



New Advances in self-consistent Green's function with Gorkov propagators (and applications)

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*Alternative title: “An approach to combine
**pp-ladders (GT), rings (GW) and
superfluidity—for nuclear physics”***





 frontiers Research Topics

Editors: L. Coraggio, S. Pastore, CB

FRONTIERS topical review (doi: 10.3389/fphy.2020.626976) :

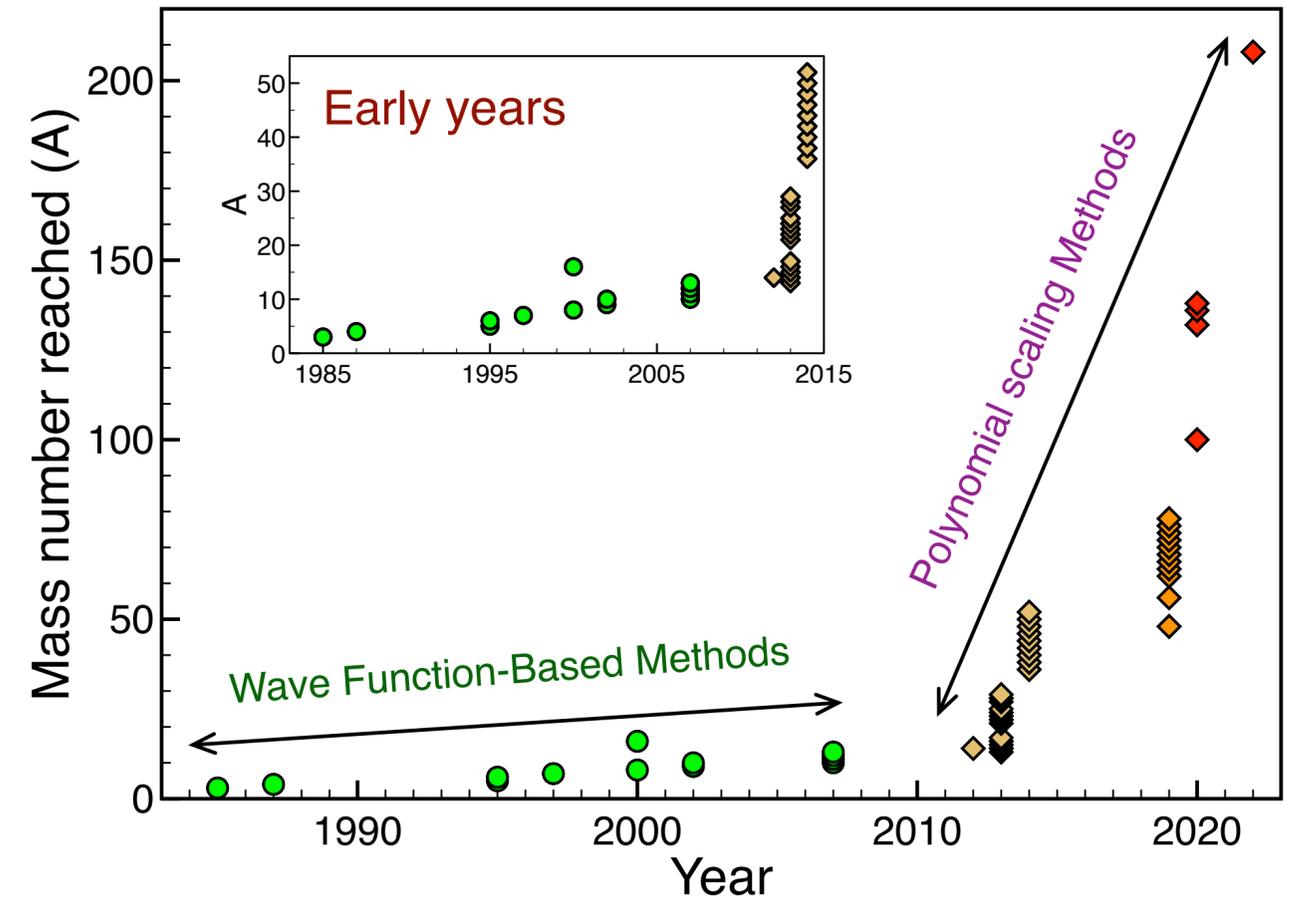
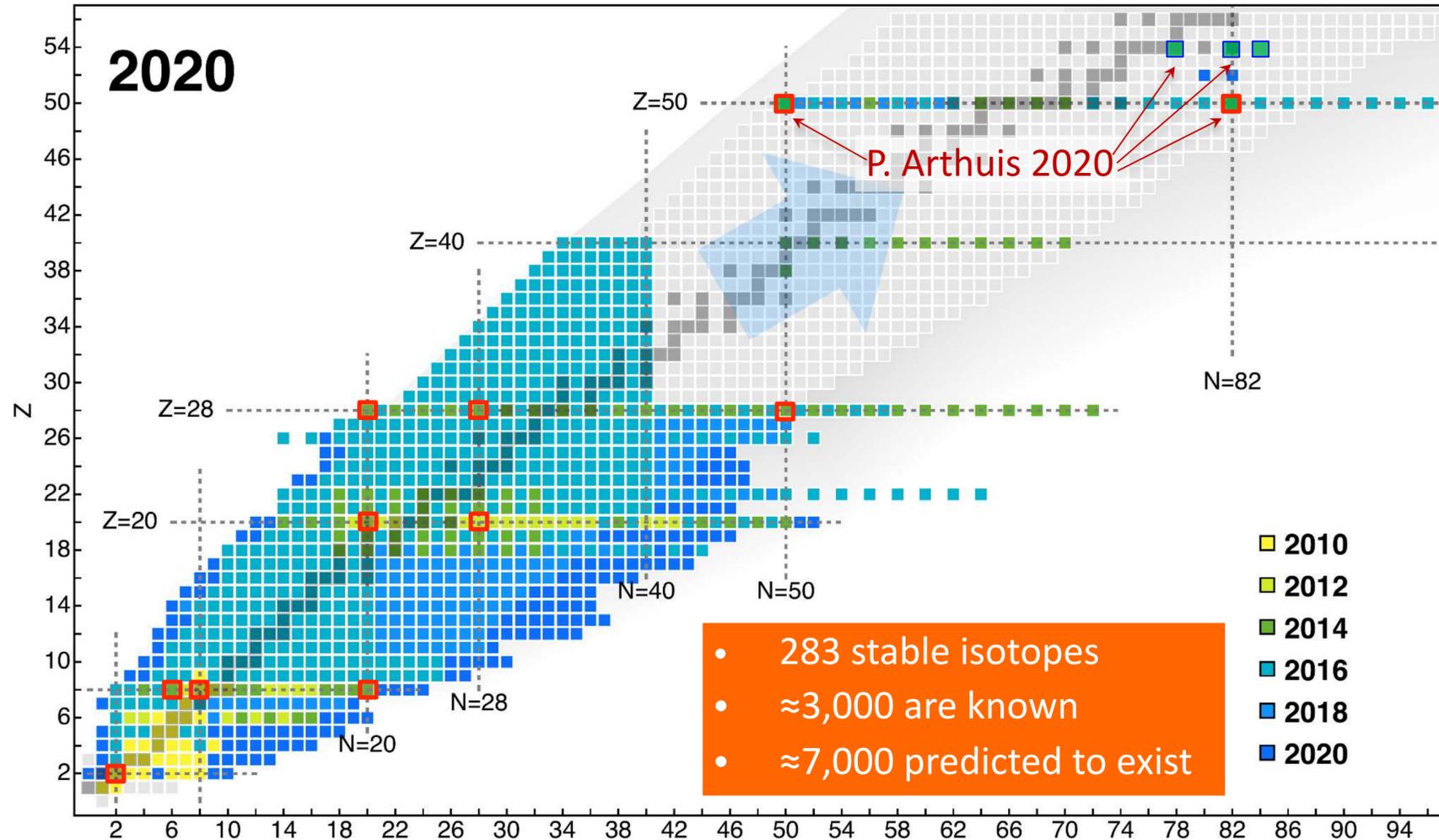
Frontiers in Physics 8, 626976 (2021)



Reach of ab initio methods across the nuclear chart

Extension beyond few-nucleons thanks to:

- Soft (nearly perturbative) effective nuclear forces
- Diagrammatic many-body approaches



Open challenges:

- Accuracy (better theory of nuclear forces)
- Mass number limit (optimised model spaces)
- Precision & scattering (high-order diag. resummations)

H. Hergert, *Frontiers in Phys* **8**, 379 (2020)

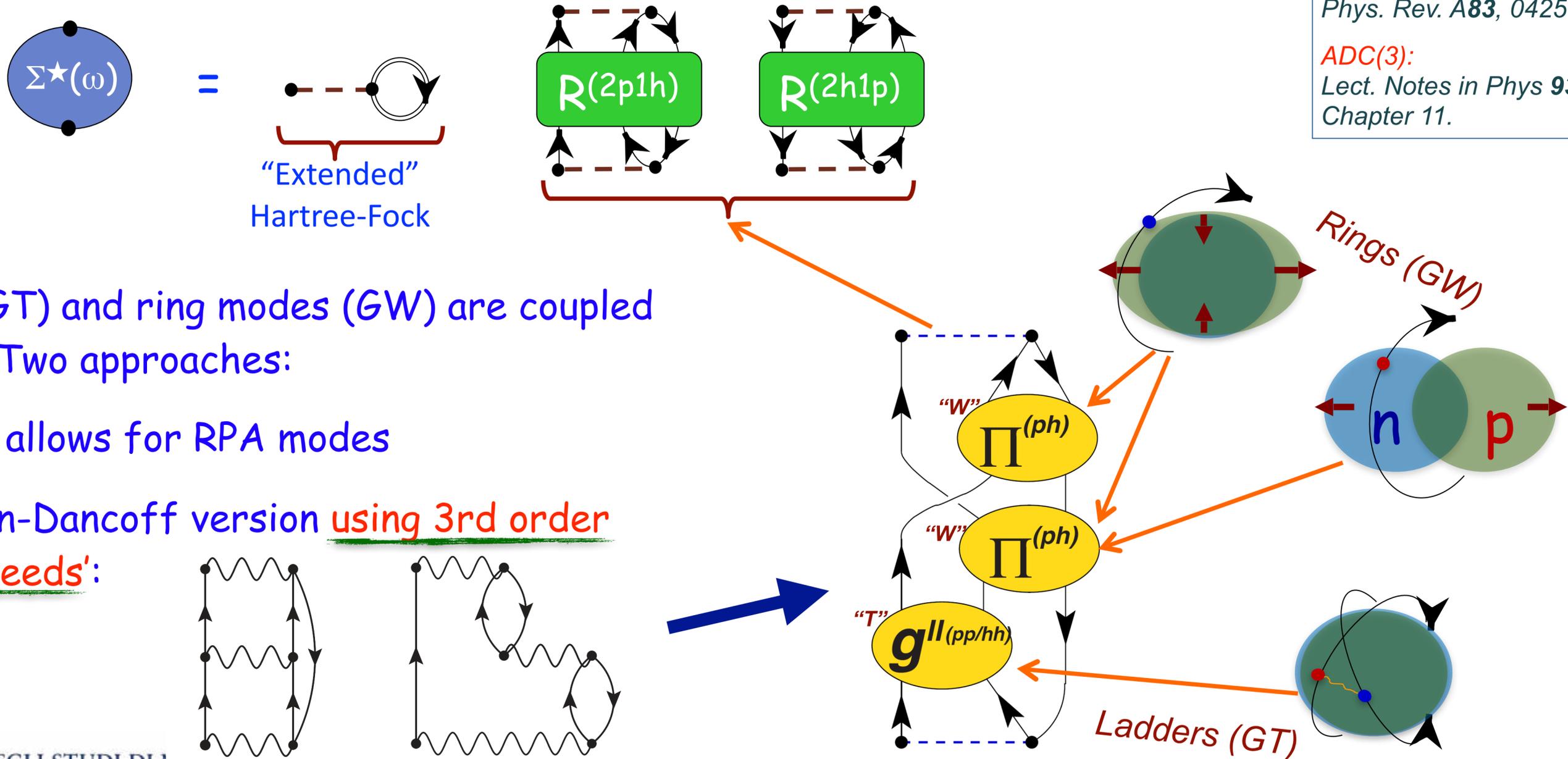
L. Coraggio, S. Pastore, **CB**, *Frontiers in Phys* **8**, 626976 (2021)



The Faddev-RPA and ADC(3) methods in a few words

Compute the nuclear self energy to extract both scattering (optical potential) and spectroscopy.

Both ladders and rings are needed for atomi nuclei:

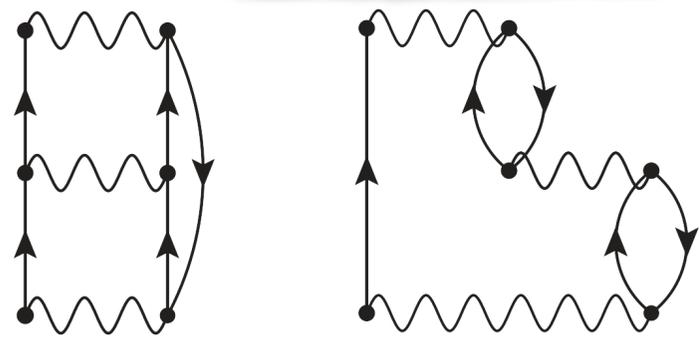


F-RPA:
 Phys. Rev. C **63**, 034313 (2001)
 Phys. Rev. A **76**, 052503 (2007)
 Phys. Rev. A **83**, 042517 (2011)

ADC(3):
 Lect. Notes in Phys **936** (2017)-
 Chapter 11.

All Ladders (GT) and ring modes (GW) are coupled to all orders. Two approaches:

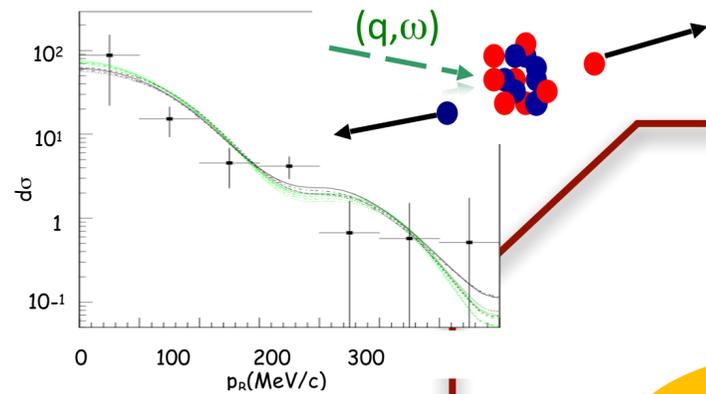
- Faddev-RPA allows for RPA modes
- ADC(3) Tamn-Dancoff version using 3rd order diagrams as 'seeds':



The Self-Consistent Green's Function with Faddeev-RPA

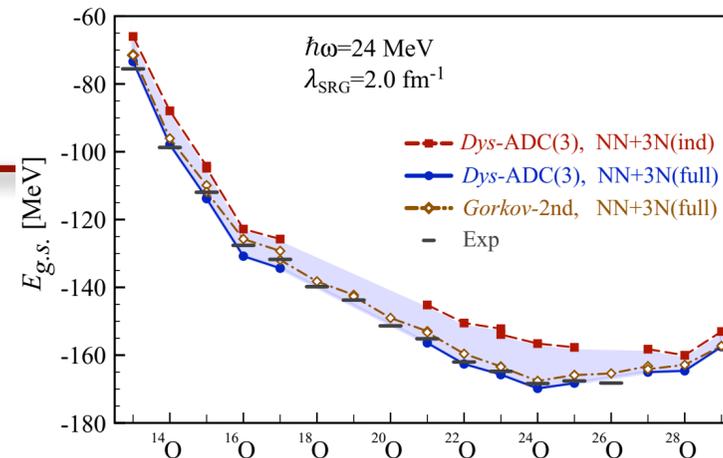
Two-nucleon emission: $^{16}\text{O}(e,e'pn)^{14}\text{N}$

[Eur. Phys. J. A43, 137 (2010)]



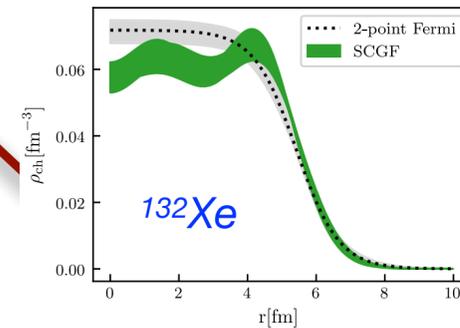
Binding energies

Oxygen drip line
[Phys. Rev. Lett. 111, 062501 (2013)]



Charge & matter distribution

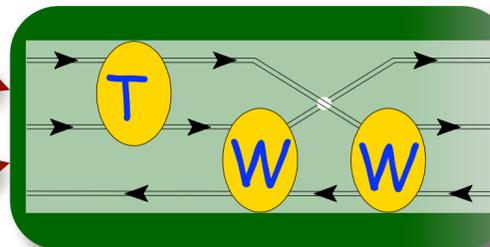
Neutron skins [Phys. Rev. Lett. 125, 182501 (2020)]



	SCGF	Exp.
^{100}Sn	4.525 – 4.707	
^{132}Sn	4.725 – 4.956	4.7093
^{132}Xe	4.700 – 4.948	4.7859
^{136}Xe	4.715 – 4.928	4.7964
^{138}Xe	4.724 – 4.941	4.8279

T ladders

W rings



Dyson Eq.

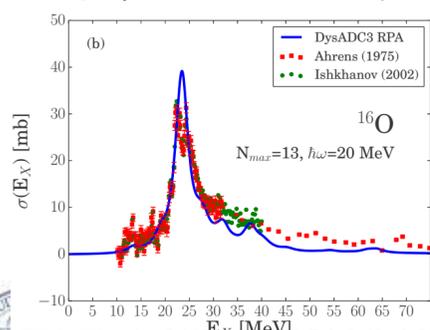
Spectroscopy

Ionisation energies and affinities for simple atoms and molecules
[Phys. Rev. A. 83, 042517 (2011); 85, 012501 (2012)]

Level	ADC(3)	FRPA	FRPA(c)	Expt.
HF				
1π	16.48	16.05	16.35	16.05
3σ	20.36	20.03	20.24	20.0
CO				
5σ	13.94	14.37	13.69	14.01
1π	16.98	16.95	16.84	16.91
4σ	20.19	19.46	19.59	19.72
H ₂ O				
1b ₁	12.86	12.62	12.67	12.62
3a ₁	15.15	14.91	14.98	14.74
1b ₂	19.21	19.06	19.13	18.51
Δ (eV)	0.30(0.30)	0.25(0.23)	0.31(0.26)	
Δ _{max} (eV)	0.70(0.70)	0.73(0.73)	0.88(0.62)	

Nuclear ELM response and dipole polarisability, α_D

[Phys. Rev. C77, 024304 (2008)]

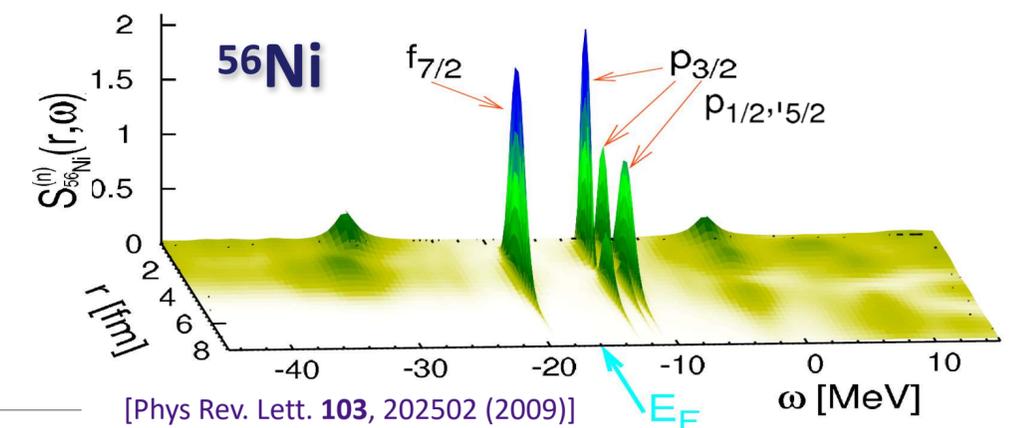
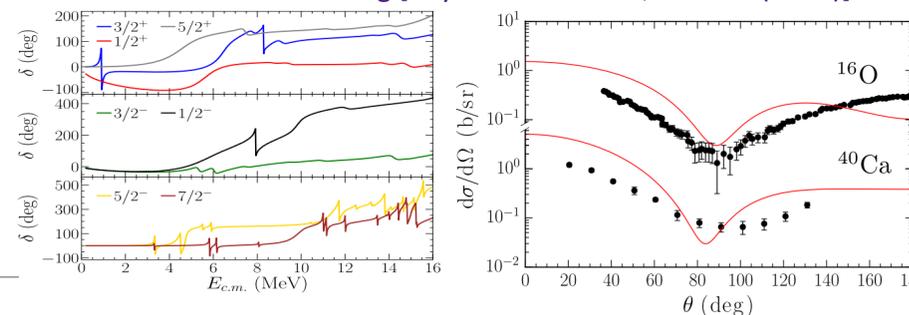


