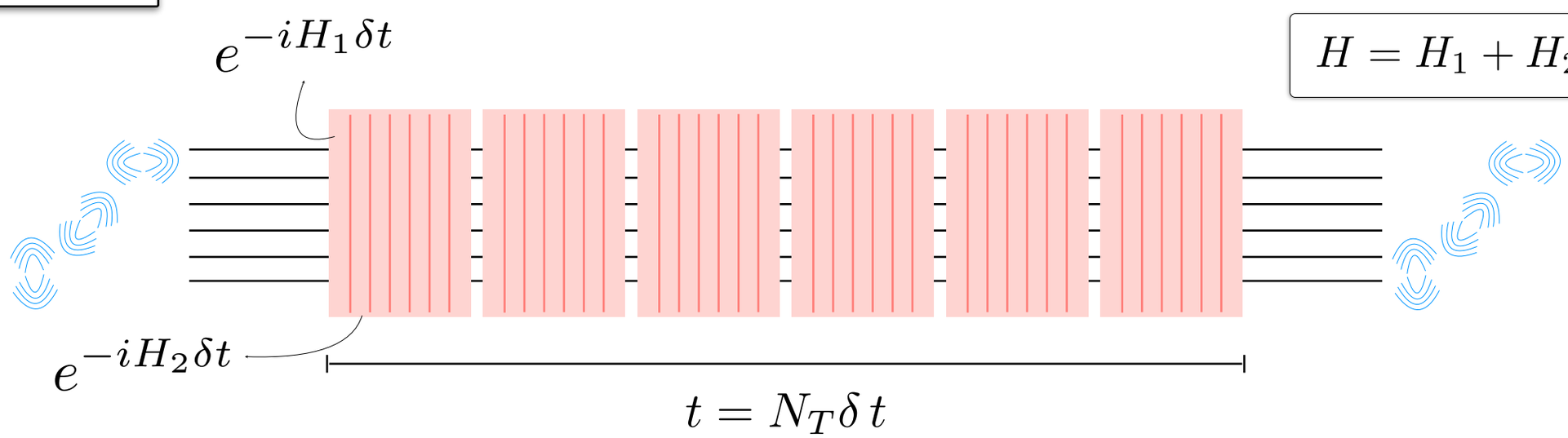
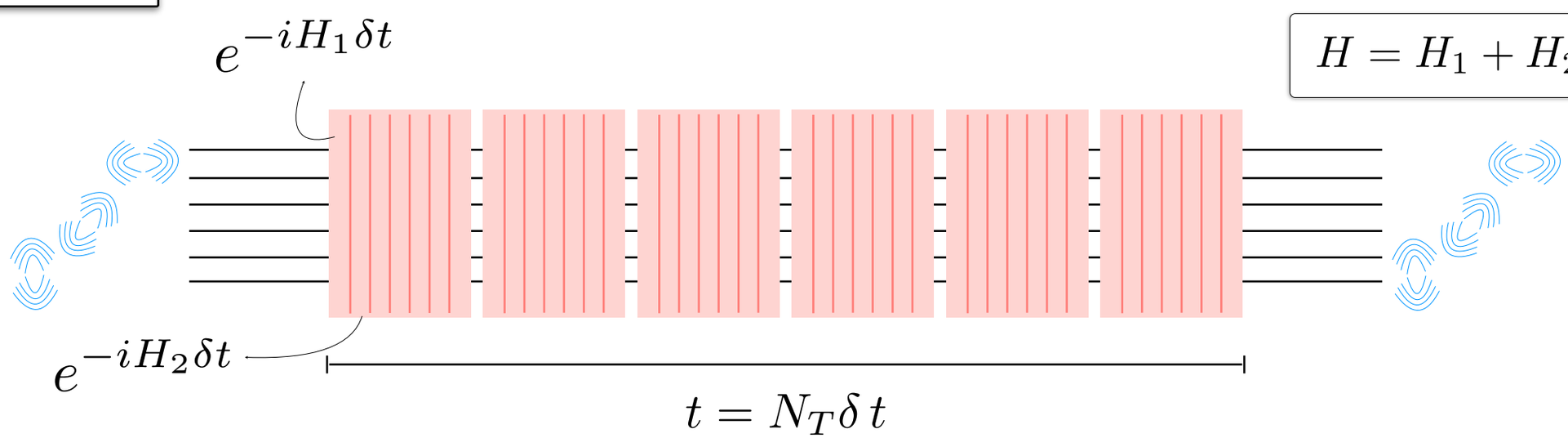
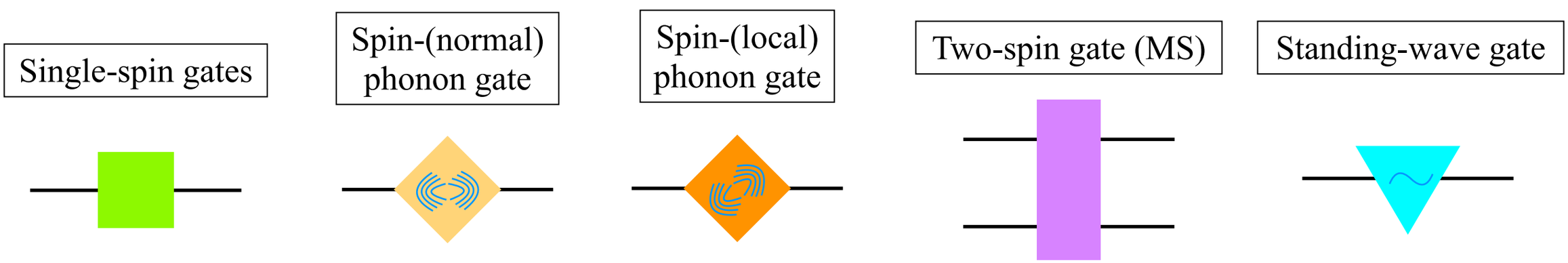
Analog-Digital



Analog-Digital



$$H = H_1 + H_2 + \cdots$$



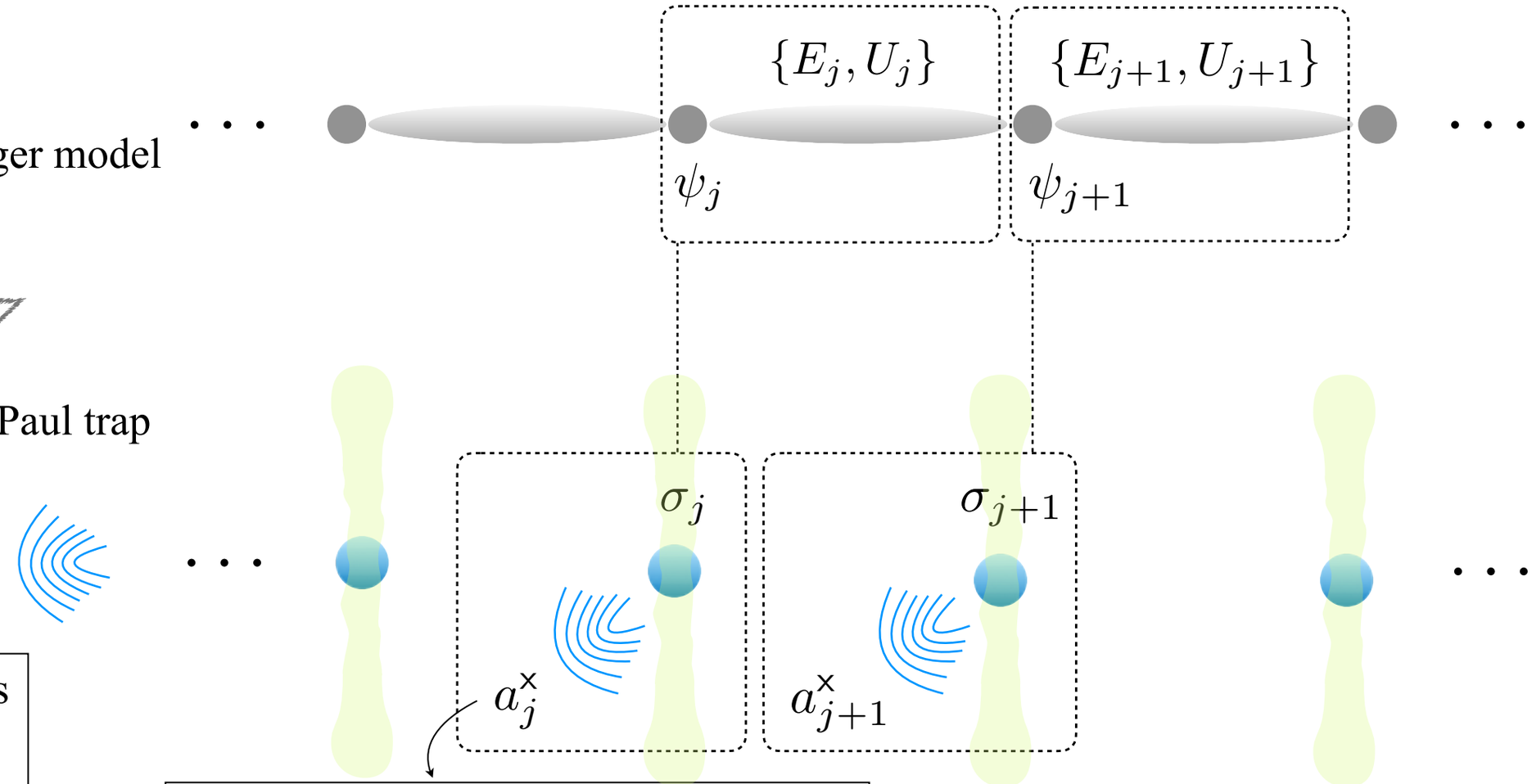
ZD, Linke, Pagano, Phys. Rev. Research 3, 043072 (2021).

