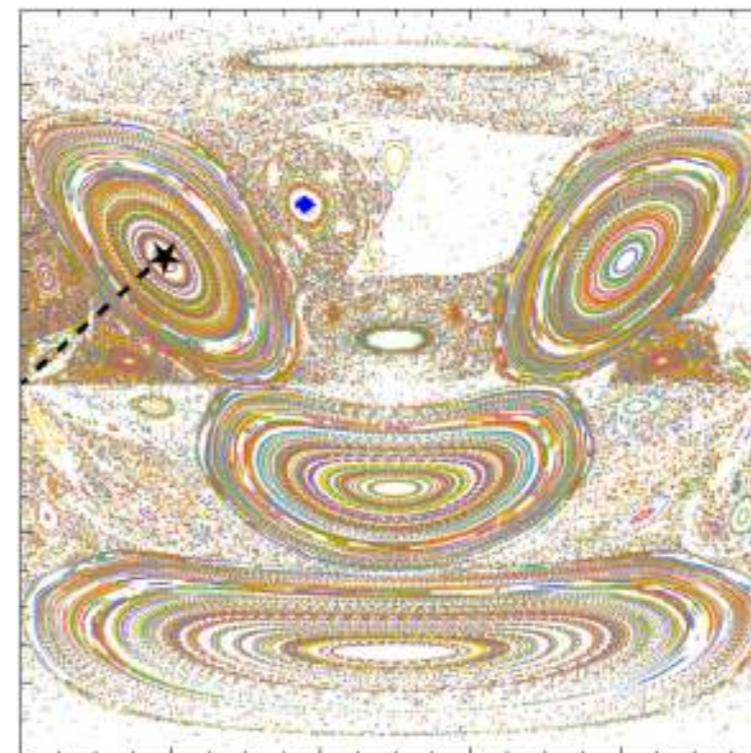
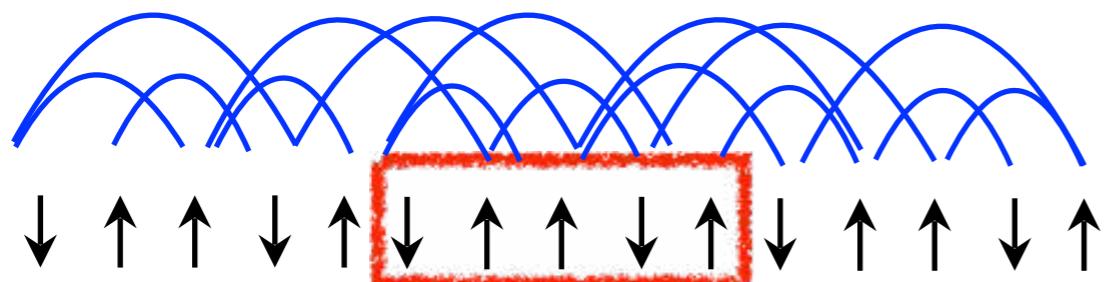