^{68}Ni :

	SCGF	Exp
E_{PDR} (MeV)	10.68	9.55(17)
	10.92	
E_{GDR} (MeV)	18.1	17.1(2)
α_D (fm ³)	3.60	3.40(23)
		3.88(31)

Optical potential

Elastic neutron scattering [Phys. Rev. Lett. 123, 092501 (2013)]

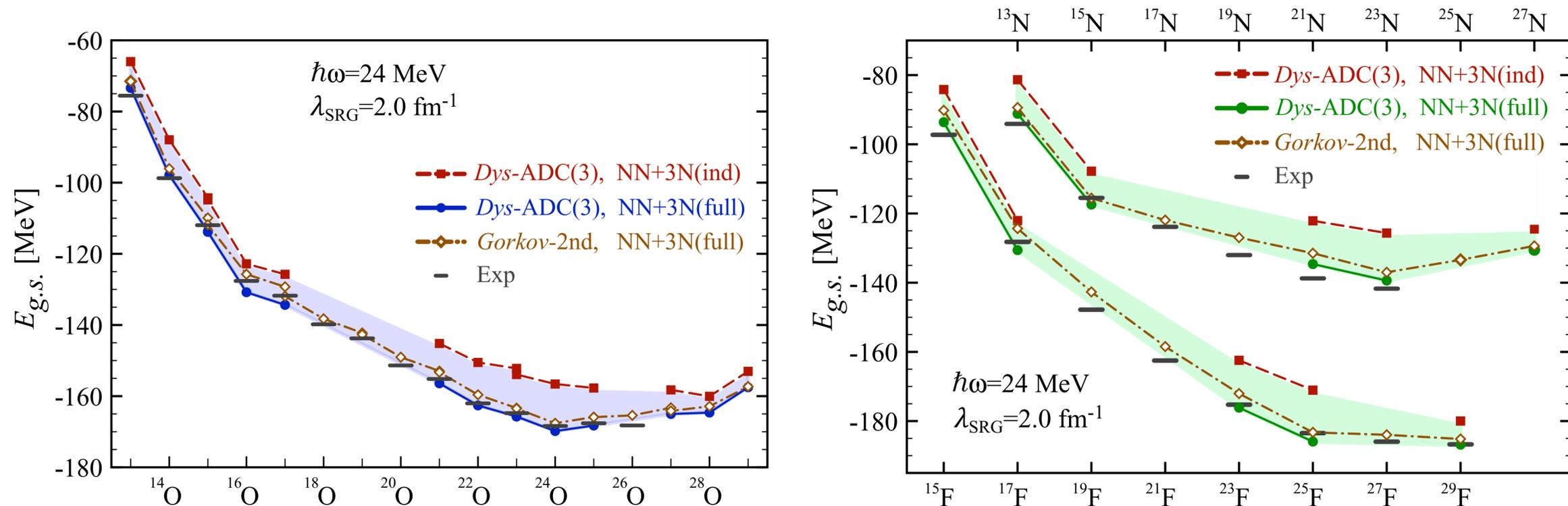


[Phys. Rev. Lett. 103, 202502 (2009)]



Results for the N-O-F chains

A. Cipollone, CB, P. Navrátil, Phys. Rev. Lett. **111**, 062501 (2013)
and Phys. Rev. C **92**, 014306 (2015)



→ 3NF crucial for reproducing binding energies and driplines around oxygen

→ cf. microscopic shell model [Otsuka et al, PRL **105**, 032501 (2010).]



Ab-initio Nuclear Computation & BcDor code

BoccaDorata code: <https://gitlab.com/cbarbieri/BoccaDorata>

- C++ class library for handling many-body propagators (MPI & OpenMP based).
- Computation of nuclear spectral functions, many-body propagators, RPA responses, coupled cluster equations and effective interaction/charges for the shell model.

Code history:

2006

core functions and FRPA

shell model charges&interactions (lowest order)

2010

new Gorkov formalism for open-shell nuclei (at 2nd order) (V. Soma, 2010–)

2012

Coupled clusters equations

2013

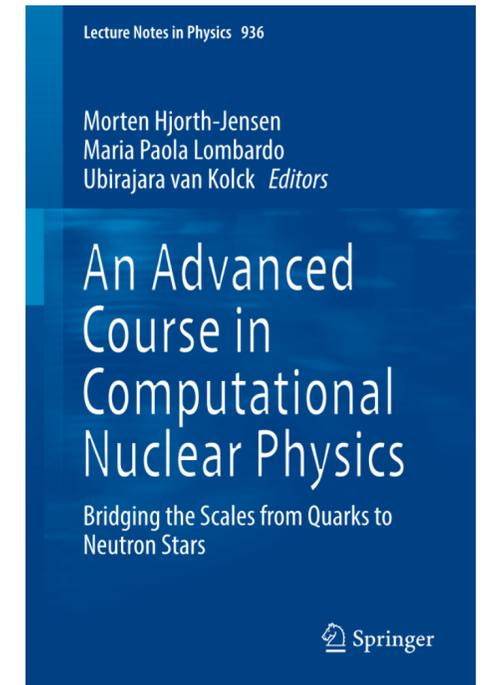
Three-nucleon forces (A. Cipollone, 2011–2015)

2014

Gorkov at 3rd order (will become massively parallel...)

2022

applications →



Self-consistent Green's function formalism and methods for Nuclear Physics

Reaching open-shell nuclei:

The Gorkov GF approach

V. Somà, T. Duguet, CB, Phys. Rev. C **84**, 064317 (2011)
V. Somà, CB, T. Duguet, Phys. Rev. C **87**, 011303R (2013)
V. Somà, CB, T. Duguet, Phys. Rev. C **89**, 024323 (2014)
CB, T. Duguet, V. Somà, Phys. Rev. C **105**, 044330 (2014)



Gorkov ansatz... for atomic nuclei

- ✨ Most atomic nuclei: partially filled orbits couple to collective vibrations

Expansion breaks down with vanishing *particle-hole* gaps

Pairing (two-nucleon) mitigates instabilities

Explicit configuration mixing

Single-reference: Bogoliubov (Gorkov)

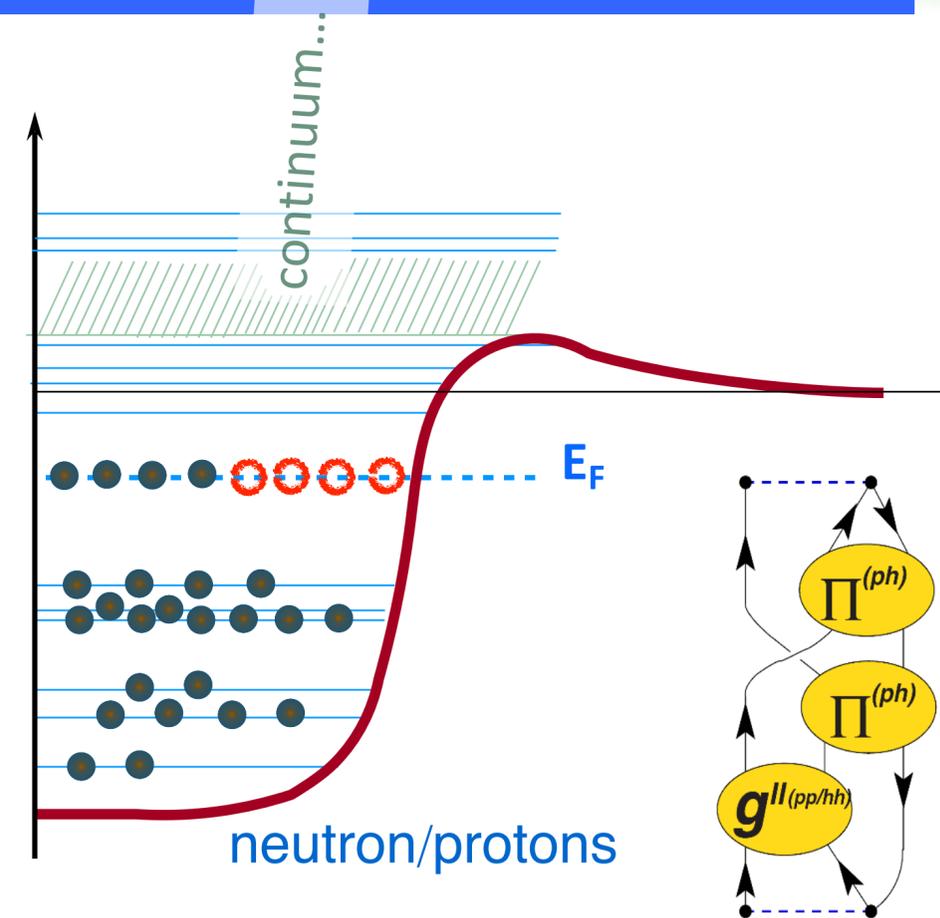
- Ansatz $E_0^{N\pm 2n} \approx E_0^N \pm 2n\mu \pm n\Delta\varepsilon_p$ for $n = 1, 2, 3, \dots$

- Auxiliary many-body state $|\Psi_0\rangle = \sum_{n=0}^{\infty} c_{2n} |\psi^{2n}\rangle$

Mixes various particle numbers

Introduce a “grand-canonical” potential $\Omega \equiv H - \mu N$

$|\Psi_0\rangle$ minimizes $\Omega_0 = \min_{|\Psi_0\rangle} \{\langle \Psi_0 | \Omega | \Psi_0 \rangle\}$ under the constraint $N = \langle \Psi_0 | N | \Psi_0 \rangle$



V. Somà, T. Duguet, CB, Phys. Rev. C **84**, 064317 (2011)
 CB, T. Duguet, V. Somà, Phys. Rev. C **105**, 044330 (2014)

Gorkov Green's functions and equations

Set of 4 Gorkov Green's functions:

$$\mathbf{G}_{\alpha\beta}(t, t') \equiv \begin{pmatrix} G_{\alpha\beta}^{11}(t, t') \equiv -i \langle \Psi_0 | T [c_\alpha(t) c_\beta^\dagger(t')] | \Psi_0 \rangle \equiv \text{blue double arrow} & G_{\alpha\beta}^{12}(t, t') \equiv -i \langle \Psi_0 | T [c_\alpha(t) c_\beta(t')] | \Psi_0 \rangle \equiv \text{black double arrow} \\ G_{\alpha\beta}^{21}(t, t') \equiv -i \langle \Psi_0 | T [c_\alpha^\dagger(t) c_\beta^\dagger(t')] | \Psi_0 \rangle \equiv \text{black double arrow} & G_{\alpha\beta}^{22}(t, t') \equiv -i \langle \Psi_0 | T [c_\alpha^\dagger(t) c_\beta(t')] | \Psi_0 \rangle \equiv \text{black double arrow} \end{pmatrix}$$

[Gorkov 1958]

In terms of Nambu fields:

$$\mathbf{G}_{\alpha\beta}(t, t') = -i \langle \Psi_0 | T \left[\underbrace{\mathbf{A}_\alpha(t) \mathbf{A}_\beta^\dagger(t')}_{\text{Nambu fields}} \right] | \Psi_0 \rangle \quad \mathbf{A}_\alpha(t) \equiv \begin{bmatrix} c_\alpha(t) \\ c_\alpha^\dagger(t) \end{bmatrix}$$

[Anderson 1958
Nambu 1960]

Don't (anti)commute: in/out arrows and Nambu indices still matter!

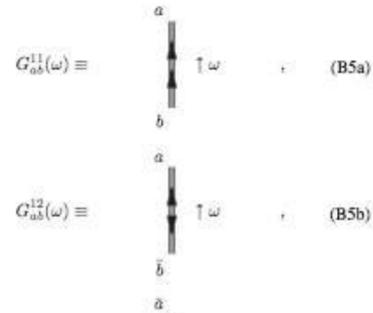


Diagrams *inflation* with (Gorkov) SCGF theory

Gorkov at
2nd order:

$$\Sigma_{\alpha\beta}^{11}(\omega) = \text{[Diagram 1]} + \text{[Diagram 2]}$$

unperturbed ones, i.e.,



propagator one obtains

$$\Sigma_{ab}^{11(1)}(\omega) = -i \int_{C^+} \frac{d\omega'}{2\pi} \sum_{cd,k} \tilde{V}_{abcd} \frac{U_d^k U_c^{k*}}{\omega' - \omega_k + i\eta} - i \int_{C^+} \frac{d\omega'}{2\pi} \sum_{cd,k} \tilde{V}_{abcd} \frac{V_d^k V_c^{k*}}{\omega' + \omega_k - i\eta} = \sum_{cd,k} \tilde{V}_{abcd} \tilde{V}_d^k \tilde{V}_c^{k*},$$

where the residue theorem has been used, i.e., the first with $+i\eta$ in the denominator, contains no pole in the u plane and thus cancels out. As in the standard case the Hartree-Fock self-energy is energy independent.

Similarly, one computes the other normal self-energy:

$$\Sigma_{ab}^{22(1)}(\omega) = \tilde{a} \text{---} \tilde{c} \text{---} \tilde{b} \text{---} \tilde{d} \text{---} \omega$$

5. Block-diagonal structure of self-energies

a. First order

The goal of this subsection is to discuss how the block-diagonal form of the propagators and interaction matrix reflects in the various self-energy contributions, starting with the first-order normal self-energy $\Sigma^{11(1)}$. Substituting Eq (C19) into Eq. (B7), and introducing the factor

$$f_{\alpha\beta\gamma\delta}^{\alpha_1, \alpha_2, \alpha_3, \alpha_4} \equiv \sqrt{1 + \delta_{\alpha\beta} \delta_{\alpha_1, \alpha_2}} \sqrt{1 + \delta_{\gamma\delta} \delta_{\alpha_3, \alpha_4}},$$

one obtains

$$\Sigma_{ab}^{11(1)} = \sum_{cd,k} \tilde{V}_{abcd} \tilde{V}_d^k \tilde{V}_c^{k*} = \sum_{\alpha, \alpha_1, \alpha_2, \gamma} \sum_{\mu, JM} \sum_{\alpha_3, \alpha_4} f_{\alpha\beta\gamma\delta}^{\alpha_1, \alpha_2, \alpha_3, \alpha_4} C_{JM}^{\alpha, \alpha_1, \alpha_2} C_{JM}^{\alpha_3, \alpha_4, \mu} \tilde{V}_{\alpha, \alpha_1, \alpha_2, \alpha_3, \alpha_4}^J \rho_{\alpha_1, \alpha_2}^{\mu, \mu*} \rho_{\alpha_3, \alpha_4}^{\mu, \mu*} = \delta_{\alpha\beta} \delta_{\alpha_1, \alpha_2} \sum_{\alpha, \alpha_1, \alpha_2, \gamma} \sum_{JM} f_{\alpha\beta\gamma\delta}^{\alpha_1, \alpha_2, \alpha_3, \alpha_4} \frac{2J+1}{2J_a+1} \tilde{V}_{\alpha, \alpha_1, \alpha_2, \alpha_3, \alpha_4}^J \rho_{\alpha_1, \alpha_2}^{\mu, \mu*} \rho_{\alpha_3, \alpha_4}^{\mu, \mu*} \equiv \delta_{\alpha\beta} \delta_{\alpha_1, \alpha_2} \Sigma_{\alpha, \alpha_1, \alpha_2}^{11(a)(1)} \equiv \delta_{\alpha\beta} \delta_{\alpha_1, \alpha_2} \Lambda_{\alpha, \alpha_1, \alpha_2}^{(a)(1)},$$

where the block-diagonal normal density matrix is introduced through $\rho_{ab} \equiv \delta_{\alpha\beta} \delta_{\alpha_1, \alpha_2} \rho_{\alpha, \alpha_1, \alpha_2}^{(a)}$, such that

$$\rho_{\alpha, \alpha_1, \alpha_2}^{(a)} = \sum_{\mu} V_{\alpha, \alpha_1}^{\mu} V_{\alpha_2, \alpha}^{\mu*}$$

and properties of Clebsch-Gordan coefficients has been used. The fact that the interaction conserves parity and charge, leading to $\delta_{\alpha\beta} = \delta_{\alpha, \beta} \delta_{\alpha_1, \alpha_2} \delta_{\alpha_3, \alpha_4}$. Similarly, for $\Sigma^{22(1)}$,

$$\Sigma_{ab}^{22(1)} = -\sum_{cd,k} \tilde{V}_{abcd} \tilde{V}_c^k \tilde{V}_d^{k*} = -\delta_{\alpha\beta} \delta_{\alpha_1, \alpha_2} \sum_{\alpha, \alpha_1, \alpha_2, \gamma} \sum_{JM} f_{\alpha\beta\gamma\delta}^{\alpha_1, \alpha_2, \alpha_3, \alpha_4} \frac{2J+1}{2J_a+1} \tilde{V}_{\alpha, \alpha_1, \alpha_2, \alpha_3, \alpha_4}^J \rho_{\alpha_1, \alpha_2}^{\mu, \mu*} \rho_{\alpha_3, \alpha_4}^{\mu, \mu*} \equiv \delta_{\alpha\beta} \delta_{\alpha_1, \alpha_2} \Sigma_{\alpha, \alpha_1, \alpha_2}^{22(a)(1)} = -\delta_{\alpha\beta} \delta_{\alpha_1, \alpha_2} \Lambda_{\alpha, \alpha_1, \alpha_2}^{(a)(1)*} = -\delta_{\alpha\beta} \delta_{\alpha_1, \alpha_2} [\Lambda_{\alpha, \alpha_1, \alpha_2}^{(a)(1)}]^{-1}.$$

Let us consider the anomalous contributions to the first-order self-energy. Substituting Eqs. (C27b) and (C19) into Eq. (B7) derives

$$\Sigma_{ab}^{12(1)} = \frac{1}{2} \sum_{cd,k} \tilde{V}_{abcd} \tilde{V}_c^k \tilde{U}_d^k = -\frac{1}{2} \sum_{\alpha, \alpha_1, \alpha_2, \gamma} \sum_{\mu, JM} \sum_{\alpha_3, \alpha_4} f_{\alpha\beta\gamma\delta}^{\alpha_1, \alpha_2, \alpha_3, \alpha_4} \eta_b \eta_c C_{JM}^{\alpha, \alpha_1, \alpha_2} C_{JM}^{\alpha_3, \alpha_4, \mu} \tilde{V}_{\alpha, \alpha_1, \alpha_2, \alpha_3, \alpha_4}^J V_{\alpha_1, \alpha_2}^{\mu, \mu*} U_{\alpha_3, \alpha_4}^{\mu, \mu*} = -\frac{1}{2} \sum_{\alpha, \alpha_1, \alpha_2, \gamma} \sum_{\mu, JM} \sum_{\alpha_3, \alpha_4} f_{\alpha\beta\gamma\delta}^{\alpha_1, \alpha_2, \alpha_3, \alpha_4} \eta_b \eta_c C_{JM}^{\alpha, \alpha_1, \alpha_2} C_{JM}^{\alpha_3, \alpha_4, \mu} \tilde{V}_{\alpha, \alpha_1, \alpha_2, \alpha_3, \alpha_4}^J V_{\alpha_1, \alpha_2}^{\mu, \mu*} U_{\alpha_3, \alpha_4}^{\mu, \mu*} = -\frac{1}{2} \sum_{\alpha, \alpha_1, \alpha_2, \gamma} \sum_{\mu, JM} f_{\alpha\beta\gamma\delta}^{\alpha_1, \alpha_2, \alpha_3, \alpha_4} \eta_b \eta_c (-1)^{2J} C_{JM}^{\alpha, \alpha_1, \alpha_2} \sqrt{2J_c+1} \tilde{V}_{\alpha, \alpha_1, \alpha_2, \alpha_3, \alpha_4}^J \rho_{\alpha_1, \alpha_2}^{\mu, \mu*} \rho_{\alpha_3, \alpha_4}^{\mu, \mu*} = \delta_{\alpha\beta} \delta_{\alpha_1, \alpha_2} \frac{1}{2} \sum_{\alpha, \alpha_1, \alpha_2, \gamma} \sum_{\mu, JM} f_{\alpha\beta\gamma\delta}^{\alpha_1, \alpha_2, \alpha_3, \alpha_4} \pi_a \pi_b (-1)^{2J} \frac{\sqrt{2J_c+1}}{\sqrt{2J_a+1}} \tilde{V}_{\alpha, \alpha_1, \alpha_2, \alpha_3, \alpha_4}^J \rho_{\alpha_1, \alpha_2}^{\mu, \mu*} \rho_{\alpha_3, \alpha_4}^{\mu, \mu*} \equiv \delta_{\alpha\beta} \delta_{\alpha_1, \alpha_2} \Sigma_{\alpha, \alpha_1, \alpha_2}^{12(a)(1)} \equiv \delta_{\alpha\beta} \delta_{\alpha_1, \alpha_2} \tilde{H}_{\alpha, \alpha_1, \alpha_2}^{(a)(1)},$$

where the block-diagonal anomalous density matrix is introduced through $\tilde{\rho}_{ab} \equiv \delta_{\alpha\beta} \delta_{\alpha_1, \alpha_2} \tilde{\rho}_{\alpha, \alpha_1, \alpha_2}^{(a)}$, such that

$$\tilde{\rho}_{\alpha, \alpha_1, \alpha_2}^{(a)} = \sum_{\mu} U_{\alpha, \alpha_1}^{\mu} V_{\alpha_2, \alpha}^{\mu*}$$

on with (Gorkov) SCGF

It is interesting to note that the first-order anomalous with a $J = 0$ many-body state. The other anomalous

$$\Sigma_{ab}^{21(1)} = \frac{1}{2} \sum_{cd,k} \tilde{V}_{cdab} \tilde{U}_c^k \tilde{V}_d^{k*} = -\frac{1}{2} \sum_{\alpha, \alpha_1, \alpha_2, \gamma} \sum_{\mu, JM} \sum_{\alpha_3, \alpha_4} f_{\alpha\beta\gamma\delta}^{\alpha_1, \alpha_2, \alpha_3, \alpha_4} \tilde{V}_{\alpha, \alpha_1, \alpha_2, \alpha_3, \alpha_4}^J V_{\alpha_1, \alpha_2}^{\mu, \mu*} U_{\alpha_3, \alpha_4}^{\mu, \mu*} = \delta_{\alpha\beta} \delta_{\alpha_1, \alpha_2} \frac{1}{2} \sum_{\alpha, \alpha_1, \alpha_2, \gamma} \sum_{\mu, JM} f_{\alpha\beta\gamma\delta}^{\alpha_1, \alpha_2, \alpha_3, \alpha_4} \tilde{V}_{\alpha, \alpha_1, \alpha_2, \alpha_3, \alpha_4}^J V_{\alpha_1, \alpha_2}^{\mu, \mu*} U_{\alpha_3, \alpha_4}^{\mu, \mu*} \equiv \delta_{\alpha\beta} \delta_{\alpha_1, \alpha_2} \Sigma_{\alpha, \alpha_1, \alpha_2}^{21(a)(1)} \equiv \delta_{\alpha\beta} \delta_{\alpha_1, \alpha_2} \tilde{H}_{\alpha, \alpha_1, \alpha_2}^{(a)(1)*}.$$