How many qubits and gates are required to achieve accuracy ϵ in a given observables? Are there algorithms that scale optimally?

Example

Lattice Schwinger model

Ions in a linear Paul trap



Collective normal modes used to perform two-ion entangling gates.

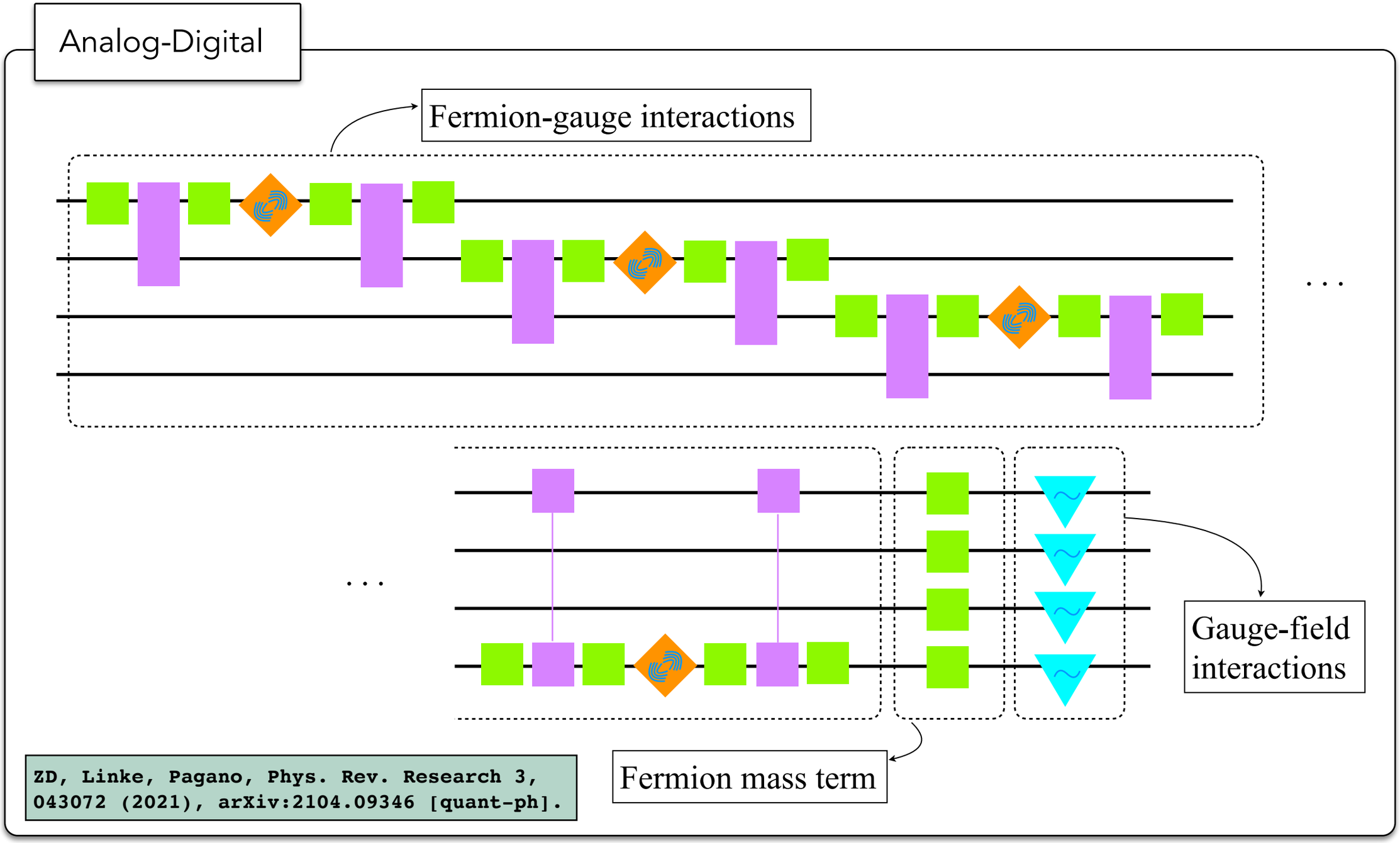
Local transverse modes used to encode the dynamic of the gauge fields.

Analog-Digital

ZD, Linke, Pagano, Phys. Rev. Research 3, 043072 (2021).

See also Casanova et al, Phys. Rev. Lett. 108, 190502 (2012), Lamata et al, EPJ Quant. Technol. 1, 9 (2014), and Mezzacapo et al, Phys. Rev. lett. 109, 200501 (2012) for analog-digital approaches to other interacting fermion-boson theories.

$$H = -ix \sum_{n=1}^{N-1} \left[\psi_n^\dagger U_n \psi_{n+1} - \text{h.c.}\right] + \sum_{n=1}^{N-1} E_n^2 + \mu \sum_{n=1}^{N} (-1)^n \psi_n^\dagger \psi_n$$



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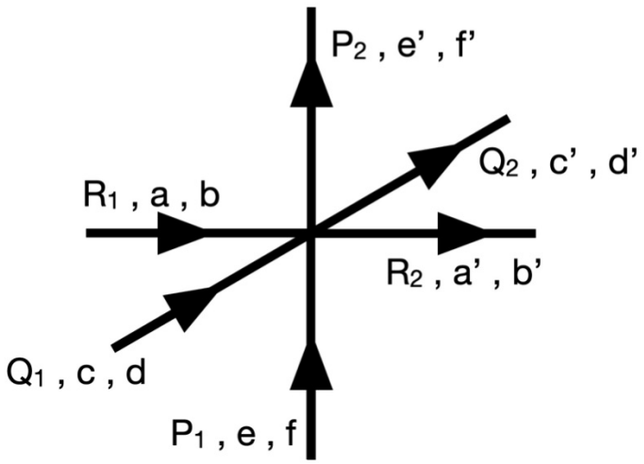
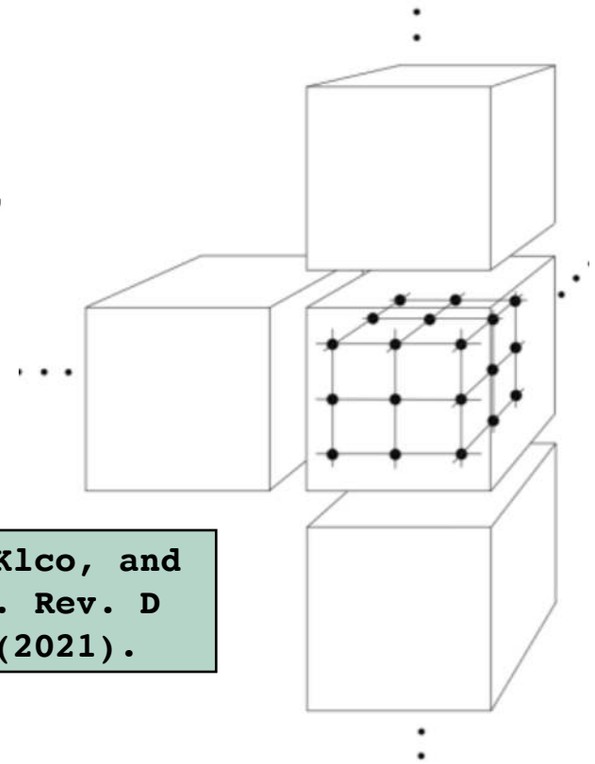
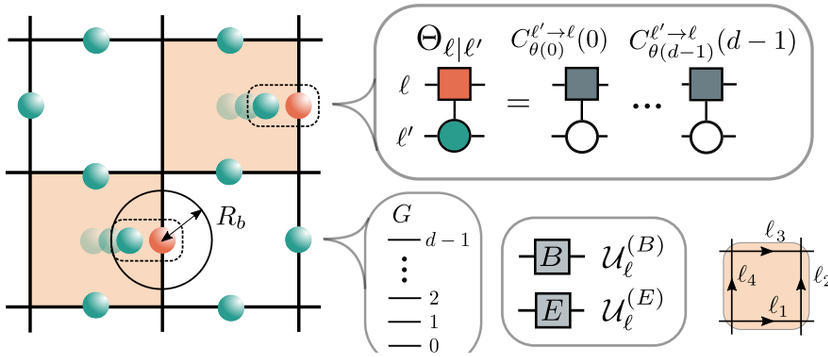
More co-design examples: Multi-dimensional local Hilbert spaces and multi-mode interactions