# Entanglement, Thermalization, and Many-Body Localization



Dima Abanin



UNIVERSITÉ  
DE GENÈVE

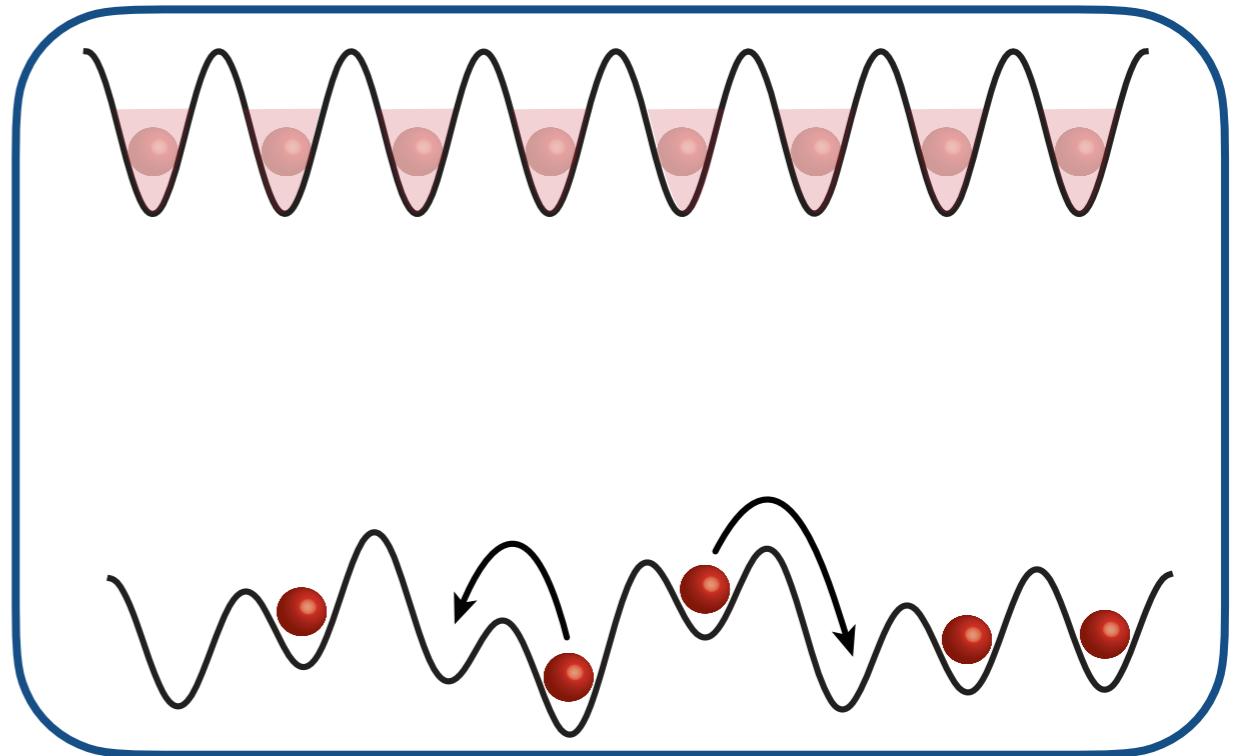
RPMBT-21  
Chapel Hill, NC  
15 September 2022

# Synthetic quantum systems



image: Munich group

Ultracold atoms



Design dimensionality, lattice, interactions,..

## Unlike “conventional” materials

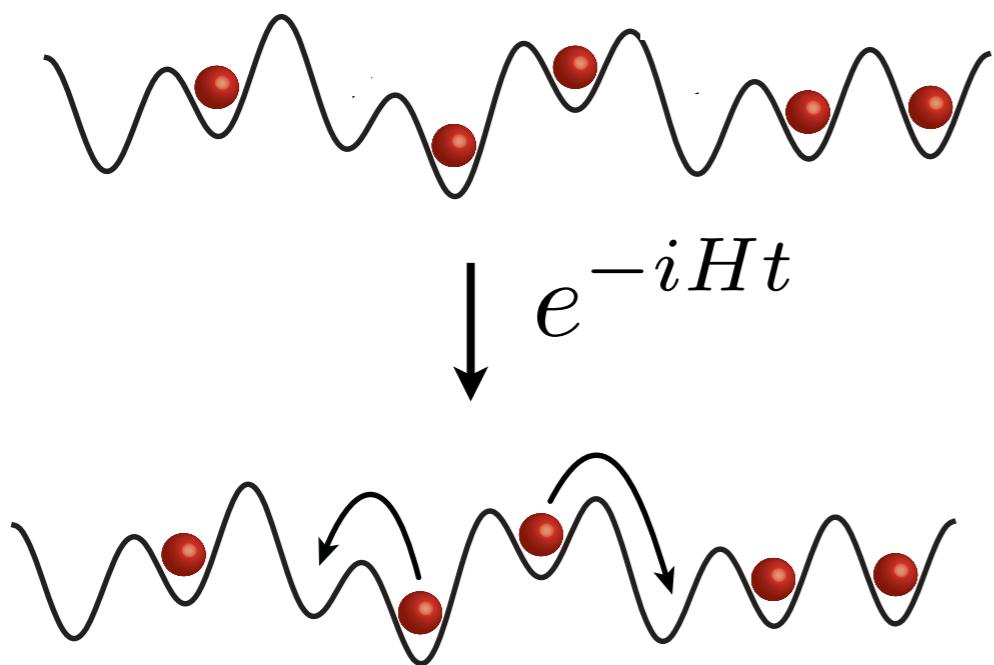
- Isolated (no phonon/heat bath)
- Long time scales  $\sim 10^{-3}s$  vs  $10^{-12}s$