Block-diagonal forms of second-order self-energy angular momentum couplings of the three quasiparticle, Q , R , and S . One proceeds first coupling particle give J_{tot} . The recoupled M term is computed as follows

$$M_a^{k_1 k_2 k_3}(J, J_{tot}) = \sum_{m_1, m_2, m_3, M} C_{J, m_1, m_2, M}^{J, M} C_{J, m_2, m_3, M}^{J, M} C_{J, m_1, m_3, M}^{J, M} \tilde{V}_{\alpha, \alpha_1, \alpha_2, \alpha_3, \alpha_4}^J U_{\alpha_1, \alpha_2}^{m_1, m_2} U_{\alpha_3, \alpha_4}^{m_3, M} = \sum_{m_1, m_2, m_3, M, r, s} C_{J, m_1, m_2, M}^{J, M} C_{J, m_2, m_3, M}^{J, M} C_{J, m_1, m_3, M}^{J, M} \delta_{\alpha, \beta} \delta_{\alpha_1, \alpha_2} \delta_{\alpha_3, \alpha_4} \delta_{m_1, m_2} \delta_{m_3, M} \times C_{J, m_1, m_2, M}^{J, M} C_{J, m_2, m_3, M}^{J, M} C_{J, m_1, m_3, M}^{J, M} = \sum_{m_1, m_2, m_3, M, J, M} \eta_b \eta_c f_{\alpha\beta\gamma\delta}^{\alpha_1, \alpha_2, \alpha_3, \alpha_4} C_{J, m_1, m_2, M}^{J, M} C_{J, m_2, m_3, M}^{J, M} C_{J, m_1, m_3, M}^{J, M} = \sum_{m_1, m_2, m_3, M, J, M} \eta_b \eta_c f_{\alpha\beta\gamma\delta}^{\alpha_1, \alpha_2, \alpha_3, \alpha_4} \frac{\sqrt{2J_c+1}}{\sqrt{2J_a+1}} \delta_{\alpha, \beta} \delta_{\alpha_1, \alpha_2} \delta_{\alpha_3, \alpha_4} \delta_{m_1, m_2} \delta_{m_3, M} \times C_{J, m_1, m_2, M}^{J, M} C_{J, m_2, m_3, M}^{J, M} C_{J, m_1, m_3, M}^{J, M} = -\delta_{J_{tot}, J} \delta_{M_{tot}, M} \sum_{\alpha, \alpha_1, \alpha_2} \pi_k f_{\alpha\beta\gamma\delta}^{\alpha_1, \alpha_2, \alpha_3, \alpha_4} \frac{\sqrt{2J_c+1}}{\sqrt{2J_a+1}} \delta_{\alpha, \beta} \delta_{\alpha_1, \alpha_2} \delta_{\alpha_3, \alpha_4} \delta_{m_1, m_2} \delta_{m_3, M} \times C_{J, m_1, m_2, M}^{J, M} C_{J, m_2, m_3, M}^{J, M} C_{J, m_1, m_3, M}^{J, M} = \delta_{J_{tot}, J} \delta_{M_{tot}, M} M_{\alpha, \alpha_1, \alpha_2}^{k_1 k_2 k_3}(J, J_{tot}),$$

where general properties of Clebsch-Gordan coefficients

$$N_{\alpha}(J, J_{tot}) = \delta_{J_{tot}, J} \delta_{M_{tot}, M} \sum_{\alpha, \alpha_1, \alpha_2} \pi_k f_{\alpha\beta\gamma\delta}^{\alpha_1, \alpha_2, \alpha_3, \alpha_4} = \delta_{J_{tot}, J} \delta_{M_{tot}, M} N_{\alpha, \alpha_1, \alpha_2}^{k_1 k_2 k_3}(J, J_{tot})$$

One can show that the same result is obtained by:

$$N_{\alpha}(J, J_{tot}) = \sum_{m_1, m_2, m_3, M} C_{J, m_1, m_2, M}^{J, M} C_{J, m_2, m_3, M}^{J, M} C_{J, m_1, m_3, M}^{J, M} N_{\alpha}^{k_1 k_2 k_3}(J, J_{tot}) = \sum_{m_1, m_2, m_3, M, r, s} C_{J, m_1, m_2, M}^{J, M} C_{J, m_2, m_3, M}^{J, M} C_{J, m_1, m_3, M}^{J, M} \delta_{\alpha, \beta} \delta_{\alpha_1, \alpha_2} \delta_{\alpha_3, \alpha_4} \delta_{m_1, m_2} \delta_{m_3, M} \delta_{\alpha, \beta} \delta_{\alpha_1, \alpha_2} \delta_{\alpha_3, \alpha_4} \delta_{m_1, m_2} \delta_{m_3, M} \eta_a \eta_b \eta_c f_{\alpha\beta\gamma\delta}^{\alpha_1, \alpha_2, \alpha_3, \alpha_4} \times C_{J, m_1, m_2, M}^{J, M} C_{J, m_2, m_3, M}^{J, M} C_{J, m_1, m_3, M}^{J, M} \tilde{V}_{\alpha, \alpha_1, \alpha_2, \alpha_3, \alpha_4}^J V_{\alpha_1, \alpha_2}^{m_1, m_2} V_{\alpha_3, \alpha_4}^{m_3, M} U_{\alpha_1, \alpha_2}^{m_1, m_2} U_{\alpha_3, \alpha_4}^{m_3, M} = \delta_{J_{tot}, J} \delta_{M_{tot}, M} S_{\alpha, \alpha_1, \alpha_2}^{k_1 k_2 k_3}(J, J_{tot})$$

$$= \sum_{\alpha, \alpha_1, \alpha_2} \sum_{\mu, \nu} \eta_a \eta_b \eta_c f_{\alpha\beta\gamma\delta}^{\alpha_1, \alpha_2, \alpha_3, \alpha_4} C_{J, m_1, m_2, M}^{J, M} C_{J, m_2, m_3, M}^{J, M} C_{J, m_1, m_3, M}^{J, M} C_{J, m_1, m_2, M}^{J, M} C_{J, m_2, m_3, M}^{J, M} C_{J, m_1, m_3, M}^{J, M} = \sum_{\alpha, \alpha_1, \alpha_2} \sum_{\mu, \nu} \eta_a \eta_b \eta_c f_{\alpha\beta\gamma\delta}^{\alpha_1, \alpha_2, \alpha_3, \alpha_4} \frac{\sqrt{2J_c+1}}{\sqrt{2J_a+1}} (-1)^{J+J_1+J_2} C_{J, m_1, m_2, M}^{J, M} C_{J, m_2, m_3, M}^{J, M} C_{J, m_1, m_3, M}^{J, M} \tilde{V}_{\alpha, \alpha_1, \alpha_2, \alpha_3, \alpha_4}^J V_{\alpha_1, \alpha_2}^{\mu, \mu*} U_{\alpha_3, \alpha_4}^{\nu, \nu*} = \delta_{J_{tot}, J} \delta_{M_{tot}, M} \sum_{\alpha, \alpha_1, \alpha_2} \sum_{\mu, \nu} \eta_a \eta_b \eta_c f_{\alpha\beta\gamma\delta}^{\alpha_1, \alpha_2, \alpha_3, \alpha_4} \frac{\sqrt{2J_c+1}}{\sqrt{2J_a+1}} (-1)^{J+J_1+J_2} \tilde{V}_{\alpha, \alpha_1, \alpha_2, \alpha_3, \alpha_4}^J V_{\alpha_1, \alpha_2}^{\mu, \mu*} U_{\alpha_3, \alpha_4}^{\nu, \nu*} = -\delta_{J_{tot}, J} \delta_{M_{tot}, M} \eta_a \eta_b N_{\alpha, \alpha_1, \alpha_2}^{k_1 k_2 k_3}(J, J_{tot}),$$

which recovers relation (72a). The remaining quantities [see Eqs. (69) and (70)] are related to $\{k_1, k_2, k_3\}$ indices and can be obtained from Eqs. (C35) and (C36) by taking into account the J_k to J_{tot} and J_c as follows:

$$P_a^{k_1 k_2 k_3}(J, J_{tot}) = \sum_{J_c} (-1)^{J+J_1+J_2+J_3} \sqrt{2J_c+1} \sqrt{2J_d+1} \begin{Bmatrix} J_k & J_k & J_c \\ J_k & J_{tot} & J_d \end{Bmatrix} M_{\alpha}(J, J_{tot}) = -\delta_{J_{tot}, J} \delta_{M_{tot}, M} \sum_{\alpha, \alpha_1, \alpha_2} \sum_{J_c} \pi_k f_{\alpha\beta\gamma\delta}^{\alpha_1, \alpha_2, \alpha_3, \alpha_4} \frac{\sqrt{2J_c+1}}{\sqrt{2J_a+1}} (2J_d+1) (-1)^{J+J_1+J_2} \times \tilde{V}_{\alpha, \alpha_1, \alpha_2, \alpha_3, \alpha_4}^J U_{\alpha_1, \alpha_2}^{m_1, m_2} U_{\alpha_3, \alpha_4}^{m_3, M} V_{\alpha_1, \alpha_2}^{m_1, m_2} V_{\alpha_3, \alpha_4}^{m_3, M} = \delta_{J_{tot}, J} \delta_{M_{tot}, M} P_{\alpha, \alpha_1, \alpha_2}^{k_1 k_2 k_3}(J, J_{tot}) Q_a^{k_1 k_2 k_3}(J, J_{tot}) = \sum_{J_c} (-1)^{J+J_1+J_2+J_3} \sqrt{2J_c+1} \sqrt{2J_d+1} \begin{Bmatrix} J_k & J_k & J_c \\ J_k & J_{tot} & J_d \end{Bmatrix} N_{\alpha}(J, J_{tot}) = \delta_{J_{tot}, J} \delta_{M_{tot}, M} \sum_{\alpha, \alpha_1, \alpha_2} \sum_{J_c} \pi_k f_{\alpha\beta\gamma\delta}^{\alpha_1, \alpha_2, \alpha_3, \alpha_4} \frac{\sqrt{2J_c+1}}{\sqrt{2J_a+1}} (2J_d+1) (-1)^{J+J_1+J_2} \times \tilde{V}_{\alpha, \alpha_1, \alpha_2, \alpha_3, \alpha_4}^J V_{\alpha_1, \alpha_2}^{m_1, m_2} U_{\alpha_3, \alpha_4}^{m_3, M} U_{\alpha_1, \alpha_2}^{m_1, m_2} U_{\alpha_3, \alpha_4}^{m_3, M} = \delta_{J_{tot}, J} \delta_{M_{tot}, M} Q_{\alpha, \alpha_1, \alpha_2}^{k_1 k_2 k_3}(J, J_{tot}) R_a^{k_1 k_2 k_3}(J, J_{tot}) = \sum_{J_c} (-1)^{2J+2J_1+2J_2} \sqrt{2J_c+1} \sqrt{2J_d+1} \begin{Bmatrix} J_k & J_k & J_c \\ J_k & J_{tot} & J_d \end{Bmatrix} M_{\alpha}(J, J_{tot}) = -\delta_{J_{tot}, J} \delta_{M_{tot}, M} \sum_{\alpha, \alpha_1, \alpha_2} \sum_{J_c} \pi_k f_{\alpha\beta\gamma\delta}^{\alpha_1, \alpha_2, \alpha_3, \alpha_4} \frac{\sqrt{2J_c+1}}{\sqrt{2J_a+1}} (2J_d+1) (-1)^{J+J_1+J_2} \times \tilde{V}_{\alpha, \alpha_1, \alpha_2, \alpha_3, \alpha_4}^J U_{\alpha_1, \alpha_2}^{m_1, m_2} U_{\alpha_3, \alpha_4}^{m_3, M} V_{\alpha_1, \alpha_2}^{m_1, m_2} V_{\alpha_3, \alpha_4}^{m_3, M} = \delta_{J_{tot}, J} \delta_{M_{tot}, M} R_{\alpha, \alpha_1, \alpha_2}^{k_1 k_2 k_3}(J, J_{tot}) S_a^{k_1 k_2 k_3}(J, J_{tot}) = \sum_{J_c} (-1)^{2J+2J_1+2J_2} \sqrt{2J_c+1} \sqrt{2J_d+1} \begin{Bmatrix} J_k & J_k & J_c \\ J_k & J_{tot} & J_d \end{Bmatrix} N_{\alpha}(J, J_{tot}) = \delta_{J_{tot}, J} \delta_{M_{tot}, M} \sum_{\alpha, \alpha_1, \alpha_2} \sum_{J_c} \pi_k f_{\alpha\beta\gamma\delta}^{\alpha_1, \alpha_2, \alpha_3, \alpha_4} \frac{\sqrt{2J_c+1}}{\sqrt{2J_a+1}} (2J_d+1) (-1)^{J+J_1+J_2} \times \tilde{V}_{\alpha, \alpha_1, \alpha_2, \alpha_3, \alpha_4}^J V_{\alpha_1, \alpha_2}^{m_1, m_2} U_{\alpha_3, \alpha_4}^{m_3, M} U_{\alpha_1, \alpha_2}^{m_1, m_2} U_{\alpha_3, \alpha_4}^{m_3, M} = \delta_{J_{tot}, J} \delta_{M_{tot}, M} S_{\alpha, \alpha_1, \alpha_2}^{k_1 k_2 k_3}(J, J_{tot})$$

These terms are finally put together to form the different contributions to second-order self-energy, as an example [see Eq. (75)]. By inserting Eqs. (C35) and (C36) and summing over all possible total and intermediate angular momenta, one has

$$\Sigma_{ab}^{11(2)} = \frac{1}{2} \sum_{J_{tot}, M_{tot}} \sum_{k_1, k_2, k_3} \left\{ \frac{M_{\alpha}^{k_1 k_2 k_3}(J, J_{tot}) (M_{\alpha}^{k_1 k_2 k_3}(J, J_{tot}))^*}{\omega - (\omega_{k_1} + \omega_{k_2} + \omega_{k_3}) + i\eta} + \frac{N_{\alpha}^{k_1 k_2 k_3}(J, J_{tot}) (N_{\alpha}^{k_1 k_2 k_3}(J, J_{tot}))^*}{\omega + (\omega_{k_1} + \omega_{k_2} + \omega_{k_3}) - i\eta} \right\}$$

$$= \delta_{\alpha\beta} \delta_{\alpha_1, \alpha_2} \frac{1}{2} \sum_{J_c} \sum_{\alpha_3, \alpha_4} \sum_{k_1, k_2, k_3} \left\{ \frac{M_{\alpha}^{k_1 k_2 k_3}(J, J_{tot}) (M_{\alpha}^{k_1 k_2 k_3}(J, J_{tot}))^*}{\omega - (\omega_{k_1} + \omega_{k_2} + \omega_{k_3}) + i\eta} + \frac{N_{\alpha}^{k_1 k_2 k_3}(J, J_{tot}) (N_{\alpha}^{k_1 k_2 k_3}(J, J_{tot}))^*}{\omega + (\omega_{k_1} + \omega_{k_2} + \omega_{k_3}) - i\eta} \right\} \equiv \delta_{\alpha\beta} \delta_{\alpha_1, \alpha_2} \Sigma_{\alpha, \alpha_1, \alpha_2}^{11(a)(2)},$$

Proceeding similarly for the other terms and defining

$$C_{\alpha, \alpha_1, \alpha_2}^{k_1 k_2 k_3}(J, J_{tot}) \equiv \frac{1}{\sqrt{6}} [M_{\alpha, \alpha_1, \alpha_2}^{k_1 k_2 k_3}(J, J_{tot}) - P_{\alpha, \alpha_1, \alpha_2}^{k_1 k_2 k_3}(J, J_{tot}) - R_{\alpha, \alpha_1, \alpha_2}^{k_1 k_2 k_3}(J, J_{tot})],$$

$$D_{\alpha, \alpha_1, \alpha_2}^{k_1 k_2 k_3}(J, J_{tot}) \equiv \frac{1}{\sqrt{6}} [M_{\alpha, \alpha_1, \alpha_2}^{k_1 k_2 k_3}(J, J_{tot}) - P_{\alpha, \alpha_1, \alpha_2}^{k_1 k_2 k_3}(J, J_{tot}) - R_{\alpha, \alpha_1, \alpha_2}^{k_1 k_2 k_3}(J, J_{tot}) - S_{\alpha, \alpha_1, \alpha_2}^{k_1 k_2 k_3}(J, J_{tot})],$$

$$\Sigma_{\alpha, \alpha_1, \alpha_2}^{11(a)(2)} = \sum_{\alpha_3, \alpha_4} \sum_{J_c} \sum_{k_1, k_2, k_3} \left\{ \frac{C_{\alpha, \alpha_1, \alpha_2}^{k_1 k_2 k_3}(J, J_{tot}) (C_{\alpha, \alpha_1, \alpha_2}^{k_1 k_2 k_3}(J, J_{tot}))^*}{\omega - (\omega_{k_1} + \omega_{k_2} + \omega_{k_3}) + i\eta} + \frac{(D_{\alpha, \alpha_1, \alpha_2}^{k_1 k_2 k_3}(J, J_{tot}))^* D_{\alpha, \alpha_1, \alpha_2}^{k_1 k_2 k_3}(J, J_{tot})}{\omega + (\omega_{k_1} + \omega_{k_2} + \omega_{k_3}) - i\eta} \right\},$$

$$\Sigma_{\alpha, \alpha_1, \alpha_2}^{12(a)(2)} = \sum_{\alpha_3, \alpha_4} \sum_{J_c} \sum_{k_1, k_2, k_3} \left\{ \frac{C_{\alpha, \alpha_1, \alpha_2}^{k_1 k_2 k_3}(J, J_{tot}) (D_{\alpha, \alpha_1, \alpha_2}^{k_1 k_2 k_3}(J, J_{tot}))^*}{\omega - (\omega_{k_1} + \omega_{k_2} + \omega_{k_3}) + i\eta} + \frac{(D_{\alpha, \alpha_1, \alpha_2}^{k_1 k_2 k_3}(J, J_{tot}))^* C_{\alpha, \alpha_1, \alpha_2}^{k_1 k_2 k_3}(J, J_{tot})}{\omega + (\omega_{k_1} + \omega_{k_2} + \omega_{k_3}) - i\eta} \right\},$$

$$\Sigma_{\alpha, \alpha_1, \alpha_2}^{21(a)(2)} = \sum_{\alpha_3, \alpha_4} \sum_{J_c} \sum_{k_1, k_2, k_3} \left\{ \frac{D_{\alpha, \alpha_1, \alpha_2}^{k_1 k_2 k_3}(J, J_{tot}) (C_{\alpha, \alpha_1, \alpha_2}^{k_1 k_2 k_3}(J, J_{tot}))^*}{\omega - (\omega_{k_1} + \omega_{k_2} + \omega_{k_3}) + i\eta} + \frac{(C_{\alpha, \alpha_1, \alpha_2}^{k_1 k_2 k_3}(J, J_{tot}))^* D_{\alpha, \alpha_1, \alpha_2}^{k_1 k_2 k_3}(J, J_{tot})}{\omega + (\omega_{k_1} + \omega_{k_2} + \omega_{k_3}) - i\eta} \right\},$$

$$\Sigma_{\alpha, \alpha_1, \alpha_2}^{22(a)(2)} = \sum_{\alpha_3, \alpha_4} \sum_{J_c} \sum_{k_1, k_2, k_3} \left\{ \frac{D_{\alpha, \alpha_1, \alpha_2}^{k_1 k_2 k_3}(J, J_{tot}) (D_{\alpha, \alpha_1, \alpha_2}^{k_1 k_2 k_3}(J, J_{tot}))^*}{\omega - (\omega_{k_1} + \omega_{k_2} + \omega_{k_3}) + i\eta} + \frac{(C_{\alpha, \alpha_1, \alpha_2}^{k_1 k_2 k_3}(J, J_{tot}))^* C_{\alpha, \alpha_1, \alpha_2}^{k_1 k_2 k_3}(J, J_{tot})}{\omega + (\omega_{k_1} + \omega_{k_2} + \omega_{k_3}) - i\eta} \right\}.$$