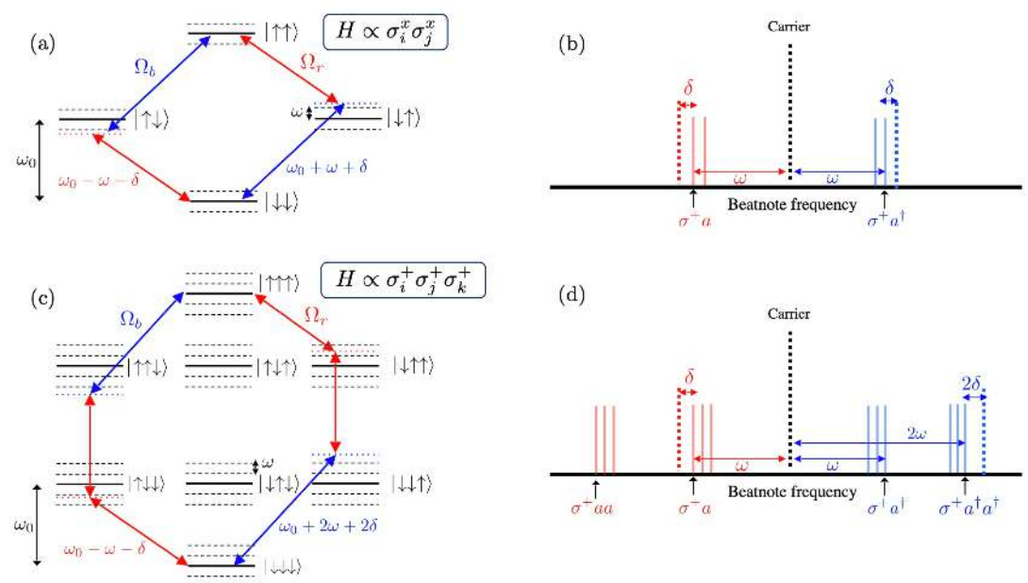




Ciavarella, Klco, and Savage, Phys. Rev. D 103, 094501 (2021).



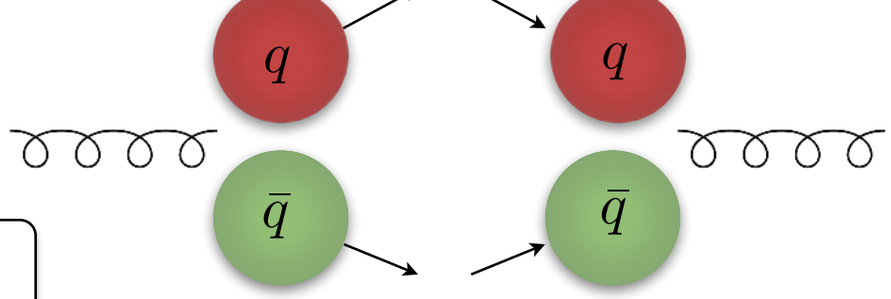
González-Cuadra, Zache, Carrasco, Kraus, Zoller, arXiv:2203.15541 [quant-ph].

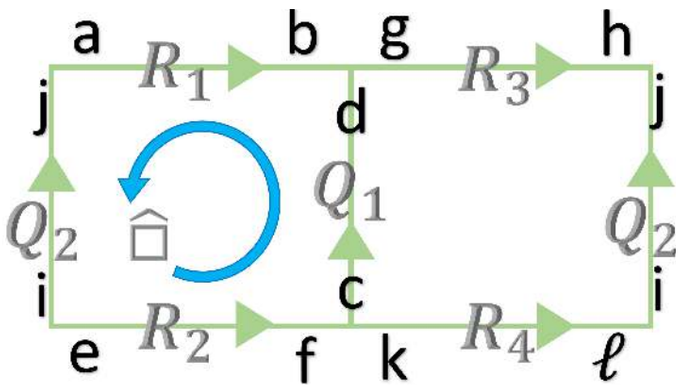


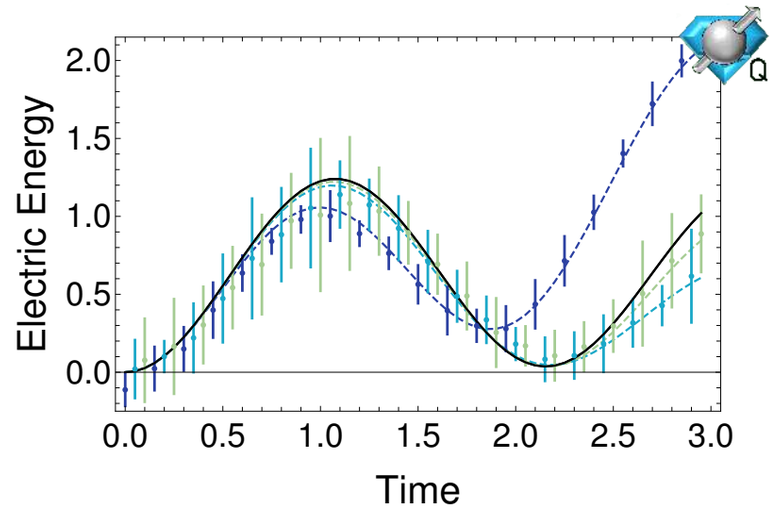
Andrade, ZD, Grass, Hafezi, Pagano, Seif, arXiv:2108.01022 [quant-ph], Bermudez et al, Pays.Rev.A79, 060303 R (2009), Katz, Centina, Monroe, arXiv:2202.04230 [quant-ph].

Finally a few more examples showcasing progress in hardware implementation of a range of QCD-inspired problems…

DIGITAL COMPUTATIONS OF NON-ABELIAN LGTs



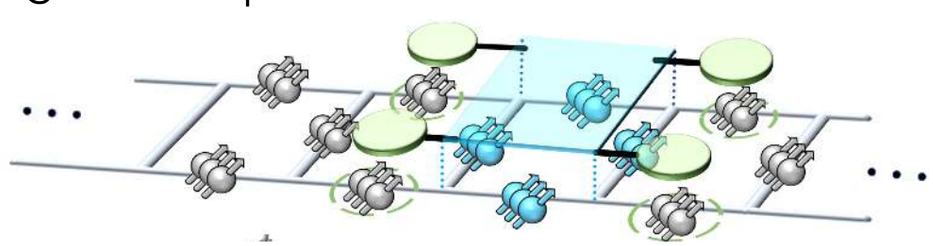


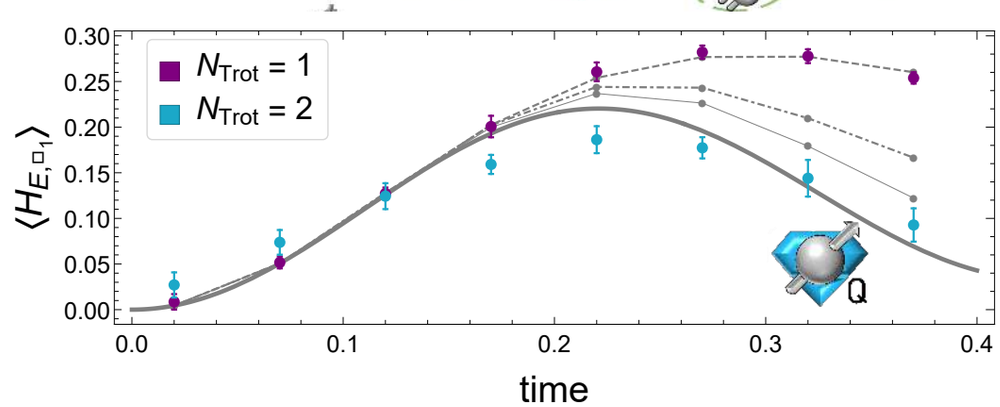


Real-time dynamic of pure SU(3) with global irrupts on IBM

Ciavarella, Klco, and Savage, Phys. Rev. D 103, 094501 (2021).