*A platform to study non-equilibrium phenomena in interacting quantum systems*

# New opportunities

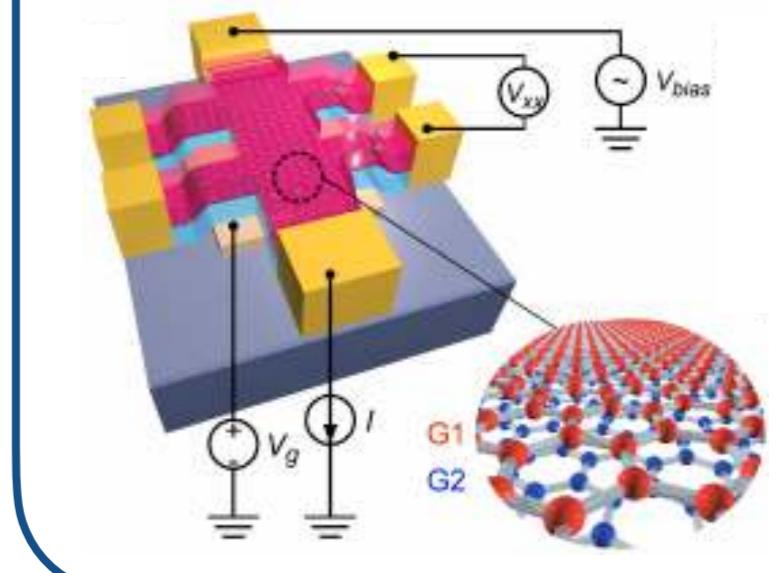
## Synthetic systems

New setups, probes



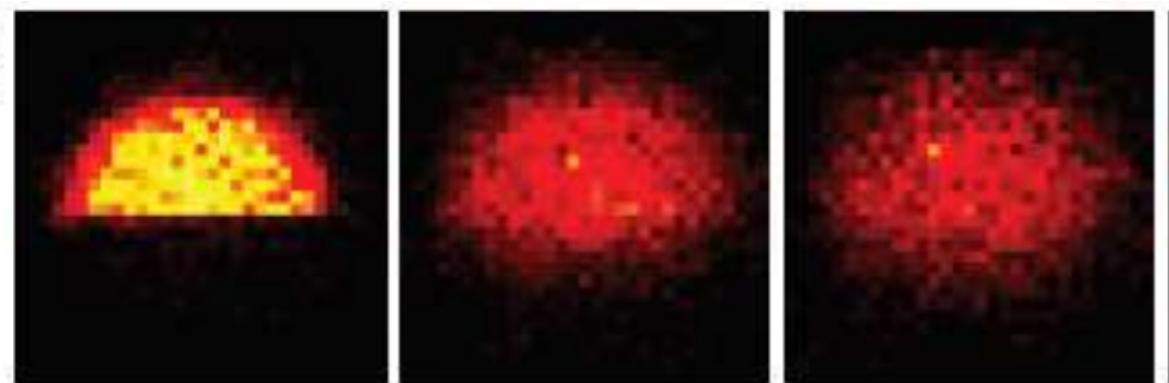
## Materials

conductivity,  
heat capacity..