6. Block-diagonal structure of Gorkov's equations

In the previous subsections it has been proven that all single-particle Green's functions and all self-energy contributions entering Gorkov's equations display the same block-diagonal structure if the systems is in a 0^+ state. Defining

$$T_{ab} - \mu \delta_{ab} \equiv \delta_{\alpha\beta} \delta_{\alpha_1, \alpha_2} [T_{\alpha, \alpha_1, \alpha_2}^{(a)} - \mu^{(a)} \delta_{\alpha, \alpha_1, \alpha_2}],$$

introducing block-diagonal forms for amplitudes W and Z through

$$W_{\alpha}(J, J_{tot}) \equiv \delta_{J_{tot}, J} \delta_{M_{tot}, M} W_{\alpha, \alpha_1, \alpha_2}^{k_1 k_2 k_3}(J, J_{tot}),$$

$$Z_{\alpha}(J, J_{tot}) \equiv -\delta_{J_{tot}, J} \delta_{M_{tot}, M} \eta_k Z_{\alpha, \alpha_1, \alpha_2}^{k_1 k_2 k_3}(J, J_{tot}),$$

with

$$(\omega_k - E_{k_1 k_2 k_3}) W_{\alpha, \alpha_1, \alpha_2}^{k_1 k_2 k_3}(J, J_{tot}) \equiv \sum_{\alpha, \alpha_1, \alpha_2} [(T_{\alpha, \alpha_1, \alpha_2}^{(a)})^* U_{\alpha, \alpha_1}^{\mu} + (D_{\alpha, \alpha_1, \alpha_2}^{k_1 k_2 k_3}(J, J_{tot}))^* V_{\alpha, \alpha_1}^{\mu}],$$

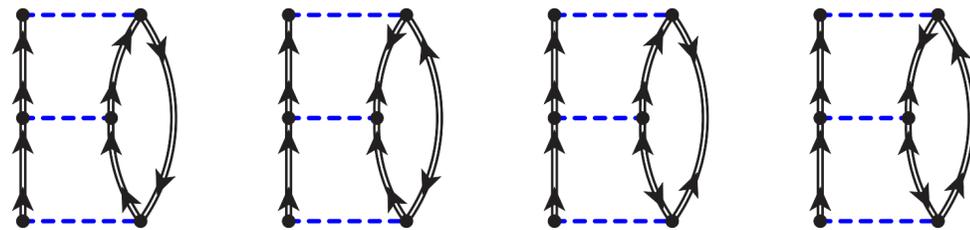
$$(\omega_k + E_{k_1 k_2 k_3}) Z_{\alpha, \alpha_1, \alpha_2}^{k_1 k_2 k_3}(J, J_{$$

Nambu-Covariant approach to build (Gorkov) propagators

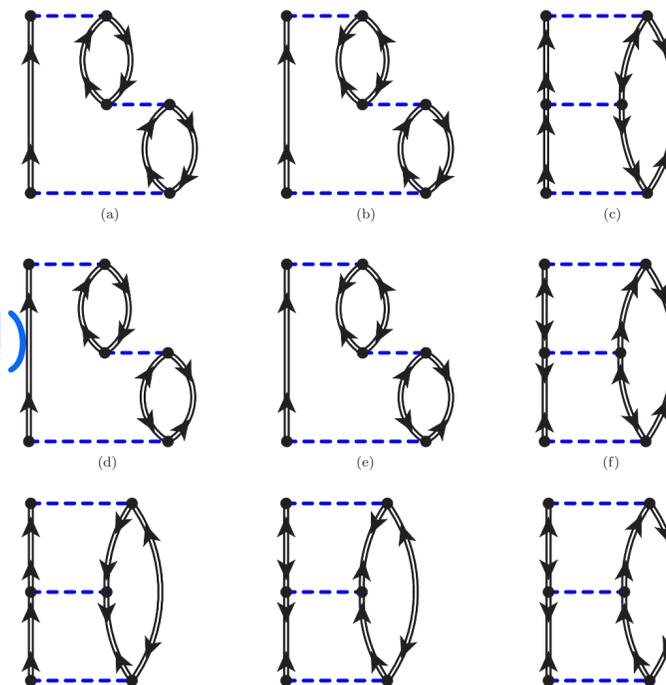
Gorkov at
2nd order:

$$\Sigma_{\alpha\beta}^{11}(\omega) = \text{[Diagram 1]} + \text{[Diagram 2]}$$

pp-ladders:



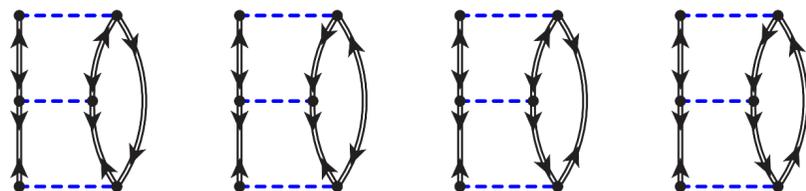
ph-rings:



Gorkov at
3rd order:

hh-interactions (hh int. among pp ladders!!!)

(ONLY NN forces)



(NN ONLY forces) LI STUDI DI MILANO



Nambu-Covariant approach to build (Gorkov) propagators

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Gorkov algebraic diagrammatic construction formalism at third order

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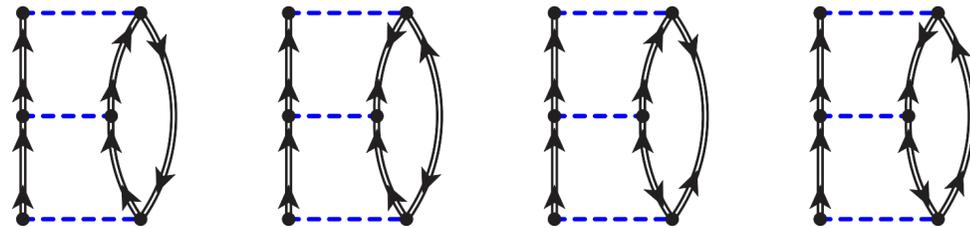
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Gorkov at
2nd order:

$$\Sigma_{\alpha\beta}^{11}(\omega) = \text{Diagram 1} + \text{Diagram 2}$$

pp-ladders:



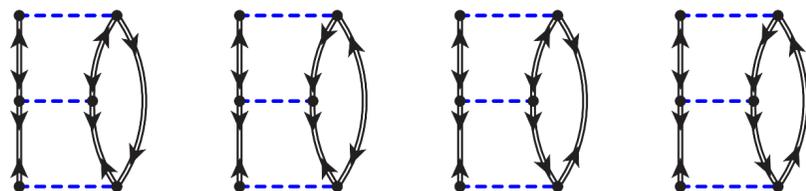
$$\tilde{\Sigma}_{\alpha\beta}^{11}(\omega) = \sum_{rr'} \left\{ C_{\alpha,r} \left[\frac{1}{\omega\mathbb{I} - \mathcal{E} + i\eta} \right]_{r,r'} C_{r',\beta}^\dagger + \bar{D}_{\alpha,r}^\dagger \left[\frac{1}{\omega\mathbb{I} + \mathcal{E}^T - i\eta} \right]_{r,r'} \bar{D}_{r',\beta} \right\}, \quad (29a)$$

$$\tilde{\Sigma}_{\alpha\beta}^{12}(\omega) = \sum_{rr'} \left\{ C_{\alpha,r} \left[\frac{1}{\omega\mathbb{I} - \mathcal{E} + i\eta} \right]_{r,r'} D_{r',\beta}^* + \bar{D}_{\alpha,r}^\dagger \left[\frac{1}{\omega\mathbb{I} + \mathcal{E}^T - i\eta} \right]_{r,r'} \bar{C}_{r',\beta}^T \right\}, \quad (29b)$$

Gorkov at
3rd order:

(ONLY NN forces)

hh-interactions (hh int. among pp ladders)



(NN ONLY forces) LI STUDI DI MILANO

$$C_{\alpha,r}^{(\text{IIa})} = \frac{1}{\sqrt{6}} \mathcal{P}_{123} \sum_{\substack{\mu\nu\lambda \\ k_4 k_5}} \frac{v_{\alpha\lambda,\mu\nu}}{2} (\bar{v}_\mu^{k_4} \bar{v}_\nu^{k_5})^* t_{k_4 k_5}^{k_1 k_2} \bar{v}_\lambda^{k_3}, \quad (43a)$$

$$C_{\alpha,r}^{(\text{IIb})} = \frac{1}{\sqrt{6}} \mathcal{P}_{123} \sum_{\substack{\mu\nu\lambda \\ k_4 k_5}} v_{\alpha\lambda,\mu\nu} (\bar{v}_\nu^{k_4} \mathcal{U}_\lambda^{k_5})^* t_{k_4 k_5}^{k_1 k_2} \mathcal{U}_\mu^{k_3}, \quad (43b)$$

$$C_{\alpha,r}^{(\text{IIc})} = \frac{1}{\sqrt{6}} \mathcal{P}_{123} \sum_{\substack{\mu\nu\lambda \\ k_4 k_5}} \frac{v_{\alpha\lambda,\mu\nu}}{2} (\bar{v}_\mu^{k_4} \bar{v}_\nu^{k_5})^* t_{k_1 k_2}^{k_4 k_5} \bar{v}_\lambda^{k_3}, \quad (47a)$$

$$C_{\alpha,r}^{(\text{IId})} = \frac{1}{\sqrt{6}} \mathcal{P}_{123} \sum_{\substack{\mu\nu\lambda \\ k_4 k_5}} v_{\alpha\lambda,\mu\nu} (\bar{v}_\nu^{k_4} \mathcal{U}_\lambda^{k_5})^* t_{k_1 k_2}^{k_4 k_5} \mathcal{U}_\mu^{k_3}, \quad (47b)$$

$$\mathcal{E}_{k_1 k_2, k_4 k_5}^{(pp)} = \sum_{\alpha\beta\gamma\delta} (\mathcal{U}_\alpha^{k_1} \mathcal{U}_\beta^{k_2})^* v_{\alpha\beta,\gamma\delta} \mathcal{U}_\gamma^{k_4} \mathcal{U}_\delta^{k_5}, \quad (45)$$

$$\mathcal{E}_{k_1 k_2, k_4 k_5}^{(hh)} = \sum_{\alpha\beta\gamma\delta} \bar{v}_\alpha^{k_1} \bar{v}_\beta^{k_2} v_{\alpha\beta,\gamma\delta} (\bar{v}_\gamma^{k_4} \bar{v}_\delta^{k_5})^*. \quad (46)$$

$$C_{\alpha,r}^{(\text{IIe})} = \frac{1}{\sqrt{6}} \mathcal{A}_{123} \sum_{\substack{\mu\nu\lambda \\ k_7 k_8}} v_{\alpha\lambda,\mu\nu} (\bar{v}_\nu^{k_7} \mathcal{U}_\lambda^{k_8})^* \mathcal{U}_\mu^{k_1} t_{k_7 k_8}^{k_3 k_2}, \quad (50a)$$

$$C_{\alpha,r}^{(\text{IIf})} = \frac{1}{\sqrt{6}} \mathcal{A}_{123} \sum_{\substack{\mu\nu\lambda \\ k_7 k_8}} v_{\alpha\lambda,\mu\nu} (\mathcal{U}_\lambda^{k_7} \bar{v}_\mu^{k_8})^* \mathcal{U}_\nu^{k_1} t_{k_7 k_8}^{k_3 k_2}, \quad (50b)$$

$$C_{\alpha,r}^{(\text{IIg})} = \frac{1}{\sqrt{6}} \mathcal{A}_{123} \sum_{\substack{\mu\nu\lambda \\ k_7 k_8}} v_{\alpha\lambda,\mu\nu} (\bar{v}_\mu^{k_7} \bar{v}_\nu^{k_8})^* \bar{v}_\lambda^{k_1} t_{k_7 k_8}^{k_3 k_2}, \quad (50c)$$

$$\mathcal{E}_{r,r'}^{(\text{Ic})} = \frac{1}{6} \mathcal{A}_{123} \mathcal{A}_{456} (\delta_{k_1, k_4} \mathcal{E}_{k_2 k_3, k_5 k_6}^{(ph)}),$$



Nambu-Covariant approach to build (Gorkov) propagators

Combine w/ dual-basis:

$$\mathcal{H}_1^e \equiv \mathcal{H}_1 \times \mathcal{H}_1^\dagger$$

$$\mathcal{B}^e \equiv \mathcal{B} \cup \bar{\mathcal{B}}$$

(contravariant) \swarrow \searrow (covariant)

$$\mathcal{B} \equiv \{ |b\rangle \} \quad \bar{\mathcal{B}} \equiv \{ \langle \bar{b} | \}$$

$$\langle \bar{b} | c \rangle = \delta_{bc}$$

Generalised states:

$$\begin{pmatrix} |\Psi_1\rangle \\ \langle \Psi'_1| \end{pmatrix}$$

Inner product:

$$g \left(\begin{pmatrix} |\Psi_1\rangle \\ \langle \Psi'_1| \end{pmatrix}, \begin{pmatrix} |\Psi_2\rangle \\ \langle \Psi'_2| \end{pmatrix} \right) \equiv \langle \Psi'_2 | \Psi_1 \rangle + \langle \Psi'_1 | \Psi_2 \rangle$$



M. Drissi (Surrey, now @ TRIUMF, Canada)

Englobe Nambu indices in the basis:

$$|\beta\rangle \equiv |b, 1\rangle \equiv \begin{pmatrix} |b\rangle \\ 0 \end{pmatrix}$$

$$|\bar{\beta}\rangle \equiv |b, 2\rangle \equiv \begin{pmatrix} 0 \\ \langle \bar{b} | \end{pmatrix}$$

Generalised Nambu operators and commutation rules:

$$\begin{aligned} A^{(b,1)} &\equiv a_b \\ A^{(b,2)} &\equiv \bar{a}_b \\ \bar{A}_{(b,1)} &\equiv \bar{a}_b \\ \bar{A}_{(b,2)} &\equiv a_b \end{aligned} \quad \longrightarrow \quad \begin{aligned} \bar{A}_\mu &= \sum_\nu g_{\mu\nu} A^\nu \\ \bar{A}_\mu &= \sum_\nu g_{\mu\nu} \bar{A}_\nu \\ A^\mu &= \sum_\nu g^{\mu\nu} \bar{A}_\nu \\ A^\mu &= \sum_\nu g^{\mu\nu} A^\nu \end{aligned} \quad \longrightarrow \quad \begin{aligned} \{ A^\mu, A^\nu \} &= g^{\mu\nu} \\ \{ A^\mu, \bar{A}_\nu \} &= g^{\mu\nu} \\ \{ \bar{A}_\mu, A^\nu \} &= g_{\mu\nu} \\ \{ \bar{A}_\mu, \bar{A}_\nu \} &= g_{\mu\nu} \end{aligned}$$



Nambu-Covariant approach to build (Gorkov) propagators

Combine w/ dual-basis:

$$\mathcal{H}_1^e \equiv \mathcal{H}_1 \times \mathcal{H}_1^\dagger$$

$$\mathcal{B}^e \equiv \mathcal{B} \cup \bar{\mathcal{B}}$$

(contravariant) \swarrow \searrow (covariant)

$$\mathcal{B} \equiv \{ |b\rangle \} \quad \bar{\mathcal{B}} \equiv \{ \langle \bar{b} | \}$$

$$\langle \bar{b} | c \rangle = \delta_{bc}$$

Generalised states:

$$\begin{pmatrix} |\Psi_1\rangle \\ \langle \Psi'_1| \end{pmatrix}$$

Inner product:

$$g \left(\begin{pmatrix} |\Psi_1\rangle \\ \langle \Psi'_1| \end{pmatrix}, \begin{pmatrix} |\Psi_2\rangle \\ \langle \Psi'_2| \end{pmatrix} \right) \equiv \langle \Psi'_2 | \Psi_1 \rangle + \langle \Psi'_1 | \Psi_2 \rangle$$



M. Drissi (Surrey, now @ TRIUMF, Canada)

Covariant (contravariant) of operators:

$$O \equiv \sum_{\substack{\mu_1 \dots \mu_k \\ \nu_1 \dots \nu_k}} o^{\mu_1 \dots \mu_k}_{\nu_1 \dots \nu_k} \bar{A}_{\mu_1} \dots \bar{A}_{\mu_k} A^{\nu_1} \dots A^{\nu_k} \quad \text{(mixed)}$$

$$\equiv \sum_{\mu_1 \dots \mu_{2k}} o_{\mu_1 \dots \mu_{2k}} A^{\mu_1} \dots A^{\mu_{2k}} \quad \text{(covariant)}$$

$$\equiv \sum_{\mu_1 \dots \mu_{2k}} o^{\mu_1 \dots \mu_{2k}} \bar{A}_{\mu_1} \dots \bar{A}_{\mu_{2k}} \quad \text{(contravariant)}$$

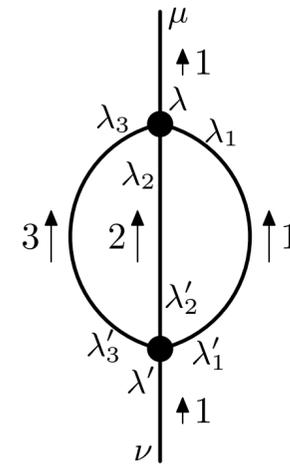
$$O_{\mu_1 \dots \mu_{2k}} = \sum_{\alpha_1 \dots \alpha_k} g_{\mu_1 \alpha_1} \dots g_{\mu_k \alpha_k} O^{\alpha_1 \dots \alpha_k}_{\mu_{k+1} \dots \mu_{2k}}$$

Observables are always scalars—they remain invariant under **any** change of basis.