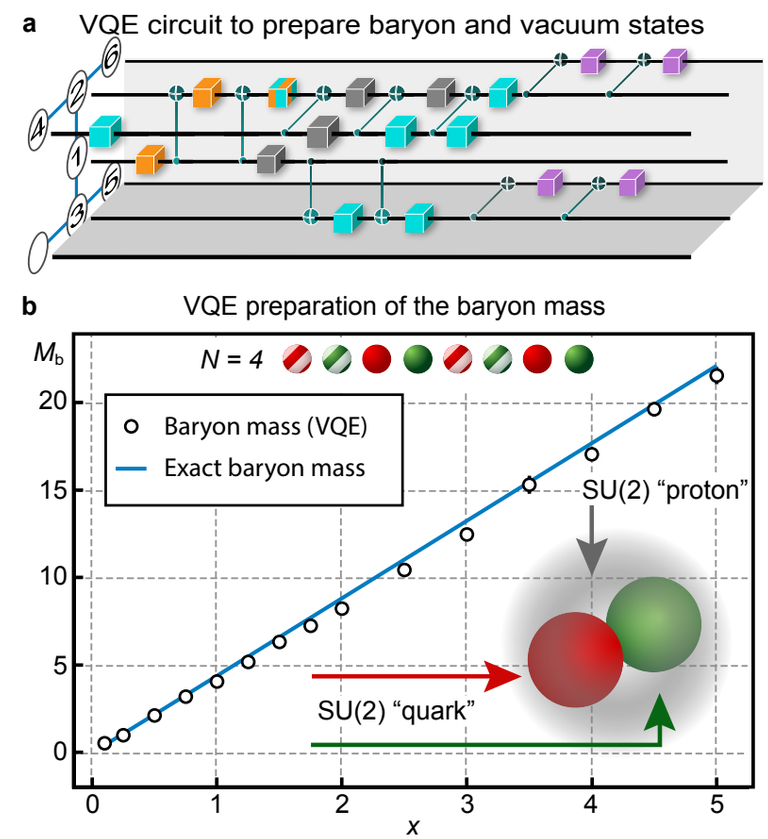
Real-time dynamic of pure SU(2) with global irreps on IBM





Klco, Savage, and Stryker, Phys. Rev. D 101, 074512 (2020).

Low-lying spectrum of SU(2) with matter in 1+1 D on IBM



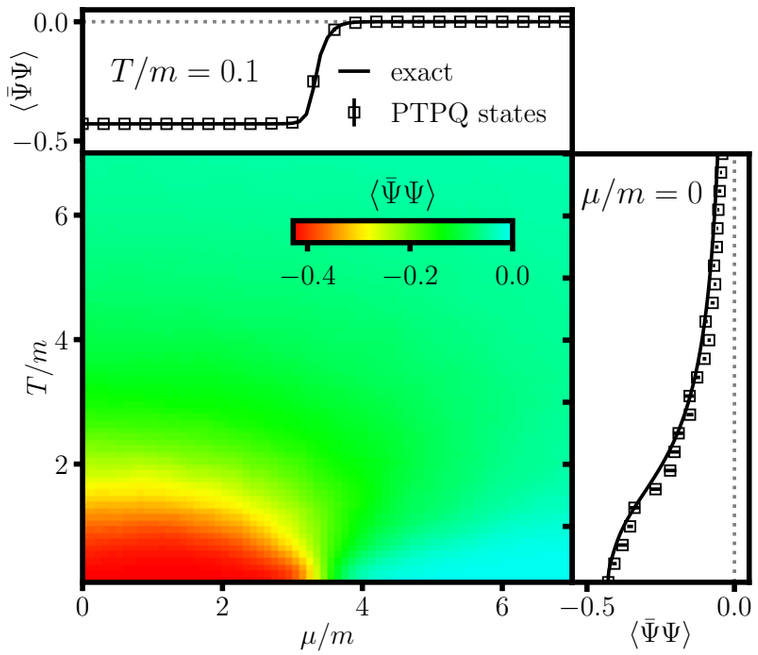
Atas et al, Nature Communications 12, 6499 (2021). SU(3) example: Atas et al: arXiv:2207.03473 [quant-ph].

See also studies on D-wave annealers: Rahman et al, Phys. Rev. D 104, 034501 (2021), Illa and Savage, arXiv:2202.12340 [quant-ph], Farrel et al, arXiv:2207.01731 [quant-ph].

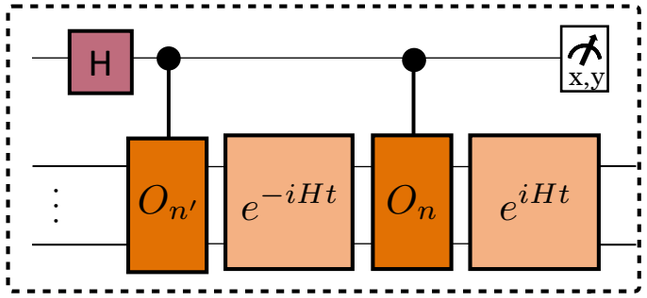
THERMAL PHASE DIAGRAM AND HEAVY QUARKONIA MOTION IN QGP?

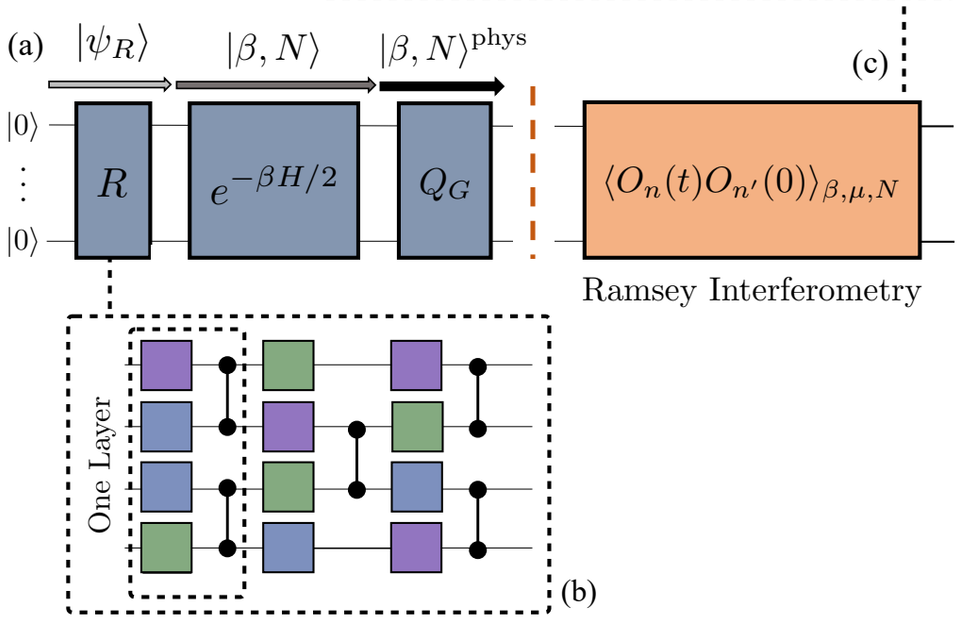
Toward Quantum Computing Phase Diagrams of Gauge Theories with Thermal Pure Quantum States, ZD, Mueller, Powers, arXiv:2208.13112 [hep-lat] (2022).