## Single-site, real-time resolution

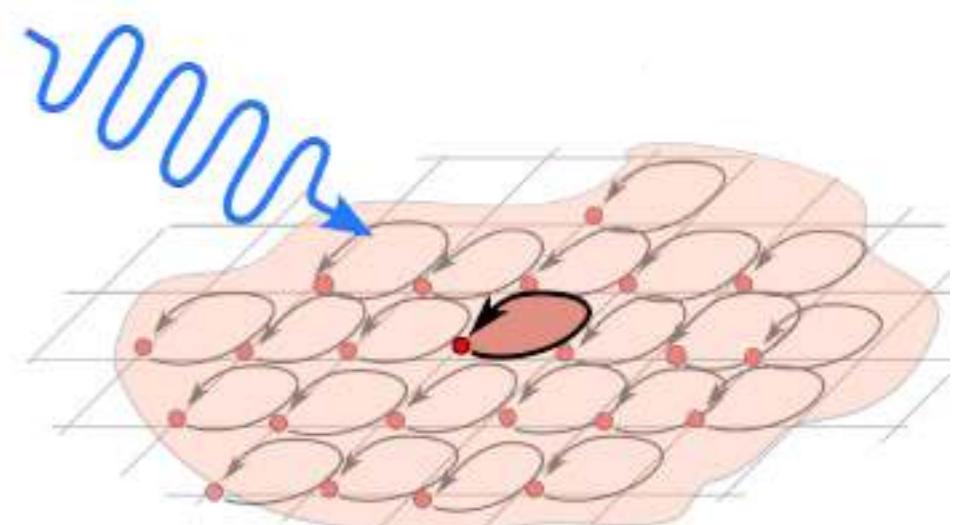
Harvard, Munich,..



time

Bloch group'16

## Periodically driven systems



# Non-equilibrium quantum matter

Cold atoms

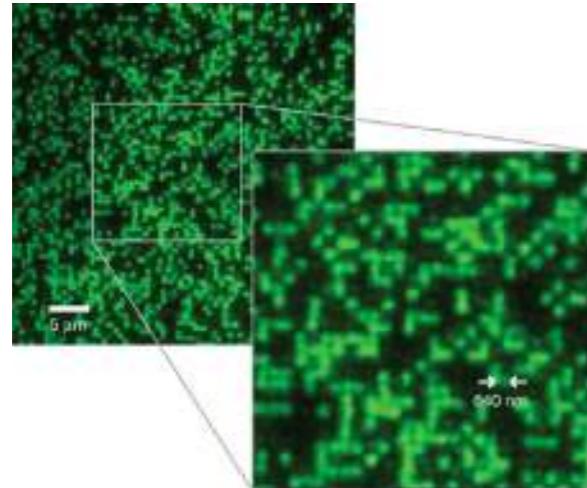


Figure: Greiner group (Harvard)