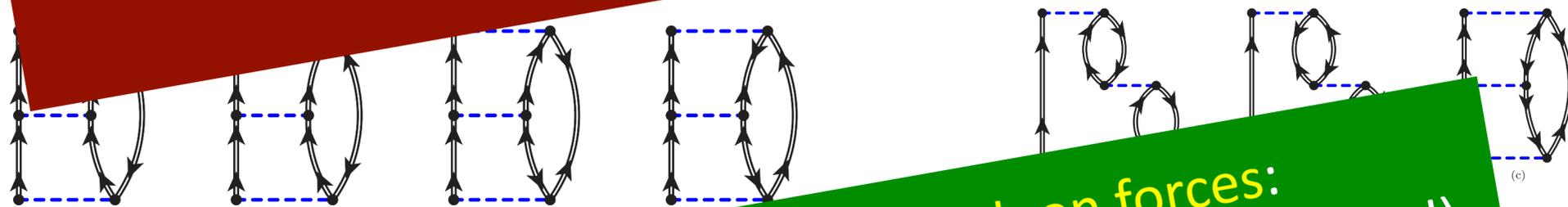
Nambu-Covariant approach to build (Gorkov) propagators

Gorkov at
2nd order:



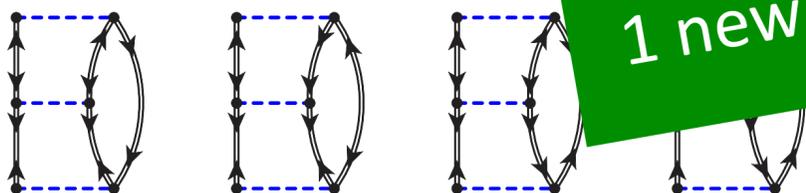
Just ONE topology at 2nd and 3rd order!
(2-body forces only)

ph-rings:

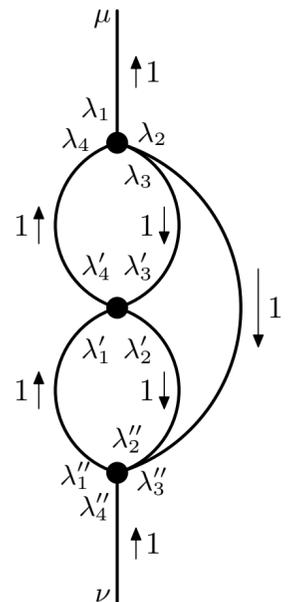
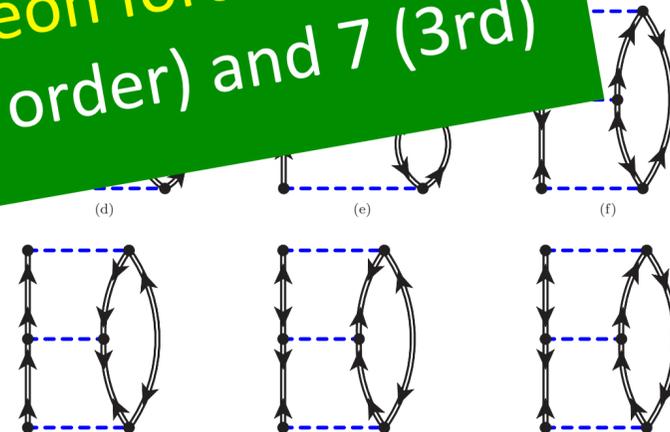


Gorkov at
3rd order:
(ONLY NN forces)

hh-interactions (hh)



For three-nucleon forces:
1 new topology (2nd order) and 7 (3rd)

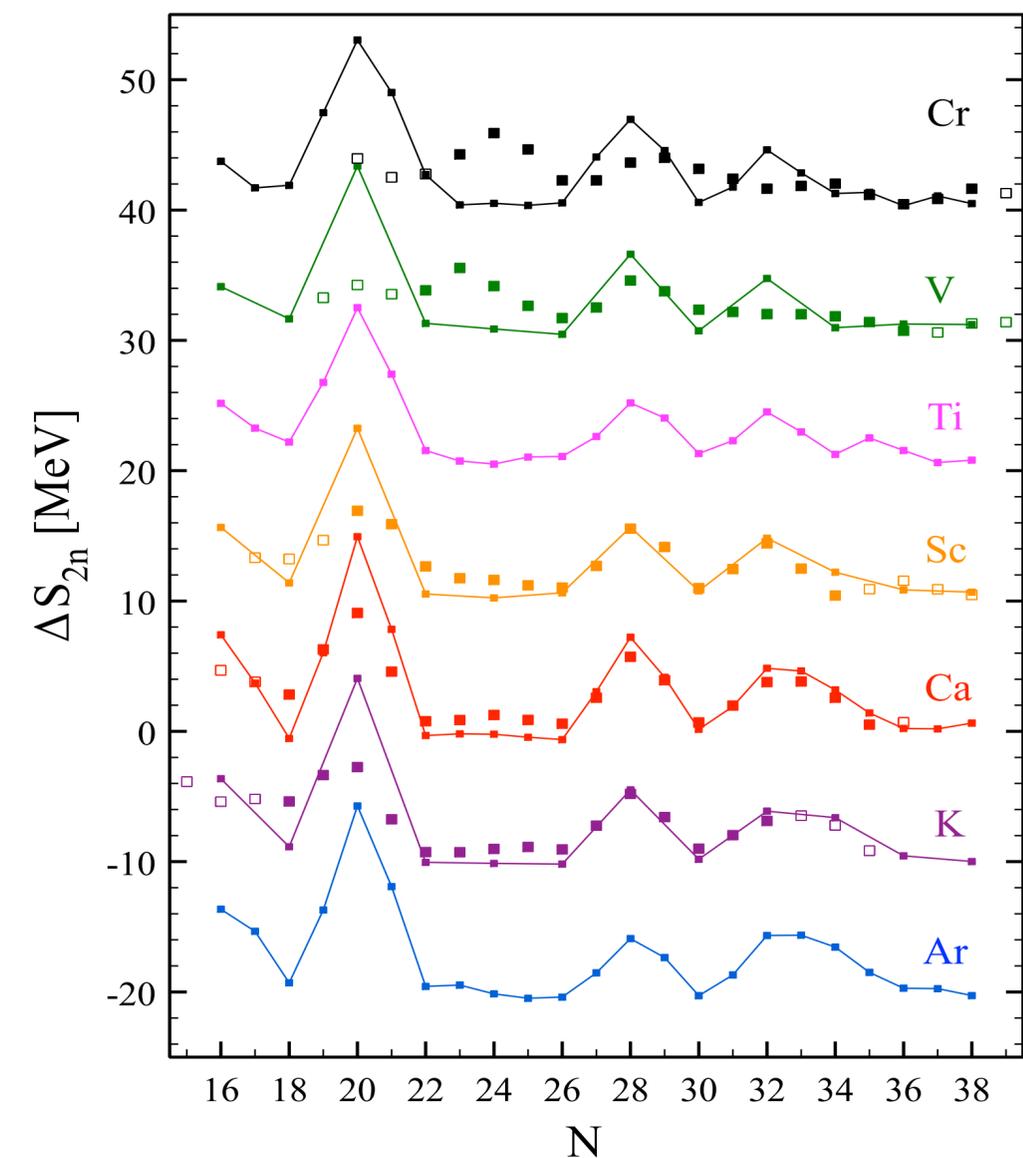
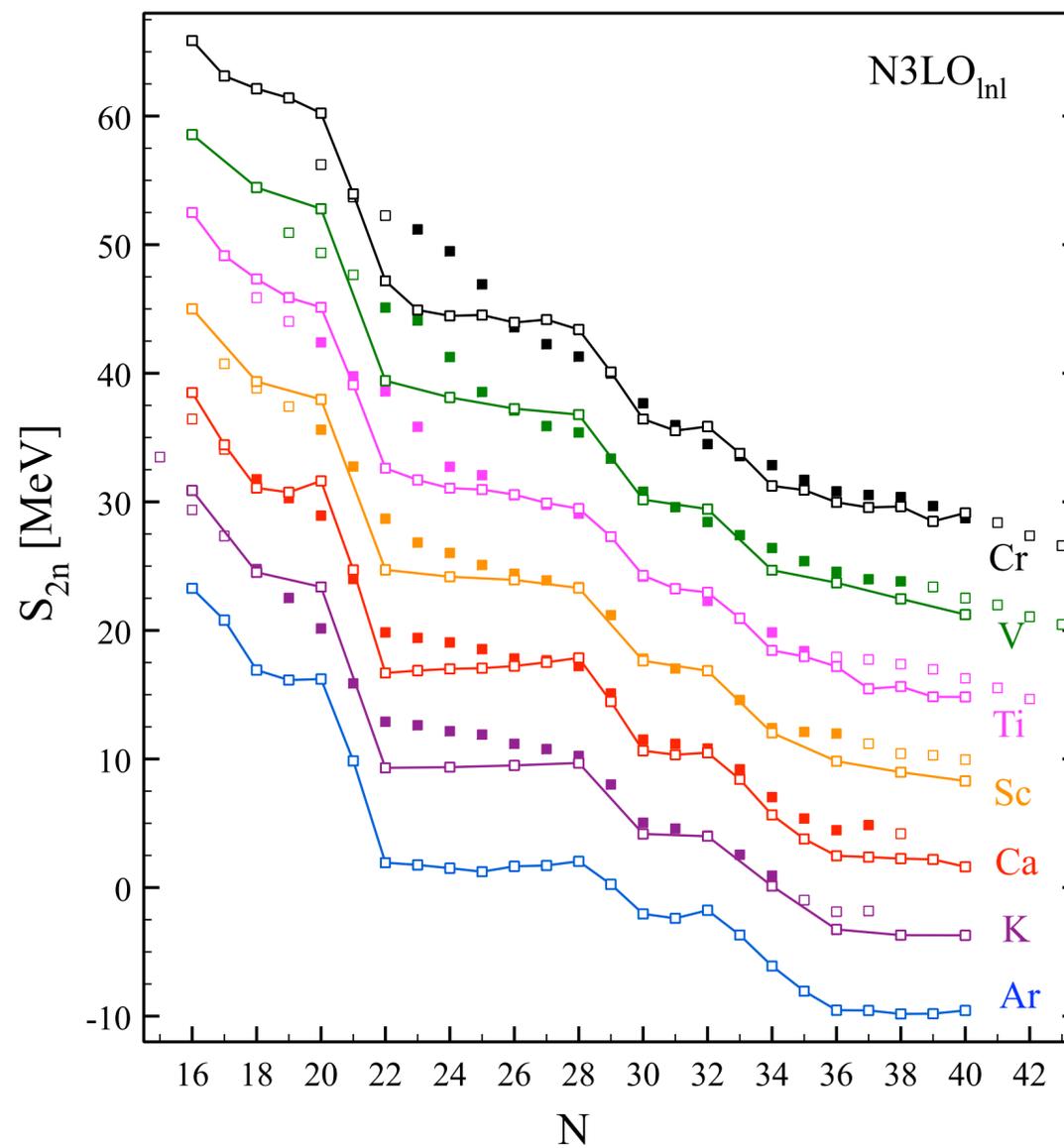
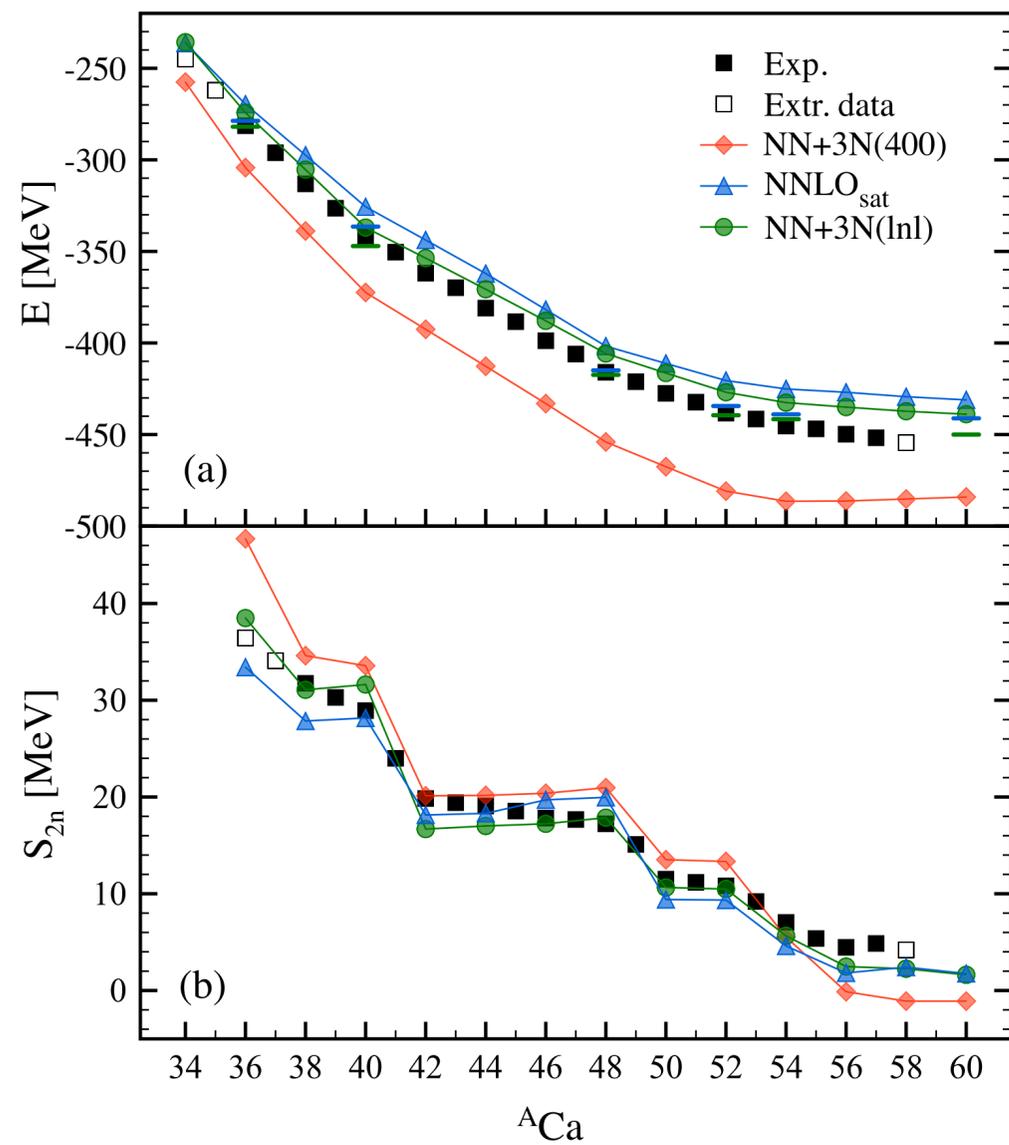


(NN ONLY forces) LI STUDI DI MILANO



N3LO(500) + nln 3NF

SCGF – Gorkov-ADC(2)



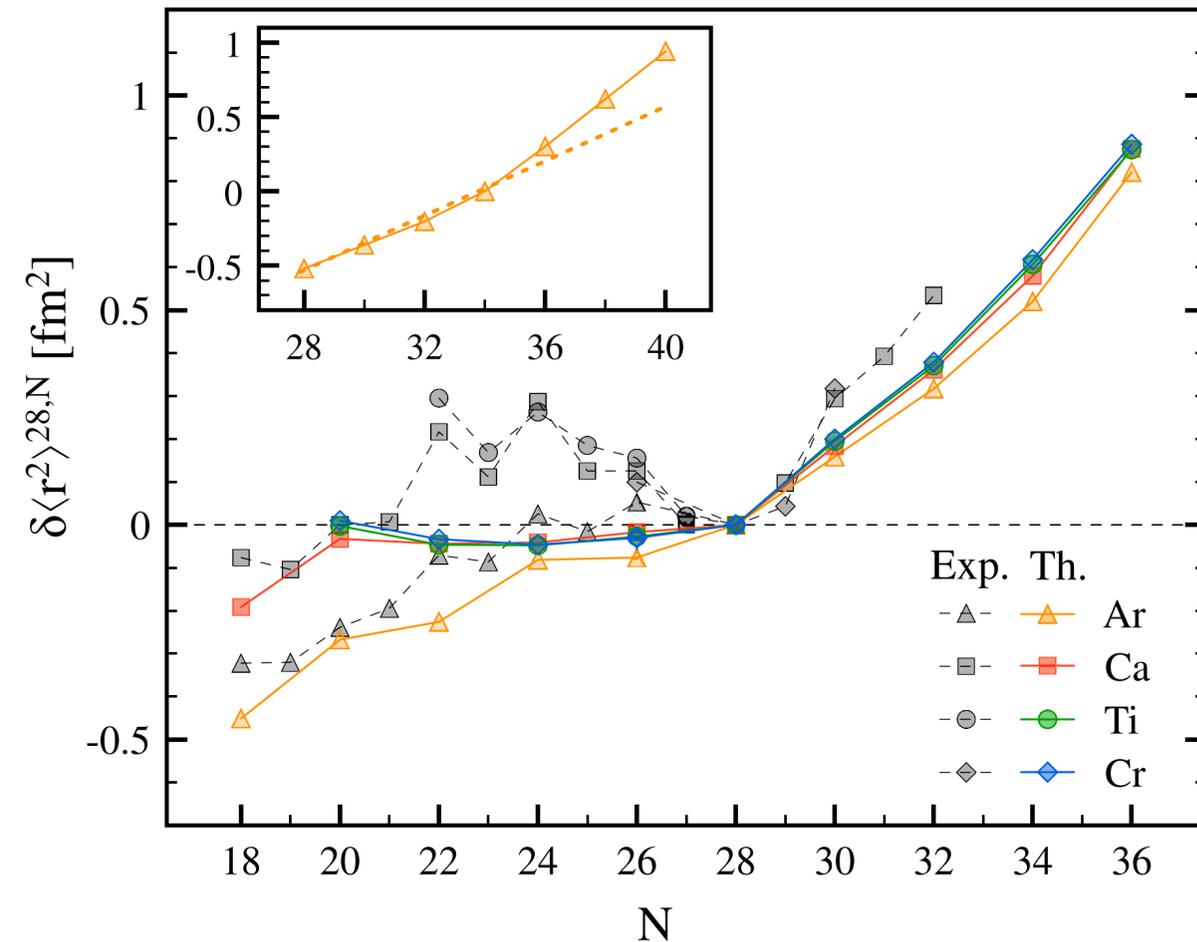
V. Somà, P. Navrátil, F. Raimondi, CB, T. Duguet – Phys. Rev. C **101**, 014318 (2020)

Eur. Phys. J. A **57** 135 (2021)



N3LO(500) + nln 3NF

SCGF – Gorkov-ADC(2)



- Bell-shaped behaviour in Ca40-48 requires particle-vibration coupling [ADC(3), FRPA]
- Universal behaviour of isotope shifts beyond $N=28$ neutrons (invariance on $20 < Z < 28$)

N3LO(500) + nln 3NF

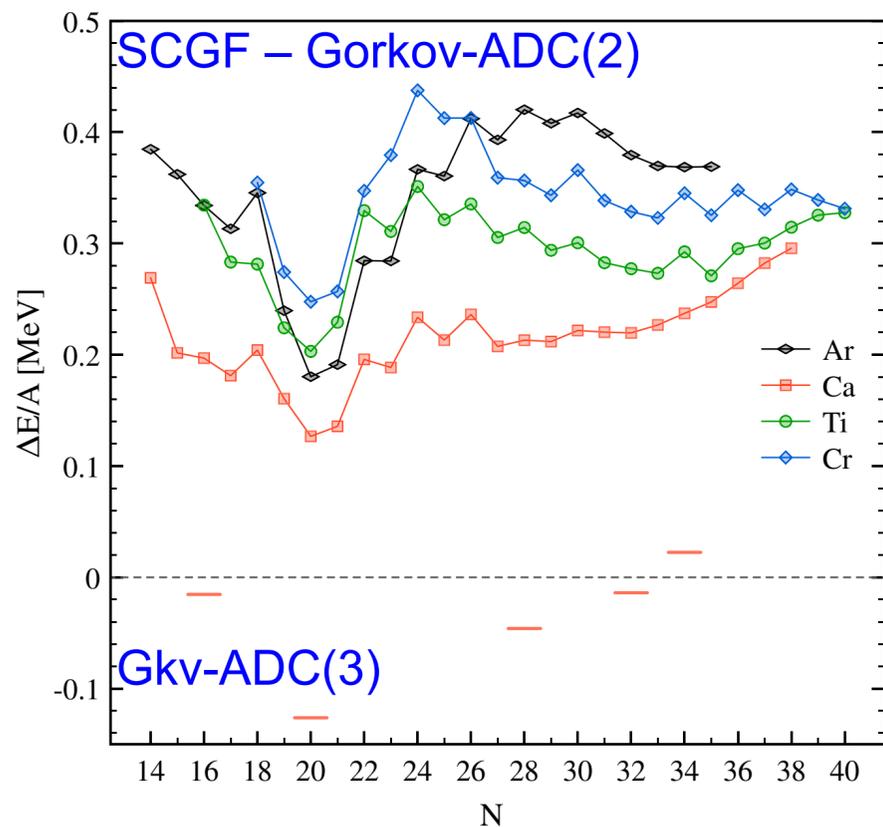
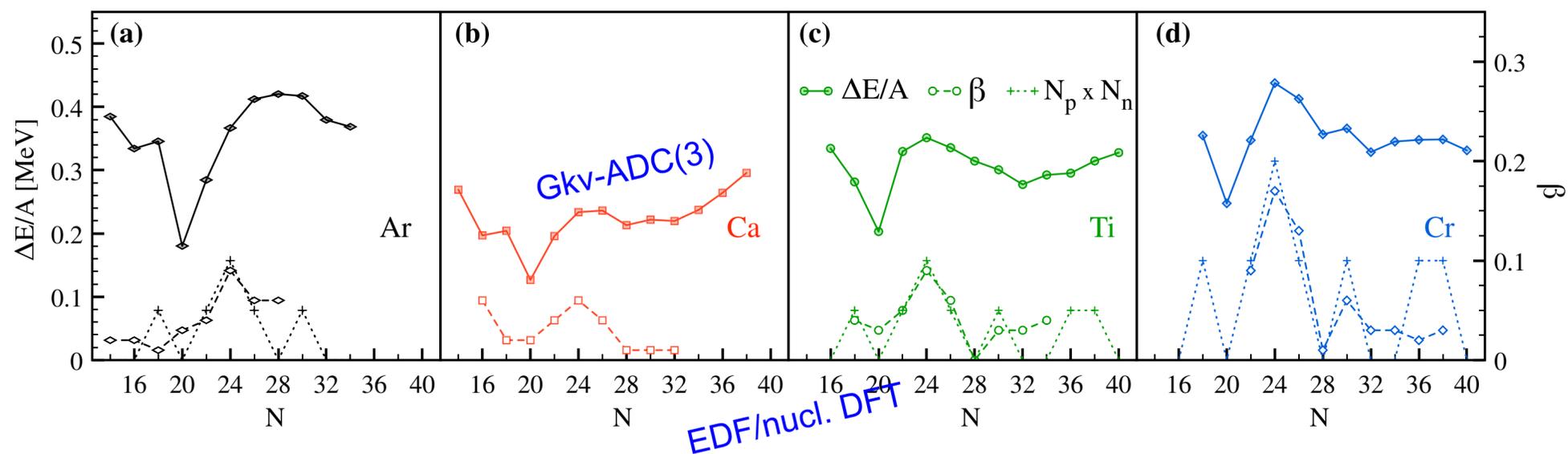