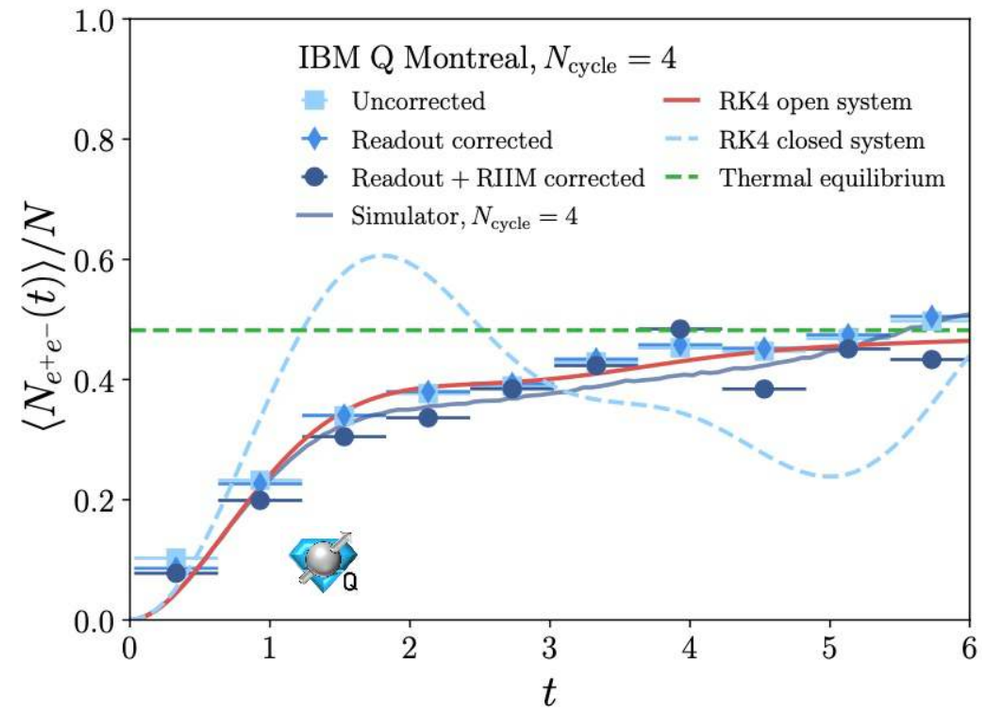
Phase diagram of Z_2^{1+1} with fermions



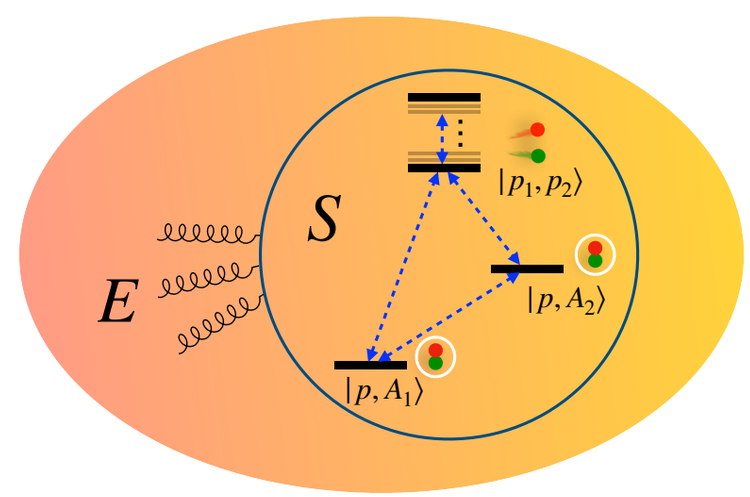
Preparing thermal states on a quantum computer