Rydberg atoms

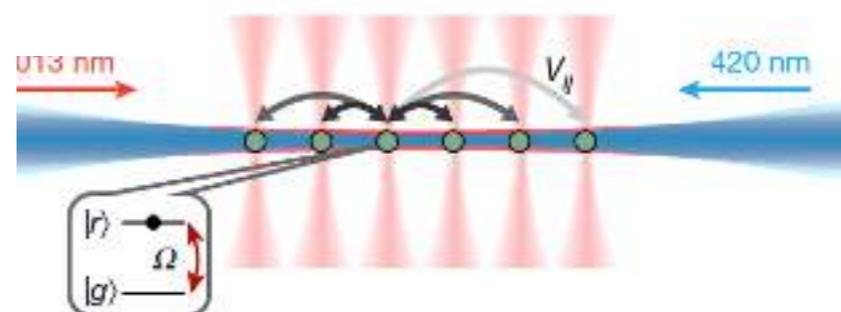


Figure: Bernien et al'17

Superconducting  
qubits

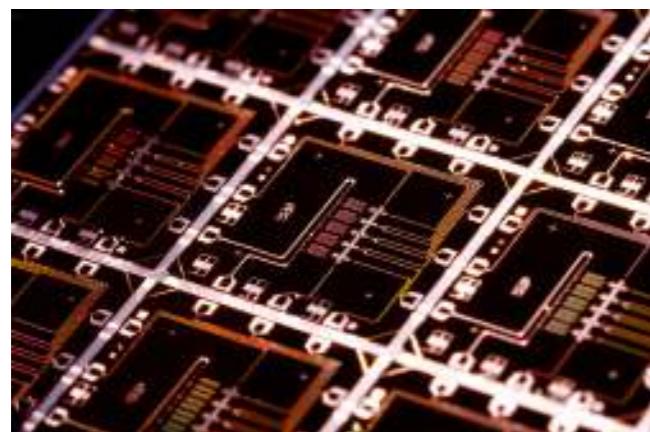


Figure: Google Quantum

Trapped ions

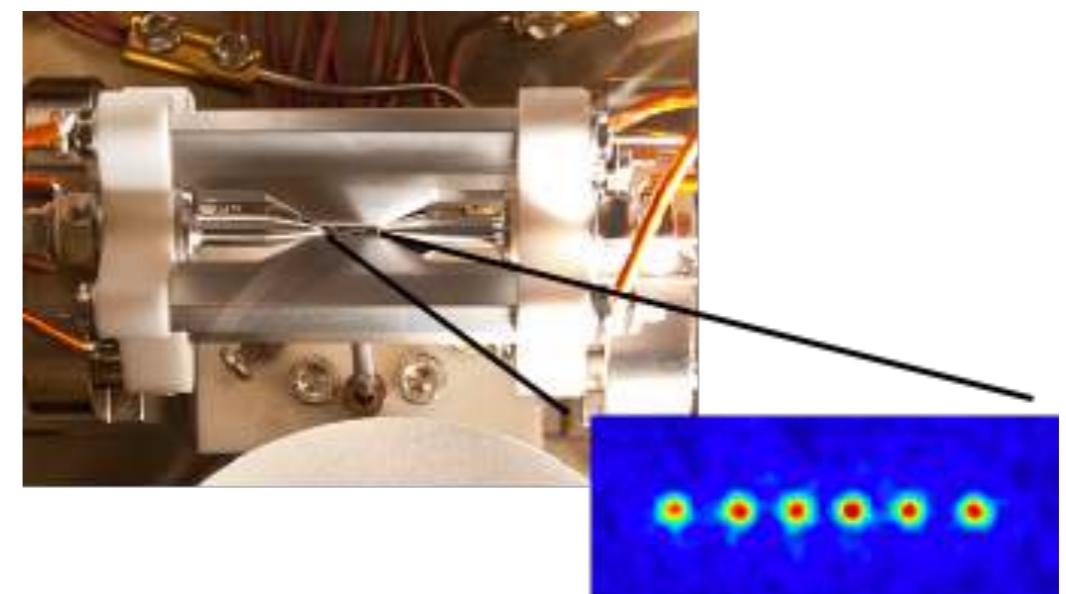
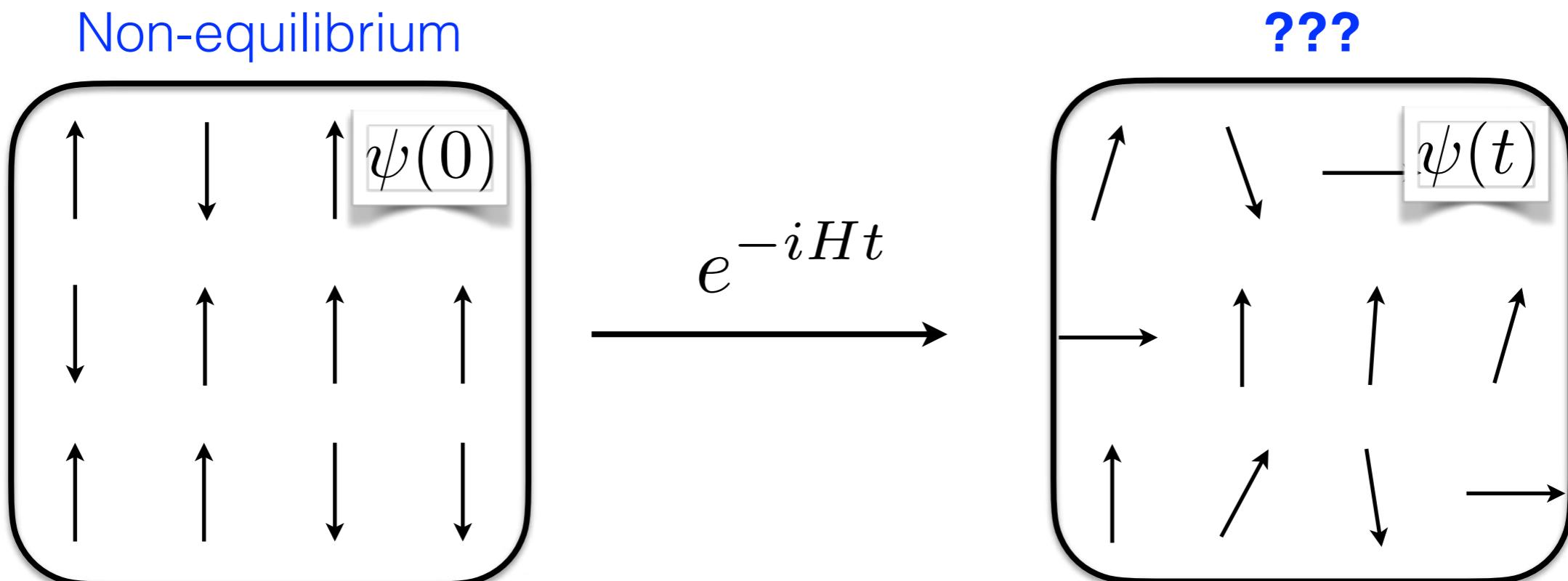