Fig. 8 Relative ADC(2) errors (theory-experiment) on total binding energies per nucleon along $Z = 18, 20, 22$ and 24 isotopic chains. ADC(3) errors are also reported for doubly closed-shell calcium isotopes and displayed as horizontal bars. Calculations and experimental data are taken from Fig. 1

- Accuracy for binding energies requires ADC(3)
- Larger discrepancies



Ab initio optical potentials from propagator theory

Relation to Feshbach theory:

Mahaux & Sartor, Adv. Nucl. Phys. 20 (1991)

Escher & Jennings Phys. Rev. C**66**, 034313 (2002)

Previous SCGF work:

CB, B. Jennings, Phys. Rev. C**72**, 014613 (2005)

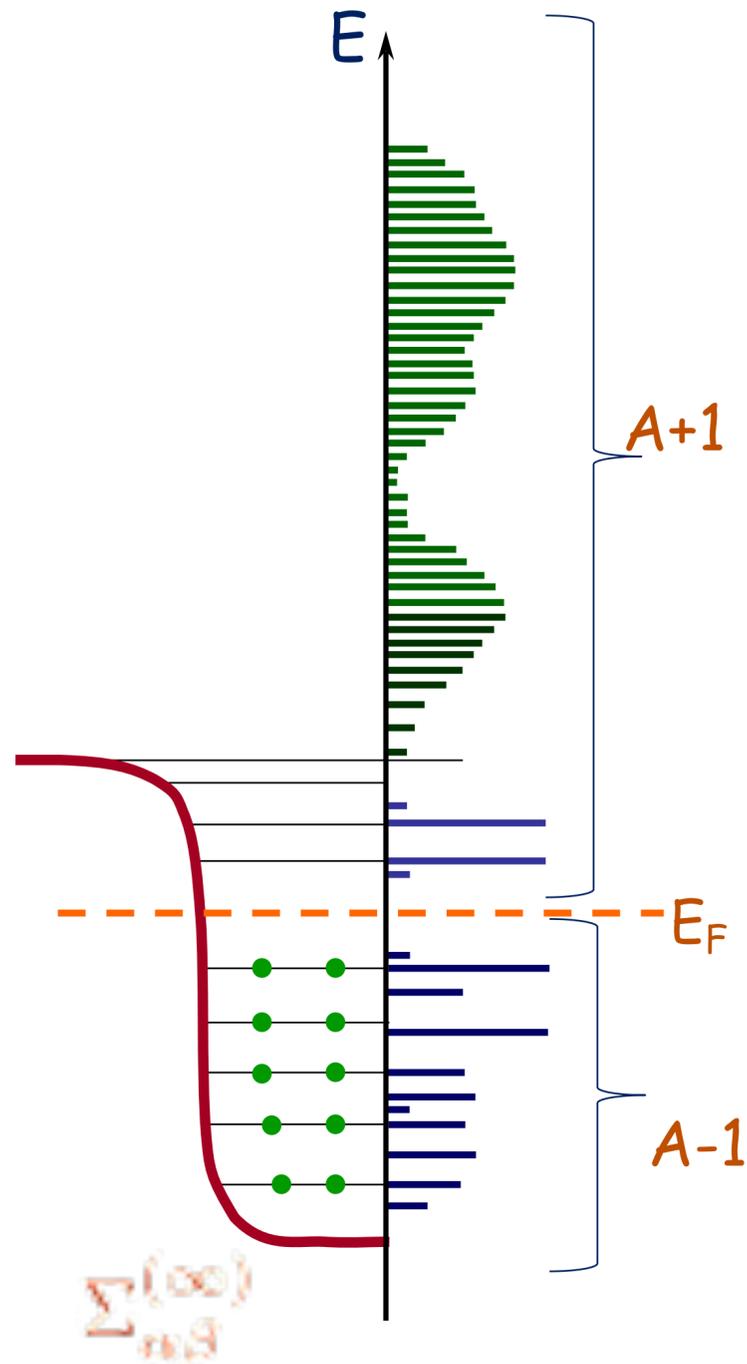
S. Waldecker, CB, W. Dickhoff, Phys. Rev. C**84**, 034616 (2011)

A. Idini, CB, P. Navrátil, Phys. Rv. Lett. **123**, 092501 (2019)

M. Vorabbi, CB, et al., in preparation



Microscopic optical potential

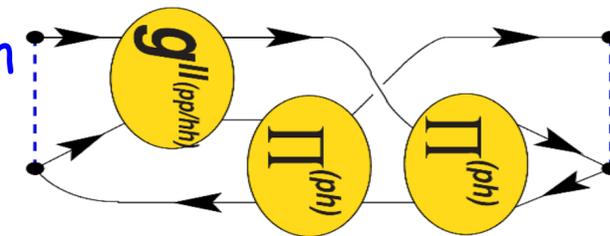


Nuclear self-energy $\Sigma^*(\mathbf{r}, \mathbf{r}'; \varepsilon)$:

- contains *both particle and hole* props.
- it is proven to be a *Feshbach opt. pot* \rightarrow in general it is *non-local* !

$$\Sigma_{\alpha\beta}^*(\omega) = \Sigma_{\alpha\beta}^{(\infty)} + \underbrace{\sum_{i,j} \mathbf{M}_{\alpha,i}^\dagger \left(\frac{1}{E - (\mathbf{K}^> + \mathbf{C}) + i\Gamma} \right)_{i,j} \mathbf{M}_{j,\beta} + \sum_{r,s} \mathbf{N}_{\alpha,r} \left(\frac{1}{E - (\mathbf{K}^< + \mathbf{D}) - i\Gamma} \right)_{r,s} \mathbf{N}_{s,\beta}^\dagger}_{\text{Particle-vibration couplings}}$$

Particle-vibration couplings:



Solve scattering and overlap functions directly in momentum space:

$$\Sigma^{*l,j}(k, k'; E) = \sum_{n, n'} R_{nl}(k) \Sigma_{n, n'}^{*l,j} R_{nl}(k')$$

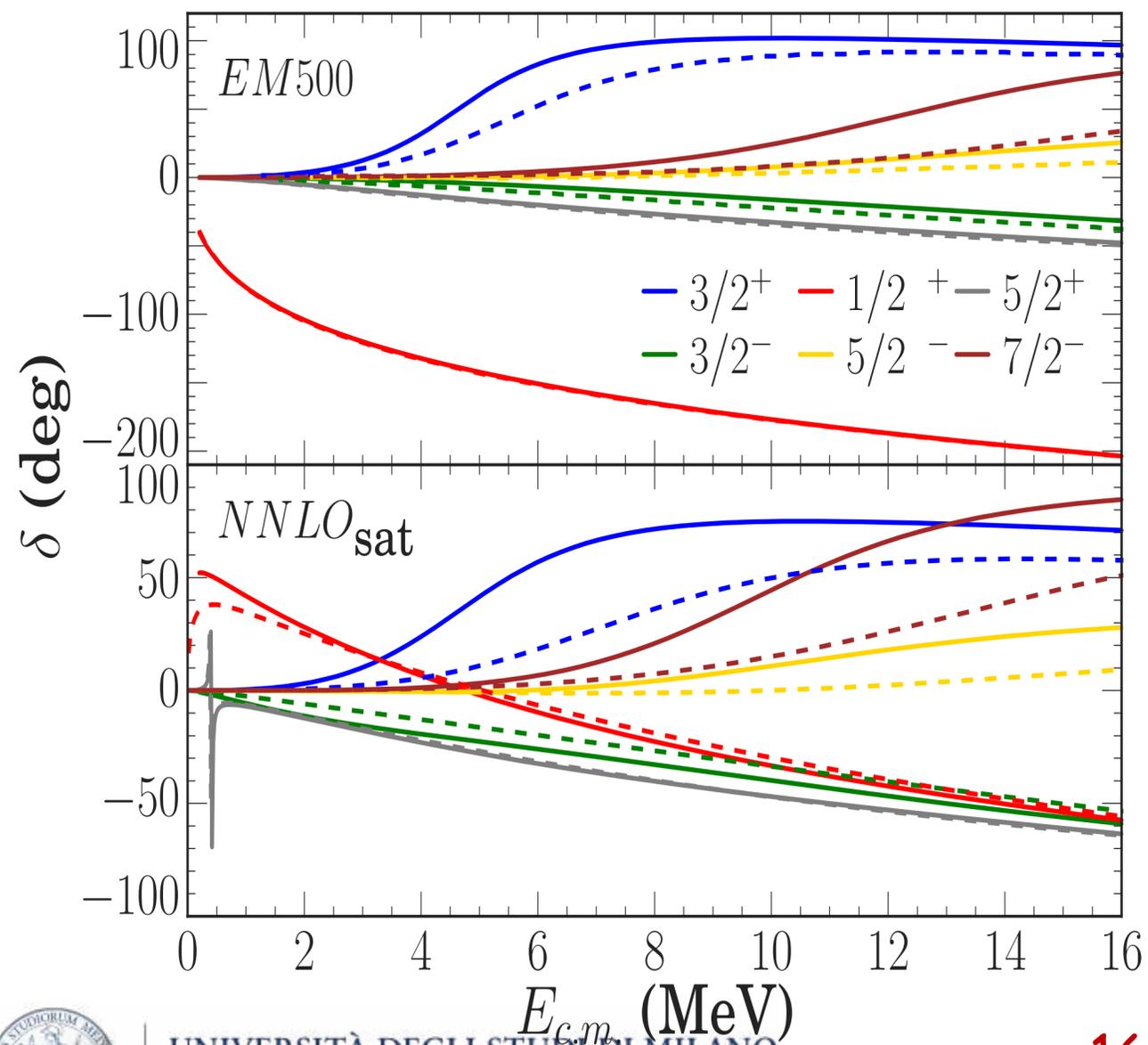
$$\frac{k^2}{2\mu} \psi_{l,j}(k) + \int dk' k'^2 \Sigma^{*l,j}(k, k'; E_{c.m.}) \psi_{l,j}(k') = E_{c.m.} \psi_{l,j}(k)$$

Low energy scattering - from SCGF

[A. Idini, CB, Navratil,
Phys. Rev. Lett. **123**, 092501 (2019)]

Benchmark with NCSM-based scattering.

Scattering from mean-field only:



----- NCSM/RGM [without core excitations]

EM500: NN-SRG $\lambda_{\text{SRG}} = 2.66 \text{ fm}^{-1}$, $N_{\text{max}}=18$ (IT)
[PRC82, 034609 (2010)]

NNLO_{sat}: $N_{\text{max}}=8$ (IT-NCSM)

———— SCGF [$\Sigma^{(\infty)}$ only], always $N_{\text{max}}=13$

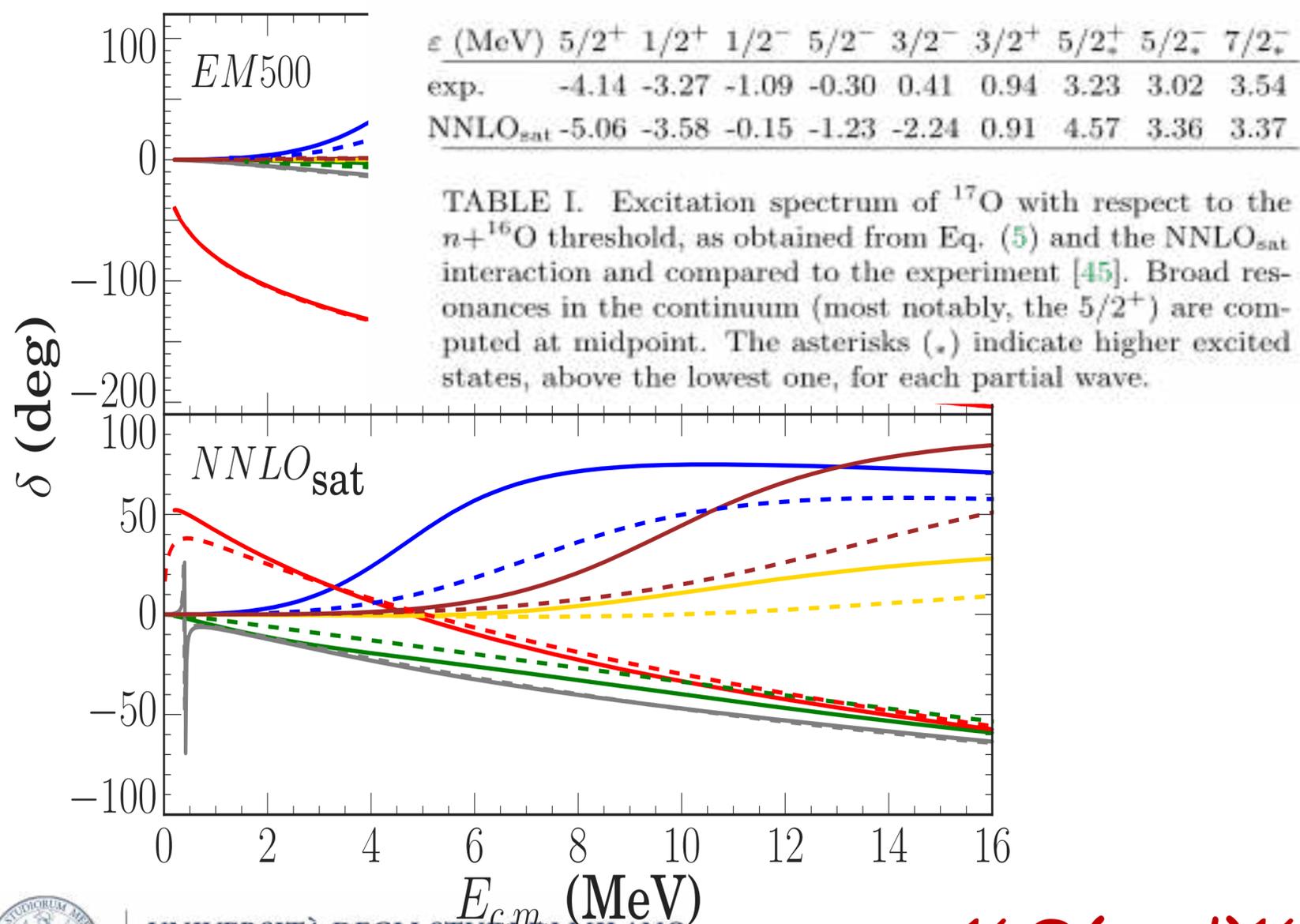


Low energy scattering - from SCGF

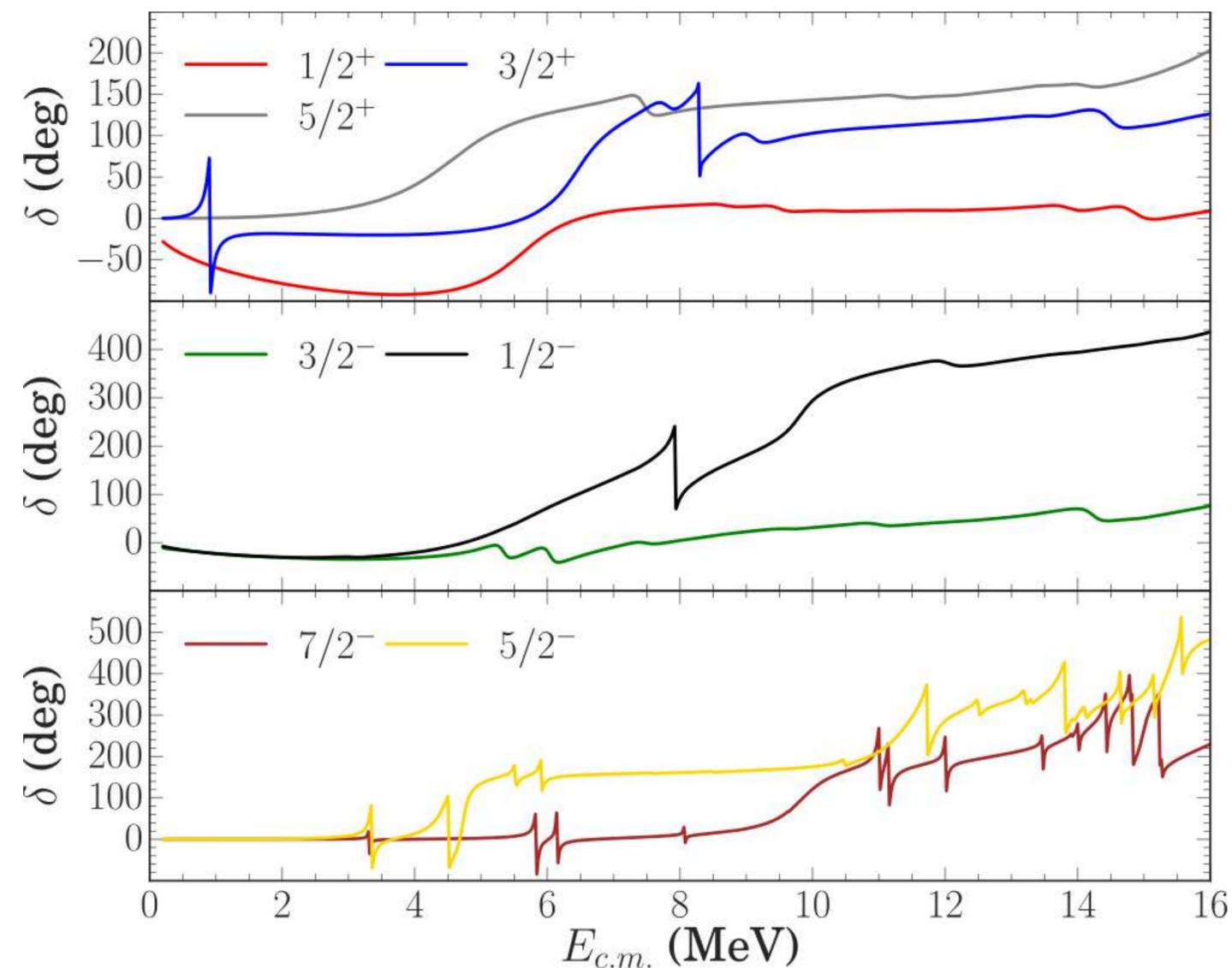
[A. Idini, CB, Navratil,
Phys. Rev. Lett. **123**, 092501 (2019)]

Benchmark with NCSM-based scattering.