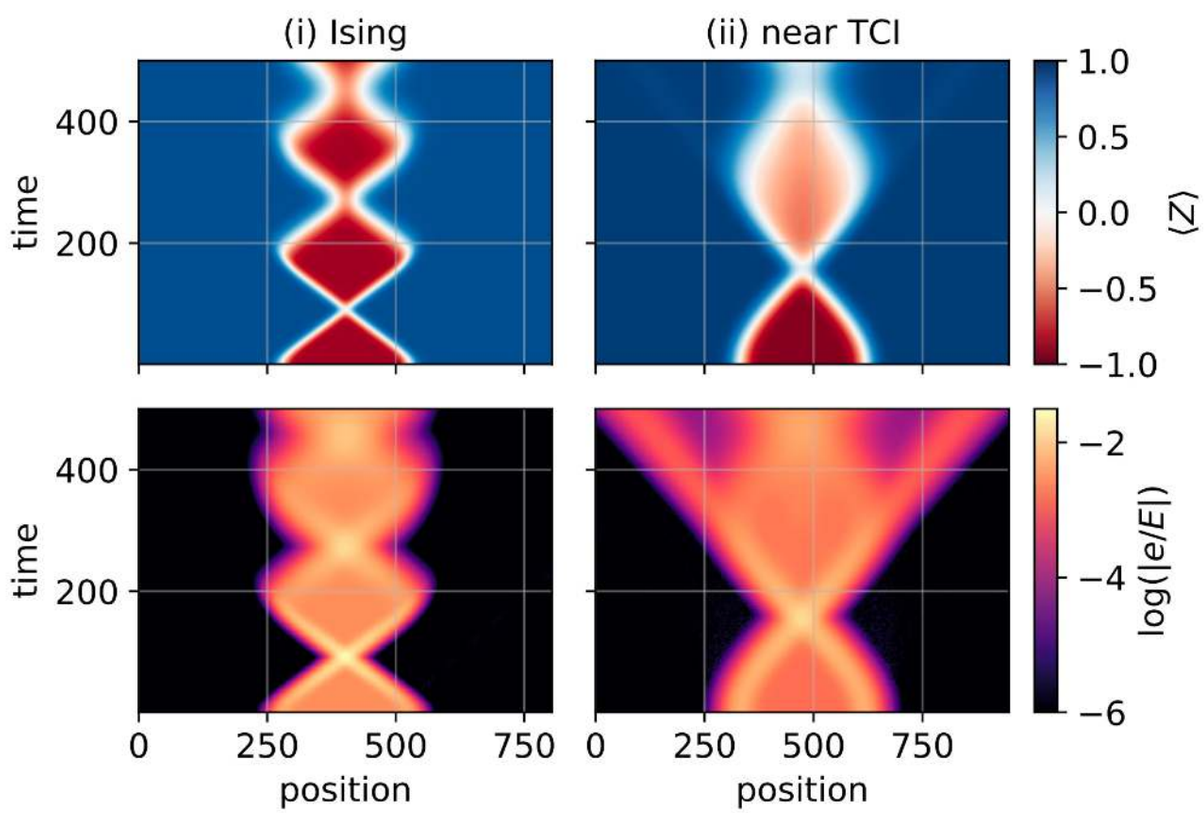


$q\bar{q}$ moving in medium

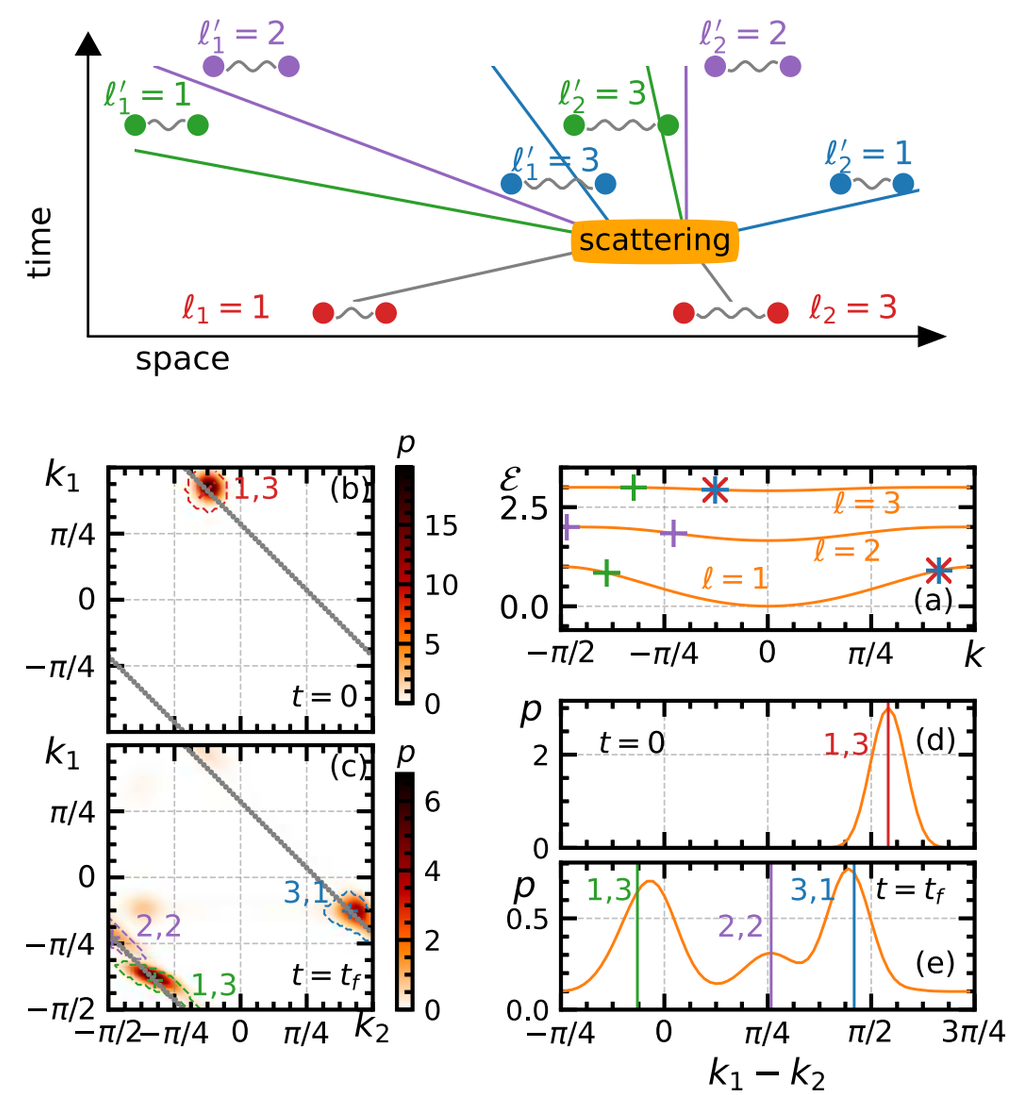


de Jong, Metcal, Mulligan, Ploskon, Ringer, and, Yao, *Phys.Rev.D* 104 (2021) 5, 051501.

FIRST STEPS TOWARD SCATTERING IN SPIN SYSTEMS
— NUMERICAL SIMULATIONS —



Ashley Milsted, Liu, John Preskill, and Vidal, PRX Quantum 3 (2022) 2, 020316.

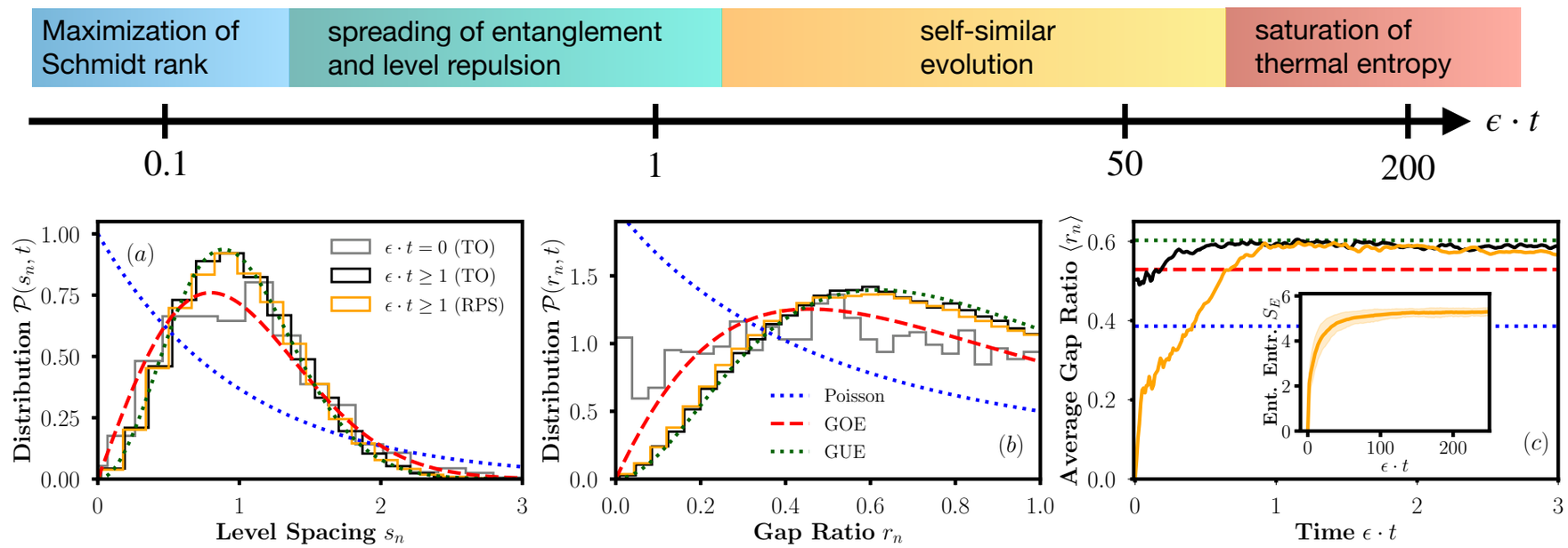


Surace, Lerose, New J. Phys. 23 (2021) 062001.

EMERGING UNDERSTANDING OF THERMALIZATION IN SIMPLE GAUGE THEORIES

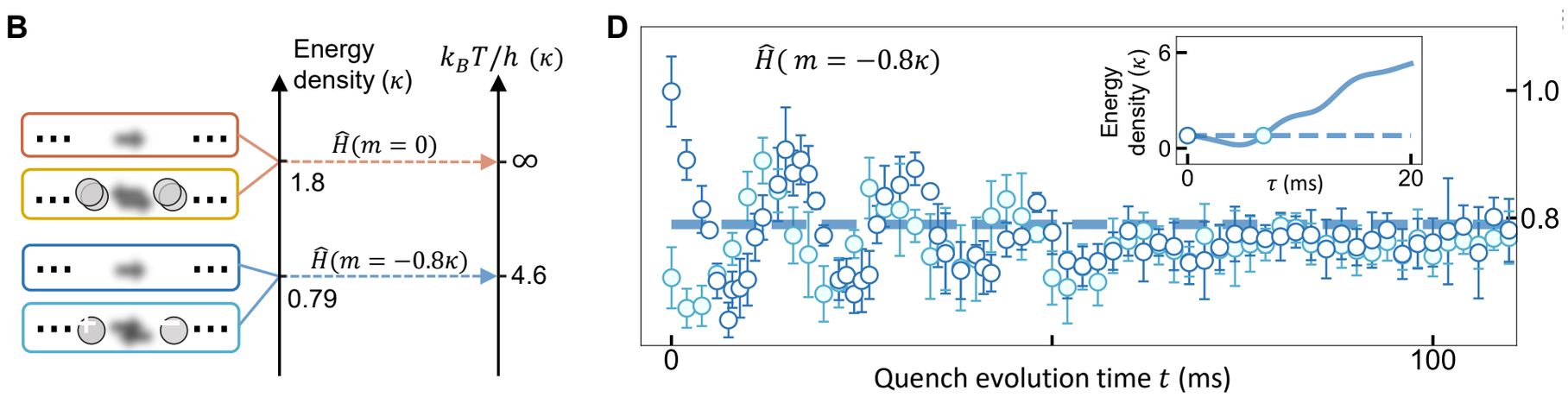
Numerical study of Z_2 LGT in 2+1 D

Mueller, Zache, Ott, Phys. Rev. Lett. 129, 011601 (2022).