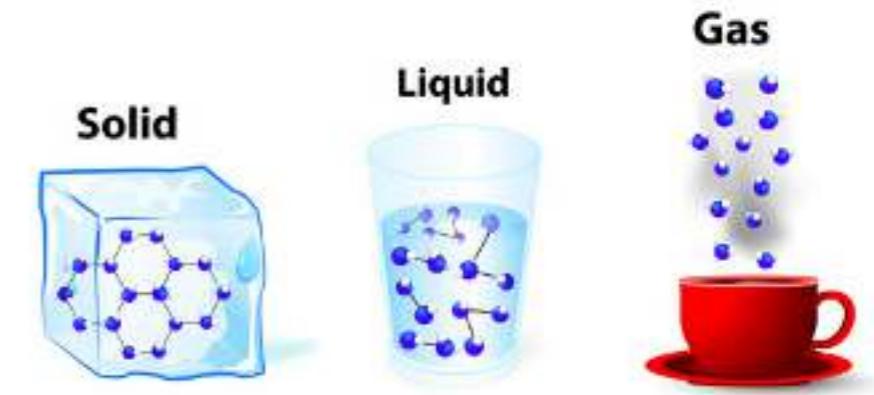


Figure: Innsbruck group

Thermalisation and why it's good to avoid it

# Dynamics of isolated quantum systems

Thermal equilibrium: universality, phase of matter  
Principle: symmetry breaking