Scattering from mean-field only:



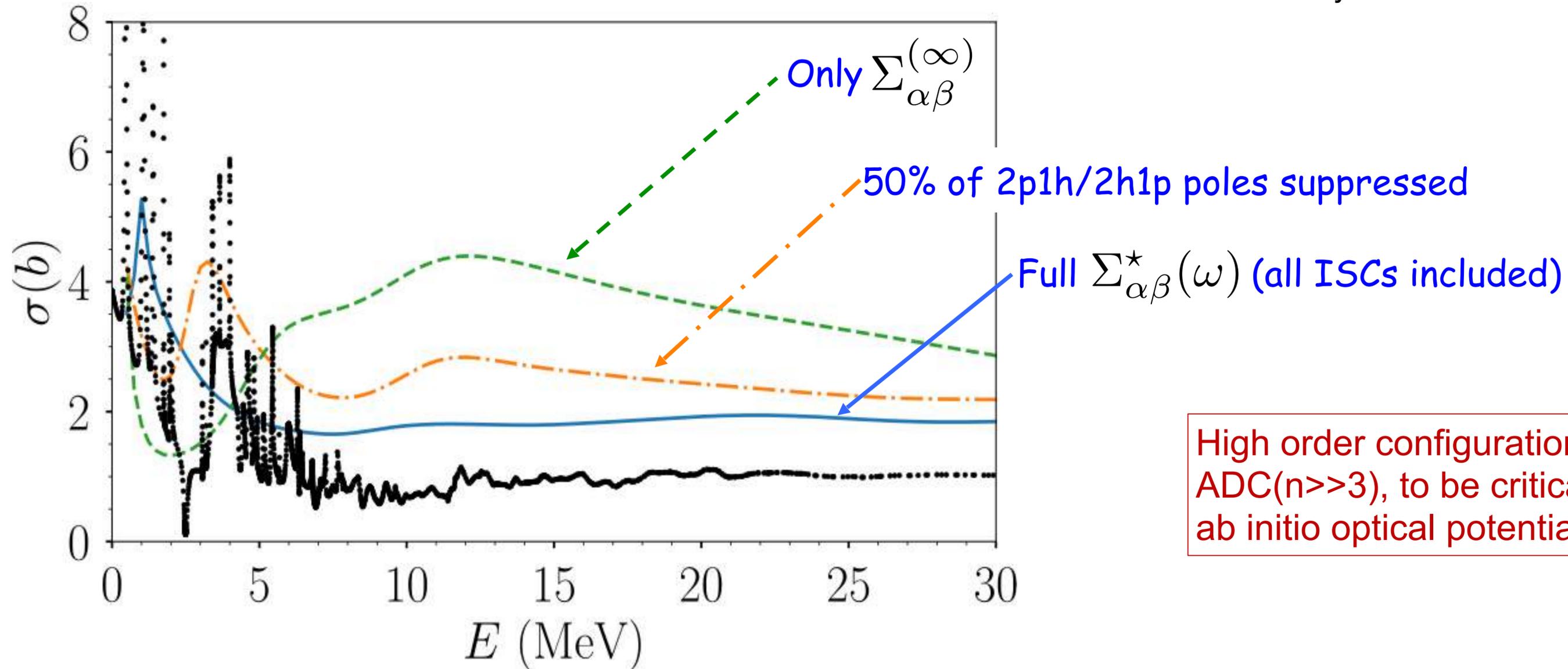
Full self-energy from SCGF:



Role of intermediate state configurations (ISCs)

n - ^{16}O , total elastic cross section

[A. Idini, CB, Navrátil,
Phys. Rev. Lett. **123**, 092501 (2019)]



High order configurations, or
ADC($n \gg 3$), to be critical for fully
ab initio optical potentials

$$\Sigma_{\alpha\beta}^*(\omega) = \Sigma_{\alpha\beta}^{(\infty)} + \underbrace{\sum_{i,j} \mathbf{M}_{\alpha,i}^\dagger \left(\frac{1}{E - (\mathbf{K}^> + \mathbf{C}) + i\Gamma} \right)_{i,j} \mathbf{M}_{j,\beta}}_{2p1h} + \underbrace{\sum_{r,s} \mathbf{N}_{\alpha,r} \left(\frac{1}{E - (\mathbf{K}^< + \mathbf{D}) - i\Gamma} \right)_{r,s} \mathbf{N}_{s,\beta}^\dagger}_{2h1p}$$

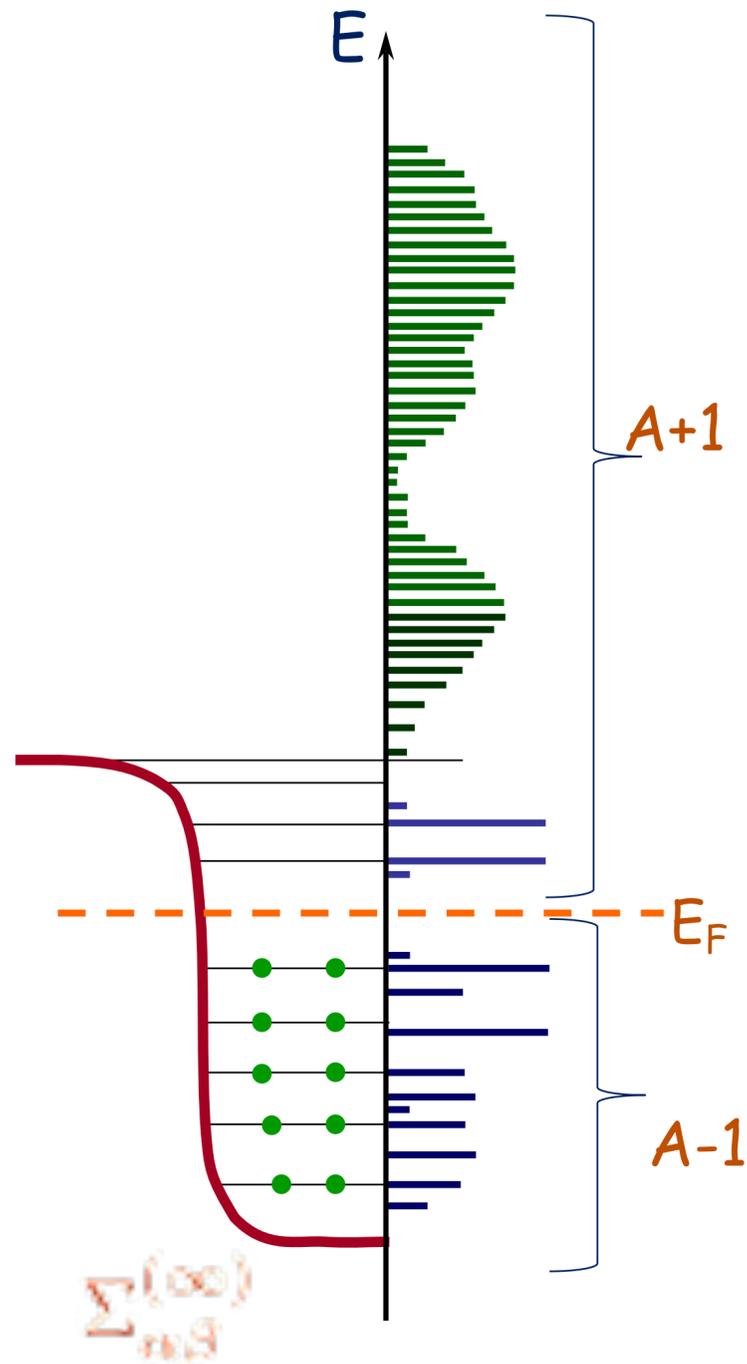


Microscopic optical potential

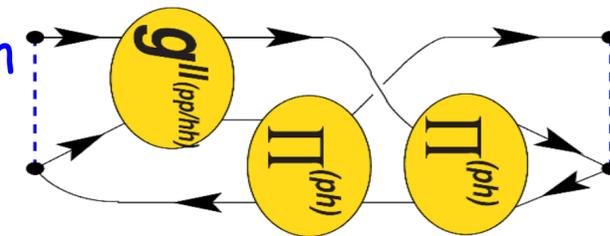
Nuclear self-energy $\Sigma^*(\mathbf{r}, \mathbf{r}'; \varepsilon)$:

- contains *both particle and hole* props.
- it is proven to be a *Feshbach opt. pot* \rightarrow in general it is *non-local* !

$$\Sigma_{\alpha\beta}^*(\omega) = \underbrace{\Sigma_{\alpha\beta}^{(\infty)}}_{\text{mean-field}} + \underbrace{\sum_{i,j} \mathbf{M}_{\alpha,i}^\dagger \left(\frac{1}{E - (\mathbf{K}^> + \mathbf{C}) + i\Gamma} \right)_{i,j} \mathbf{M}_{j,\beta} + \sum_{r,s} \mathbf{N}_{\alpha,r} \left(\frac{1}{E - (\mathbf{K}^< + \mathbf{D}) - i\Gamma} \right)_{r,s} \mathbf{N}_{s,\beta}^\dagger}_{\text{Particle-vibration couplings}}$$

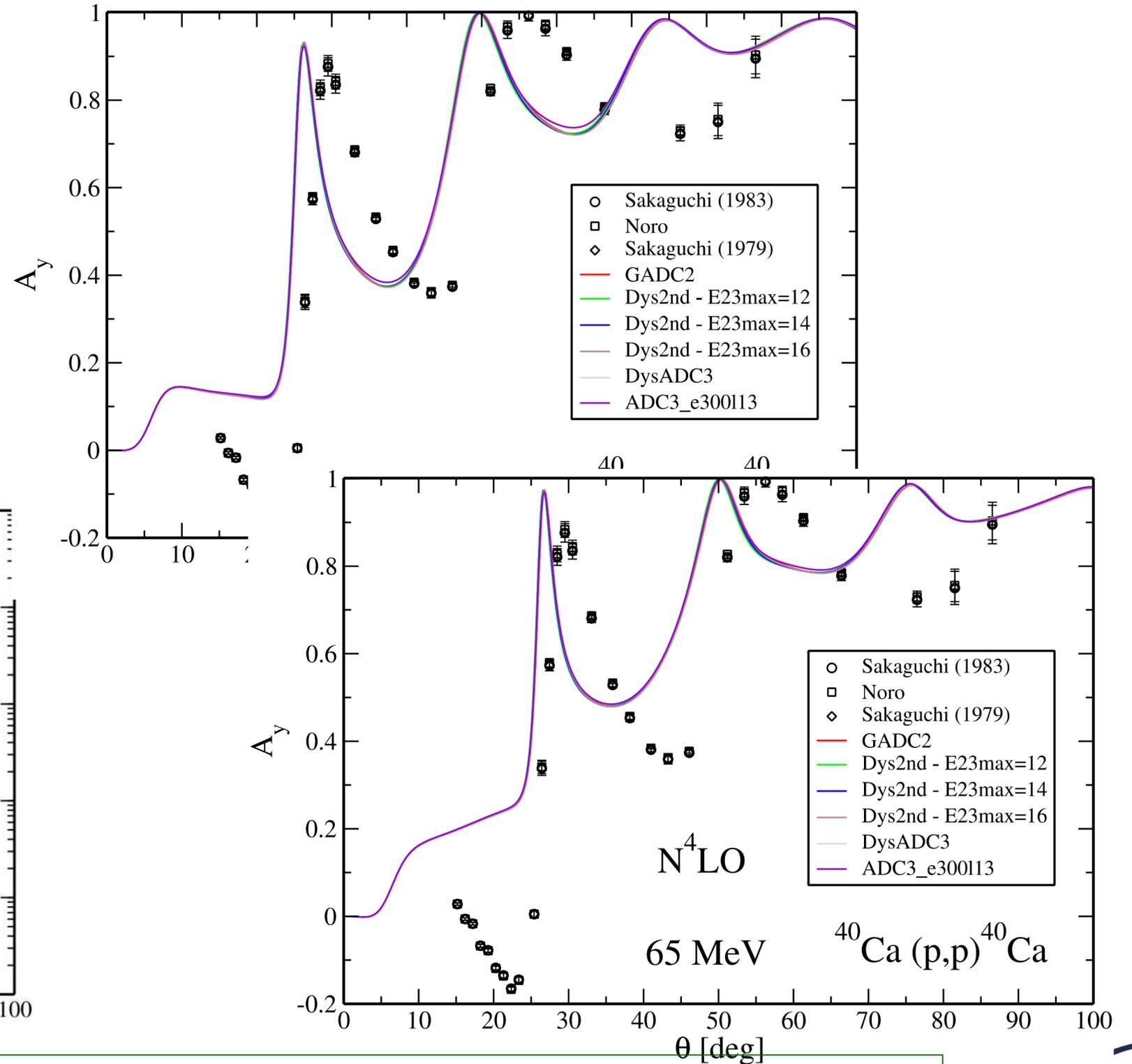
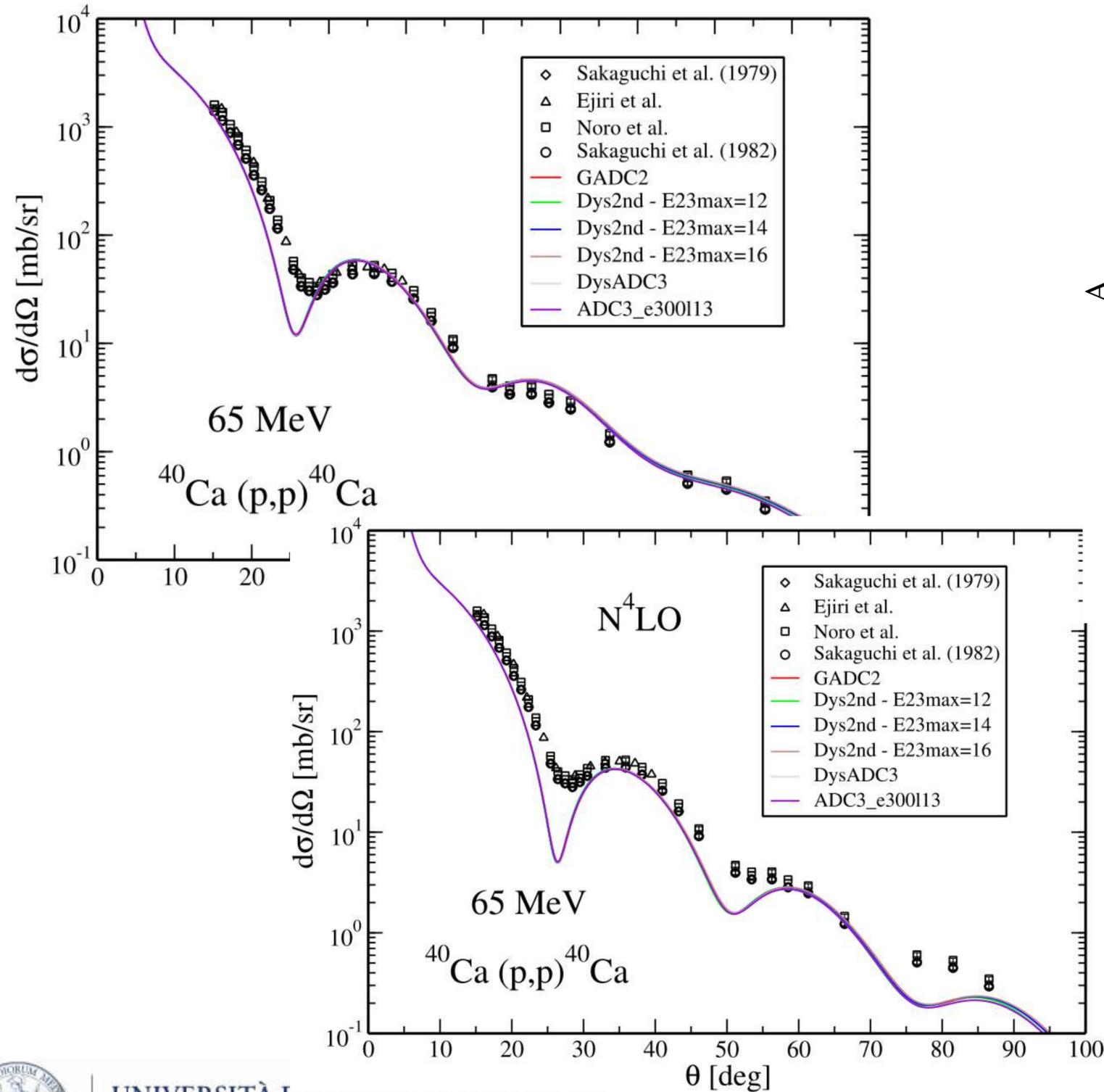


Particle-vibration couplings:



$$\Sigma_{\alpha\beta}^{(\infty)} = \text{Diagram of a self-energy loop}$$

Elastic nucleon nucleus scattering



Electron-Ion Trap colliders...

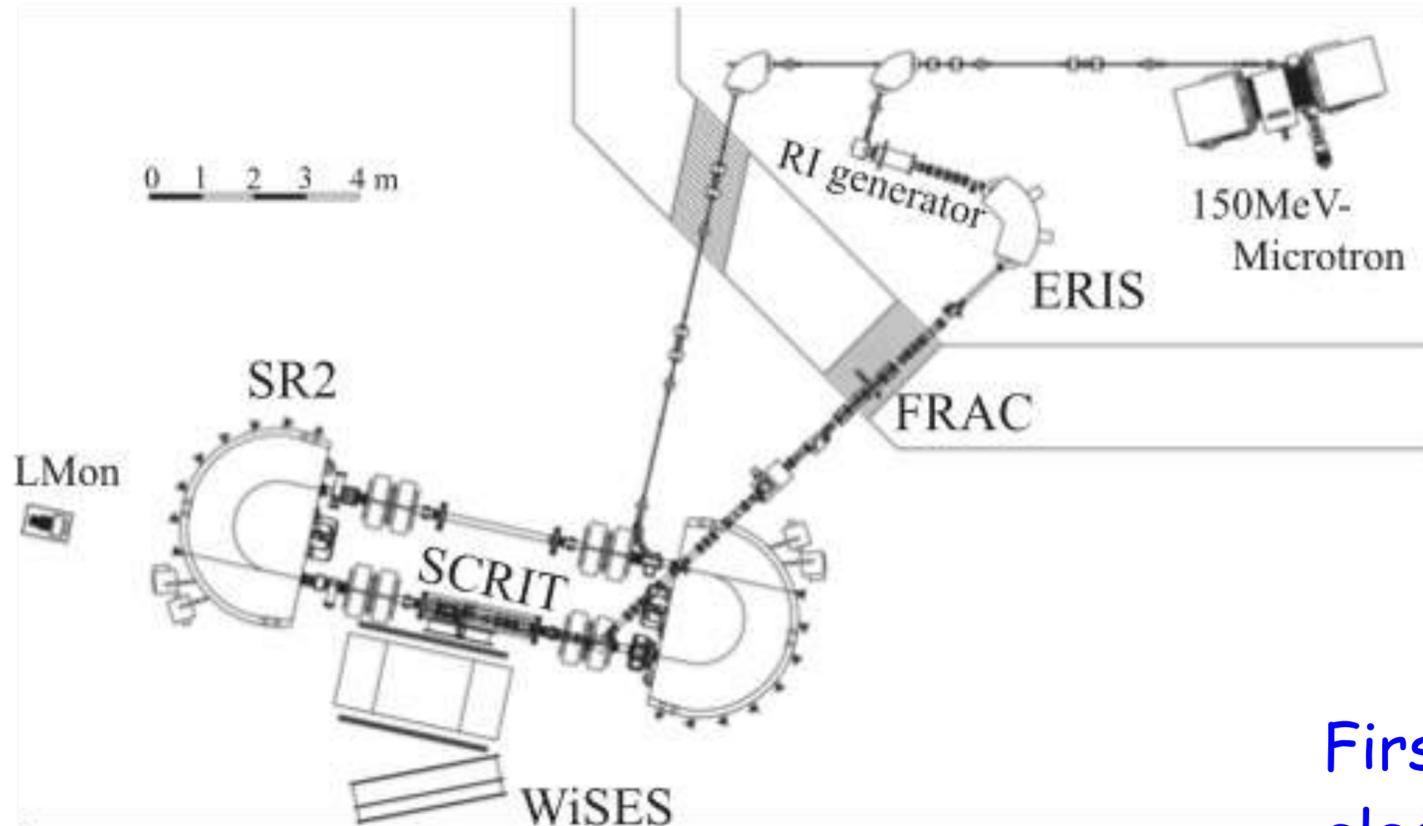


FIG. 1. Overview of the SCRIT electron scattering facility.

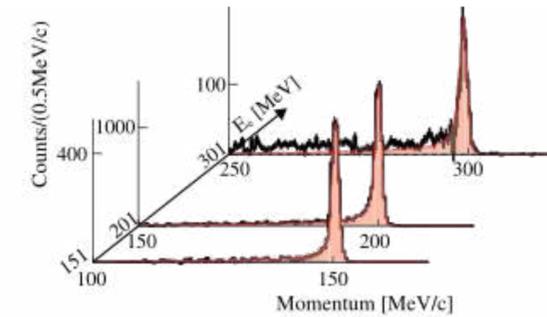
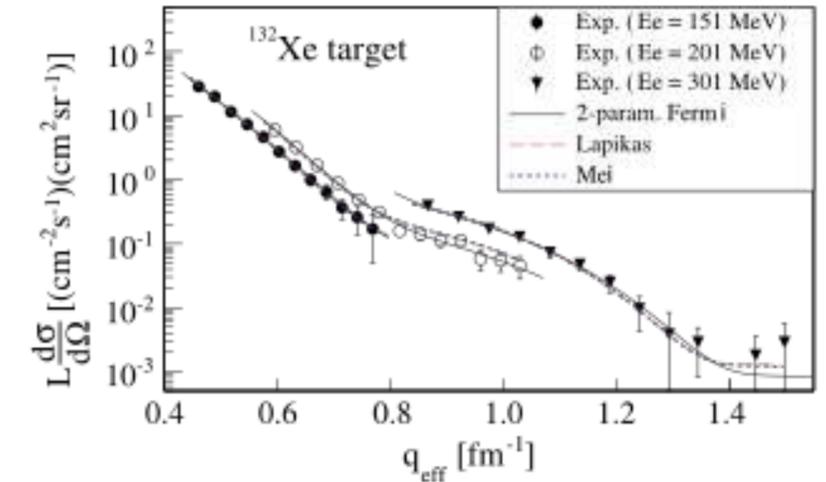


FIG. 3. Reconstructed momentum spectra of ^{132}Xe target after background subtraction. Red shaded lines are the simulated radiation tails following the elastic peaks.