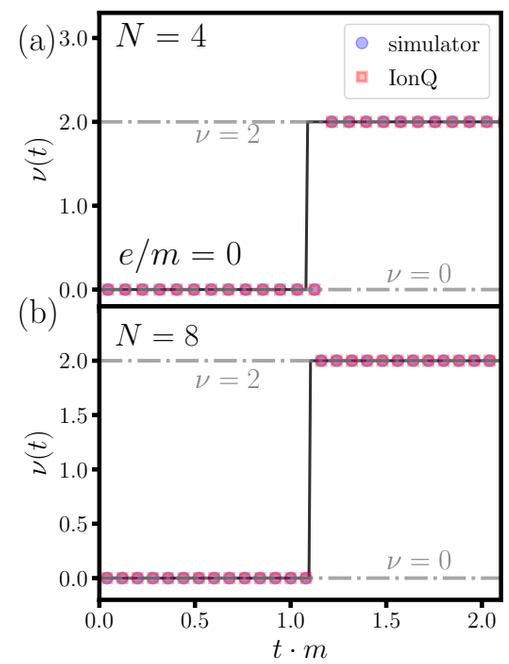
Quantum Link Model in a 70-site analog simulator

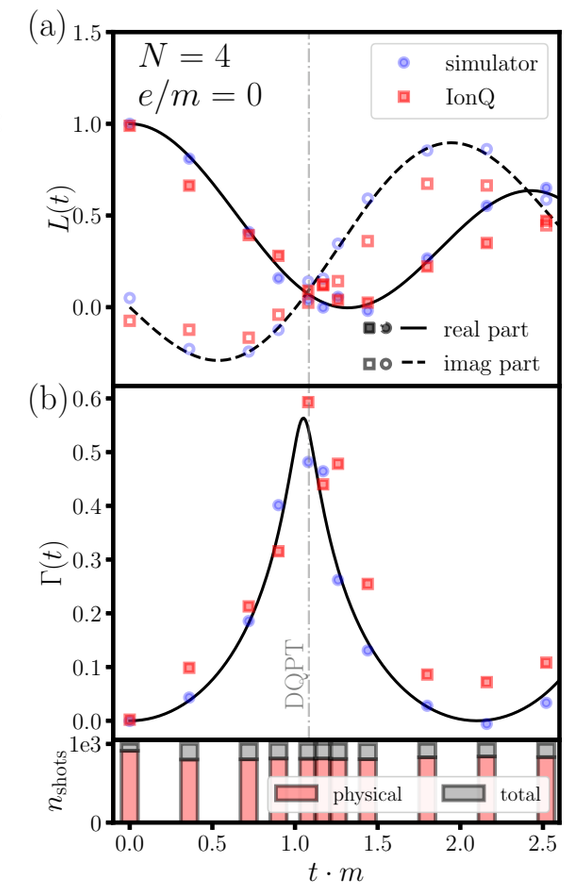
Zhou et al, *Science* 377 (2022) 6603.

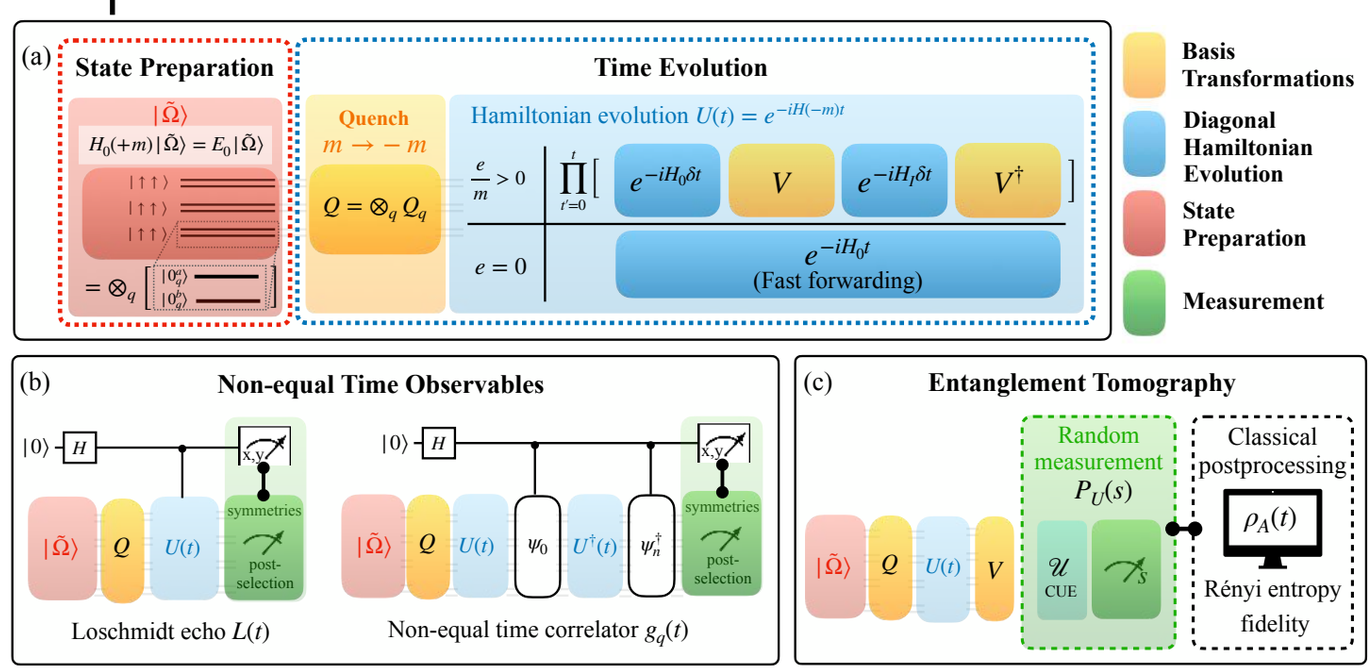


AND GENERALLY NON-EQUILIBRIUM AND TOPOLOGICAL PHENOMENA

A dynamical phase transition and topological order in lattice Schwinger model with an IonQ quantum computer:



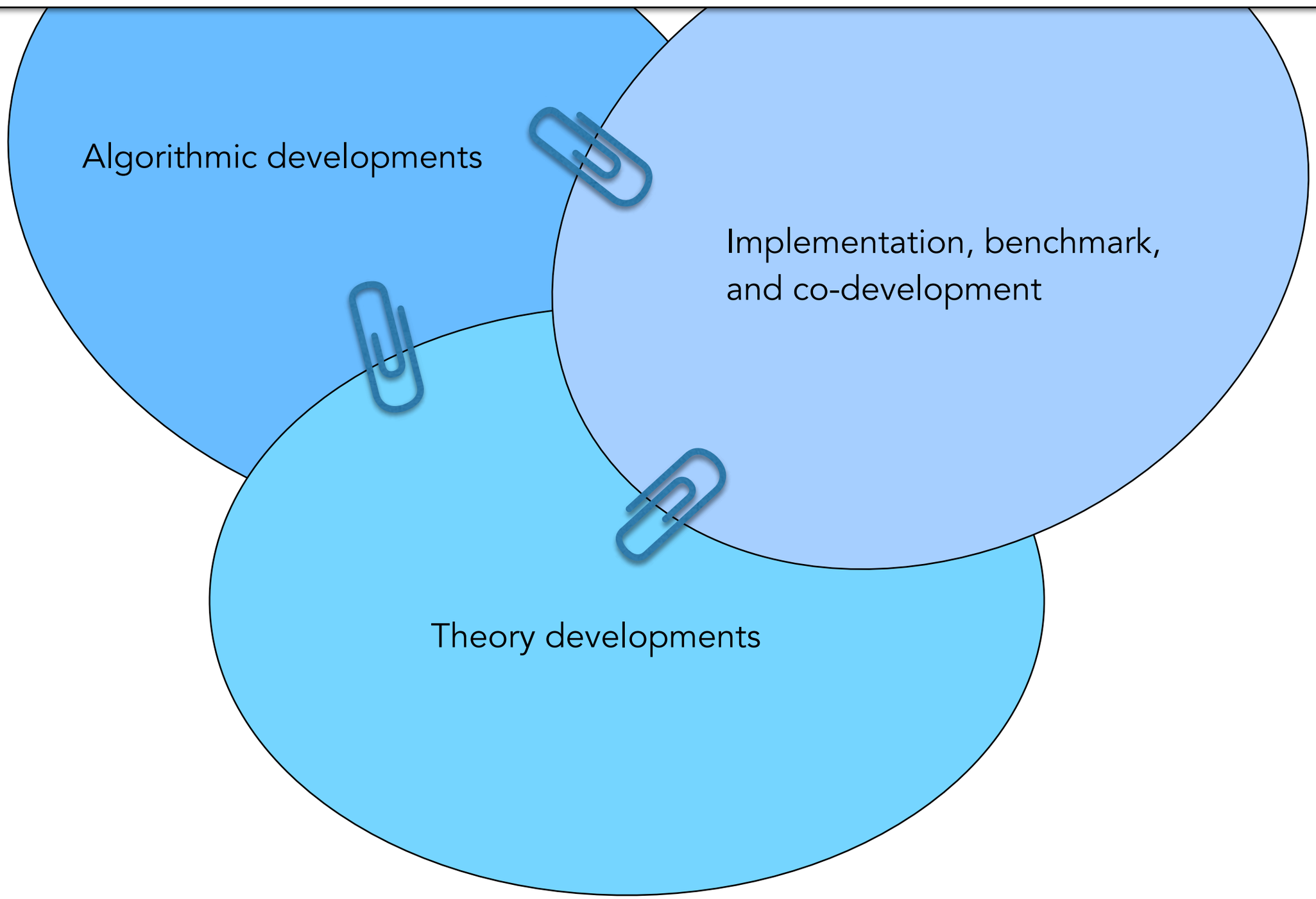




Digital quantum computation of dynamical quantum phase transitions and entanglement tomography in a Lattice Gauge Theory, Mueller, Carolan, Connelly, ZD, Dumitrescu, Mueller, Yeter-Aydeniz, to be released (2022).

See also:
Cohen, Lamm, Lawrence, Yamauchi, *Phys. Rev. D* 104 (2021) 9, 094514.

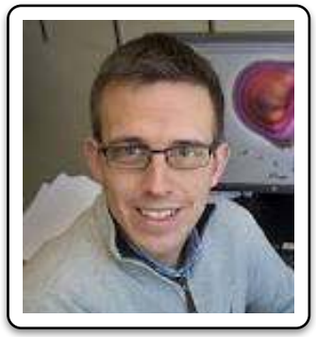
We've got a long way to go to get to **QCD** but we know what to do! If one thing we learned from the successful conventional lattice-QCD program is that **theory/algorithm/experiment** collaborations will be the key. It is even more important in the quantum-computing era since our computers are themselves physical systems!



MY COLLABORATORS IN SOME OF THESE PROJECTS…

NP

N. MUELLER (P)

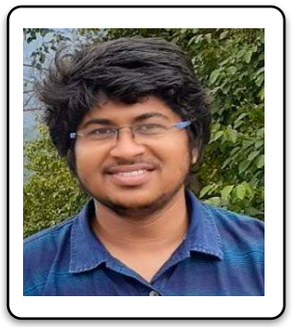


J. STRYKER (P)

A. SHAW I (S)

C. POWERS (S)

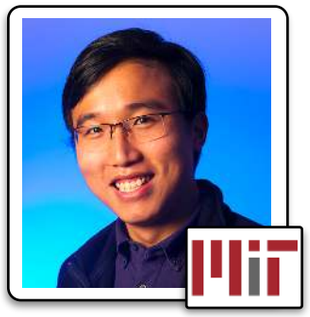
S. KADAM (S)



QIS/CS

J. BRINGEWATT (S)

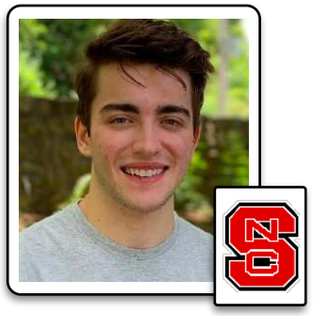
M. Tran (P)



J. CAROLAN (U)

J. WATSON (P)

A. SHAW II (S)



D. CONNELLY (U)

AMO

N. NGUYEN (S)

A. SEIF (P)

T. GRASS (SP)

B. ANDRADE (S)

I. RAYCHOWDHURY

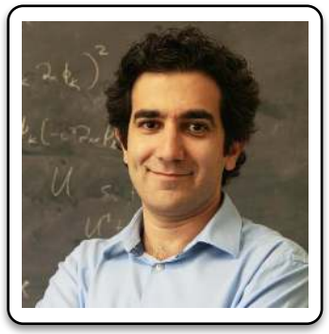
G. PAGANO

E. DUMITRESCU

A. CHILDS

A. GORSHKOV

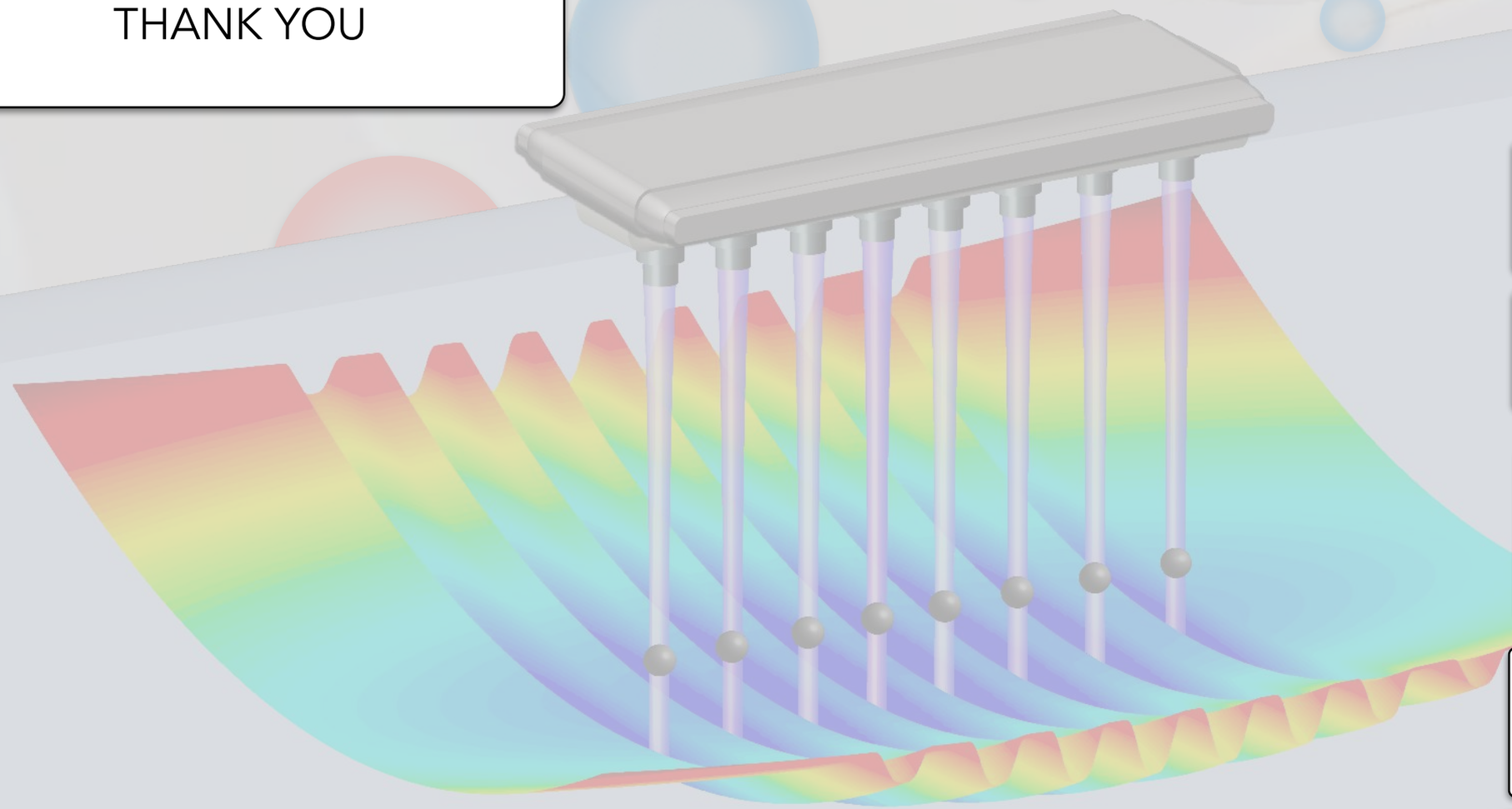
M. HAFEZI



N. LINKE

C. MONROE

THANK YOU



University of Maryland 1856

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