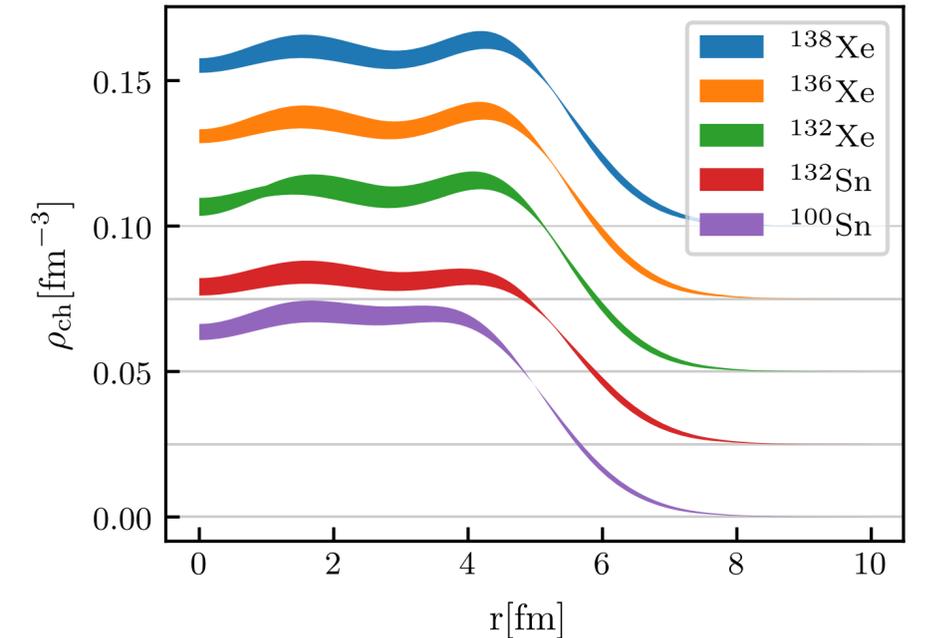
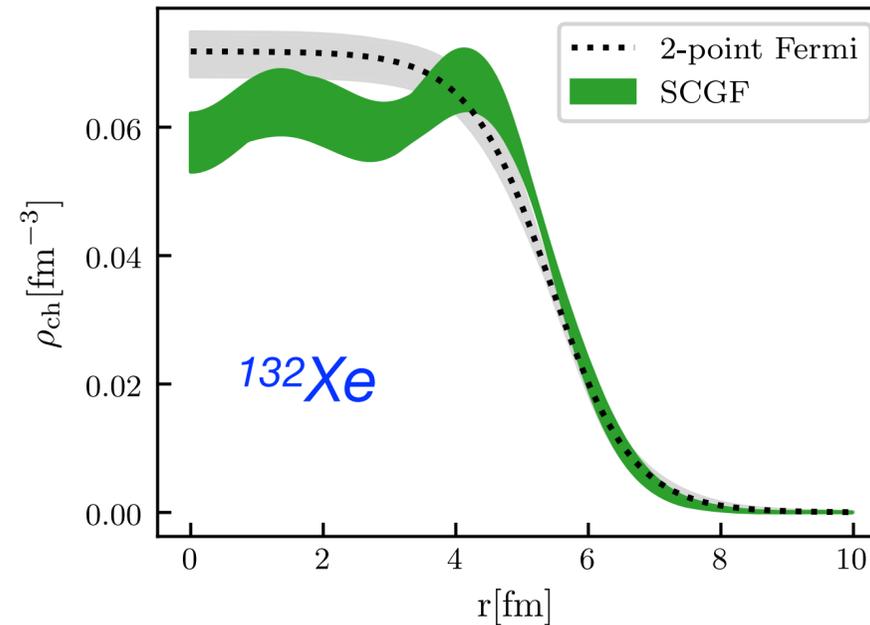
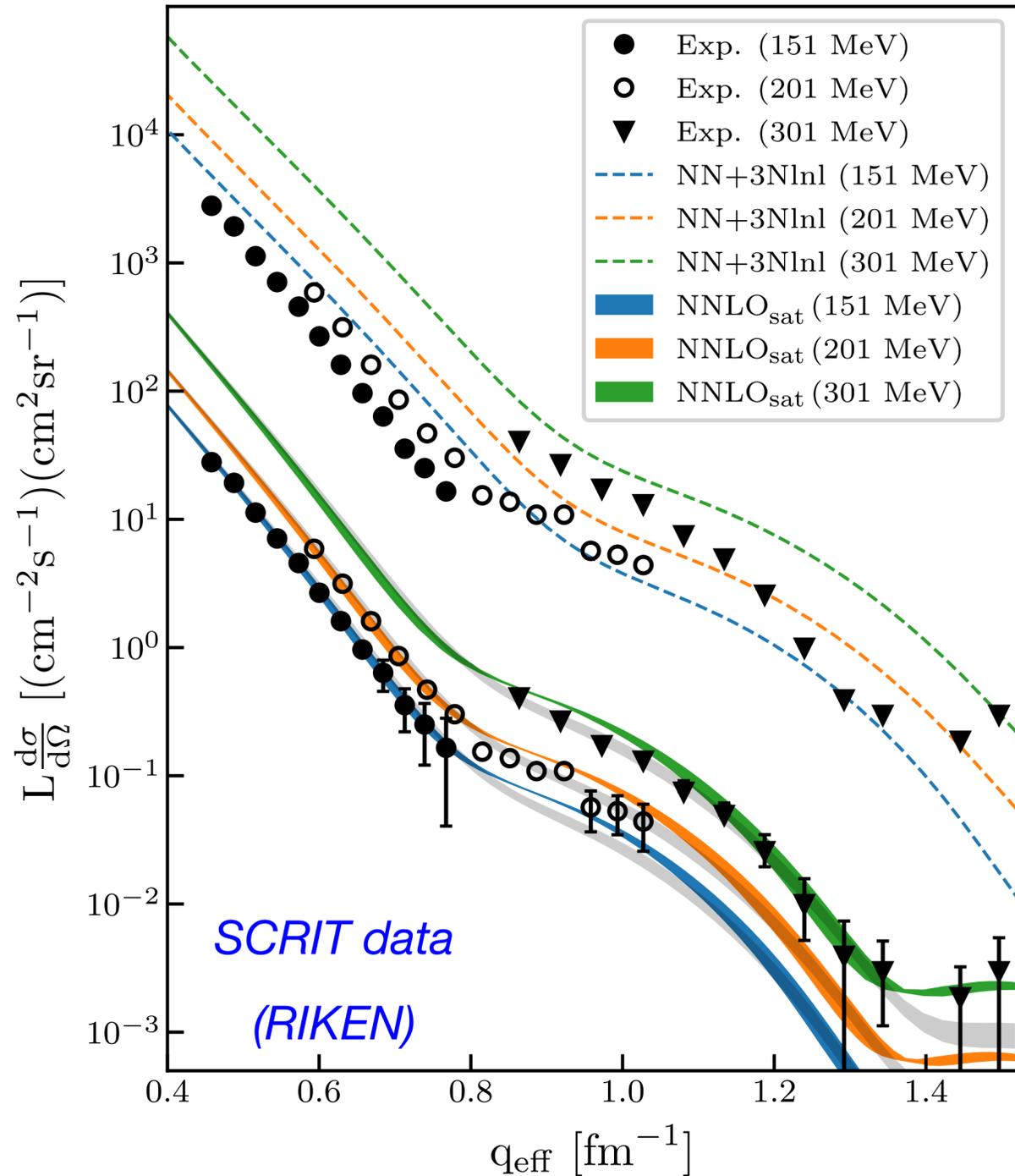


First ever measurement of charge radii through electron scattering with and ion trap setting that can be used on radioactive isotopes !!

K. Tsukada *et al.*, *Phy rev Lett* **118**, 262501 (2017)

P. Arhuis, CB, M. Vorabbi, P. Finelli,
Phys. Rev. Lett. **125**, 182501 (2020)

Charge density for Sn and Xe isotopes



	SCGF	Exp.
^{100}Sn	4.525 – 4.707	
^{132}Sn	4.725 – 4.956	4.7093
^{132}Xe	4.700 – 4.948	4.7859
^{136}Xe	4.715 – 4.928	4.7964
^{138}Xe	4.724 – 4.941	4.8279

*P. Arthuis, CB, M. Vorabbi, P. Finelli, Phys. Rev. Lett. **125**, 182501 (2020)*



Quantum Monte Carlo in configuration space

arXiv:2203.16167v2 [nucl-th]

Quantum Monte Carlo in Configuration Space with Three-Nucleon Forces

Pierre Arthuis ^{1,2,*} Carlo Barbieri ^{3,4,†} Francesco Pederiva^{5,6} and Alessandro Roggero ^{5,6,7}

¹ *Technische Universität Darmstadt, Department of Physics, 64289 Darmstadt, Germany*

² *ExtreMe Matter Institute EMMI and Helmholtz Forschungsakademie Hessen für FAIR (HFHF),
GSI Helmholtzzentrum für Schwerionenforschung GmbH, 64291 Darmstadt, Germany*

Configuration Interaction Monte Carlo (CIMC):

CI ansatz:

$$|\Psi\rangle = \sum_{\mathbf{n}} \langle \mathbf{n} | \Psi \rangle |\mathbf{n}\rangle \equiv \sum_{\mathbf{n}} \Psi(\mathbf{n}) |\mathbf{n}\rangle$$

*Immaginari time evolution
to the ground state (DMF):*

$$\Psi_{\tau+\Delta\tau}(\mathbf{m}) = \sum_{\mathbf{n}} \langle \mathbf{m} | P | \mathbf{n} \rangle \Psi_{\tau}(\mathbf{n}),$$
$$\langle \mathbf{m} | P | \mathbf{n} \rangle = \langle \mathbf{m} | e^{-\Delta\tau(H-E_T)} | \mathbf{n} \rangle$$

*Monte Carlo sampling over
configuration space:*

$$|\mathbf{m}\rangle = a_{p_1}^{\dagger} \dots, a_{p_M}^{\dagger} a_{h_1} \dots, a_{h_M} |\Phi_{\text{HF}}\rangle$$
$$\equiv |\Phi_{h_1, \dots, h_M}^{p_1, \dots, p_M}\rangle$$

The specific CIMC algorithm is build in such a way that it preserves the variational principle.



Quantum Monte Carlo in configuration space

arXiv:2203.16167v2 [nucl-th]

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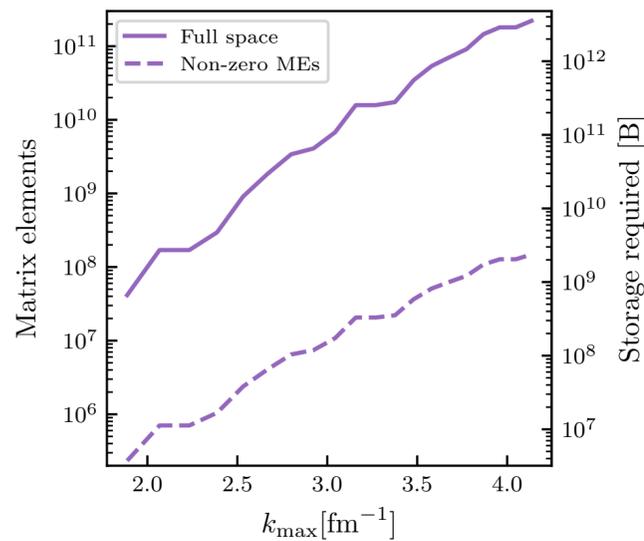
² ExtreMe Matter Institute EMMI and Helmholtz Forschungsakademie Hessen für FAIR (HFHF), GSI Helmholtzzentrum für Schwerionenforschung GmbH, 64291 Darmstadt, Germany



P. Arthuis (Surrey, now @ TU Darmstadt,

Configuration Interaction Monte Carlo (CIMC) for 3-nucleon forces:

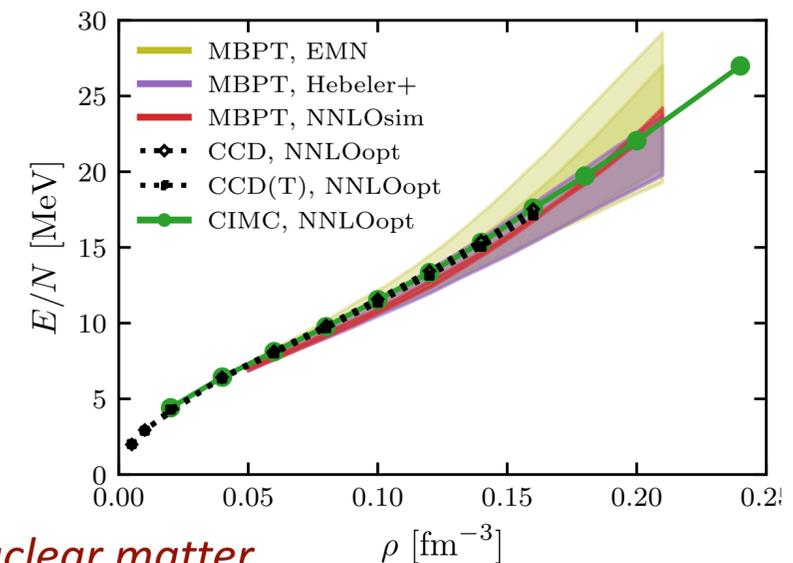
$$v_{\alpha\beta,\gamma\delta}^{(N.O.)} = \sum_{\mu\nu} v_{\alpha\beta\mu,\gamma\delta\nu}^{(3NF)} \rho_{\nu\mu}$$



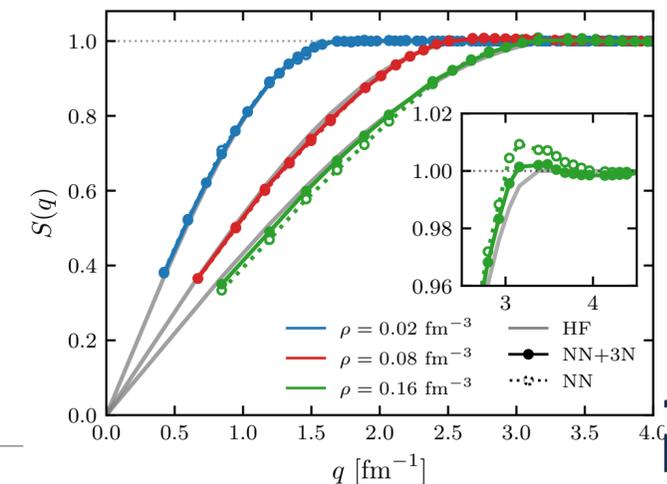
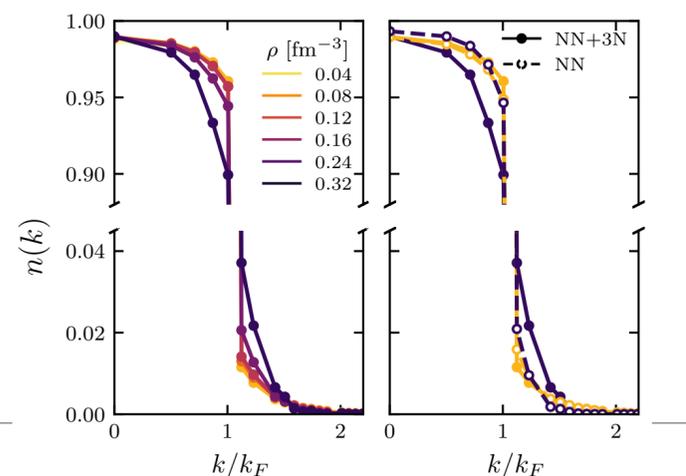
Efficient storage



of 3NF mtx elements



pure nuclear matter



66 neutrons, $\rho = 0.16 \text{ fm}^{-3}$



Nuclear Density Functional Theory



PHYSICAL REVIEW C **104**, 024315 (2021)

Nuclear energy density functionals grounded in *ab initio* calculations

F. Marino^{1,2,*}, C. Barbieri^{1,2}, A. Carbone³, G. Colò^{1,2}, A. Lovato^{4,5}, F. Pederiva^{6,5}, X. Roca-Maza^{1,2}
and E. Vigezzi²

¹Dipartimento di Fisica "Aldo Pontremoli," Università degli Studi di Milano, 20133 Milano, Italy

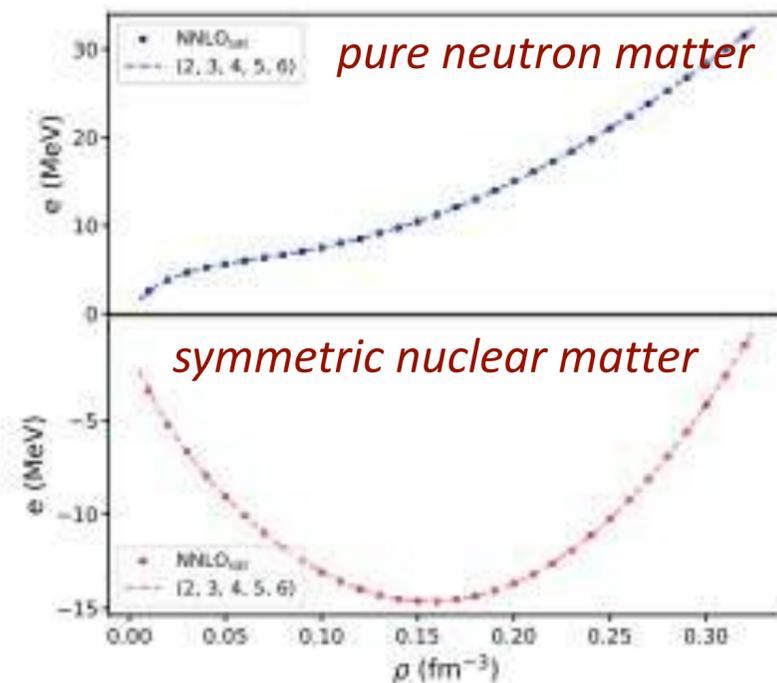
²Istituto Nazionale di Fisica Nucleare, Sezione di Milano, 20133 Milano, Italy

³Istituto Nazionale di Fisica Nucleare, CNAF, Viale Carlo Farini 612, 40137 Bologna, Italy

DFT is in principle exact – but the energy density functional (EDF) is not known

For nuclear physics this is even more demanding: need to link the EDF to theories rooted in QCD!

Machine-learn DFT functional
on the nuclear equation of state

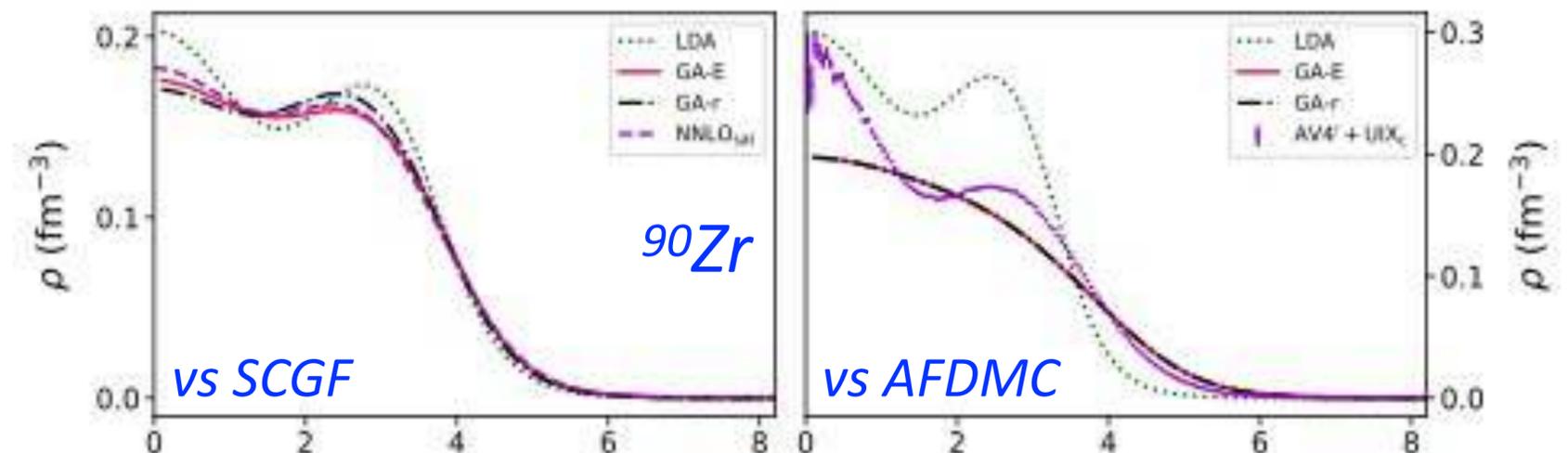


Jacob's ladder



+ approximate GA

Benchmark in finite systems



Summary

Thank you for your attention!!

- Ab initio applications to structure and reactions are becoming increasingly powerful. Systematic applications beyond testing forces and structure becoming available
- Particle-phonon coupling (ADC3/FRPA) being implemented.
- The covariant version of Nambu-Gorkov formalism in SCGF:
 - Minimises the number of diagrams to handle
 - Only basic topologies are retained.
 - Facilitates automatic diagram generation at higher orders.
- Applications... optical potentials, g.s. observables, one-nucleon spectroscopy



And thanks to my **collaborators** (over the years...)

Thank you for your attention!!



*E. Vigezzi, G. Colò, X. Roca-Maza,
F. Marino, A. Scalesi*



P. Navrátil



*A. Cipollone, A. Rios,
A. Idini, P. Arthuis, M. Drissi*



C. Giusti



energie atomique • énergies alternatives

V. Somà, T. Duguet, A. Scalesi



P. Finelli