*Universality? New out-of-equilibrium phases/properties?*

↓

*Efficient control of synthetic systems*

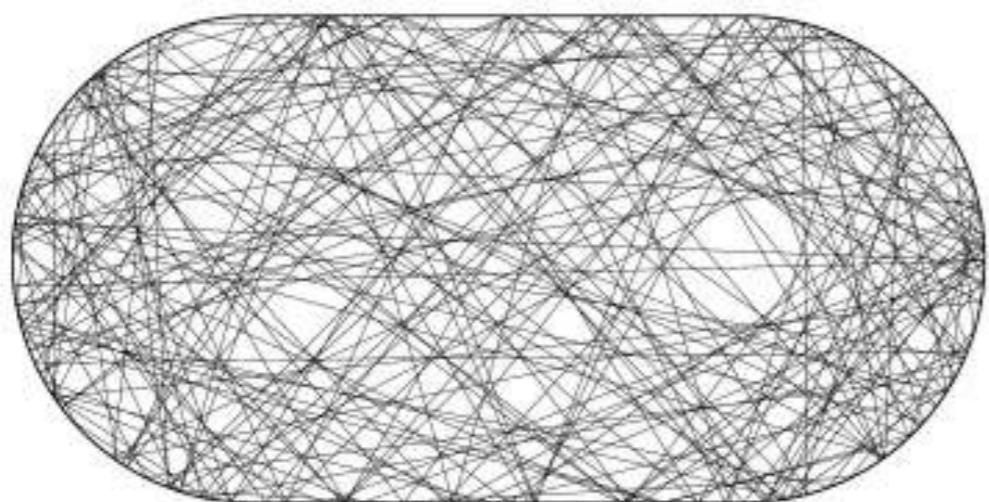
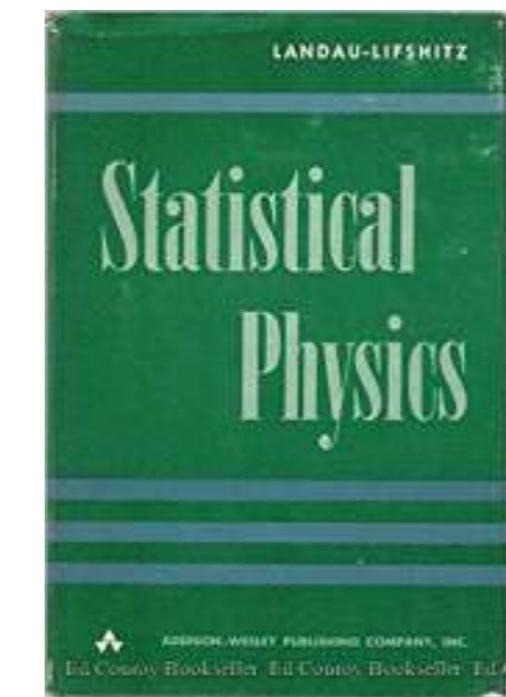
# Ergodicity



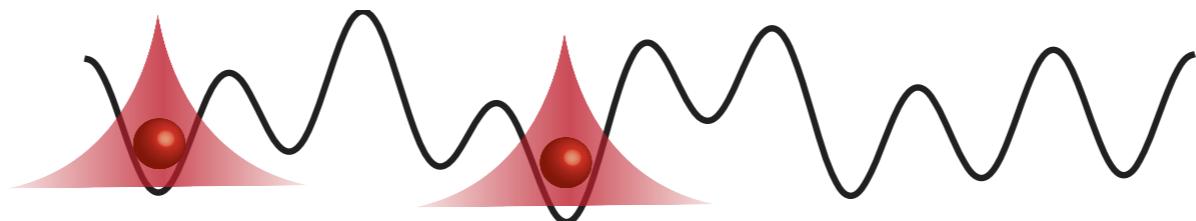
Non-equilibrium



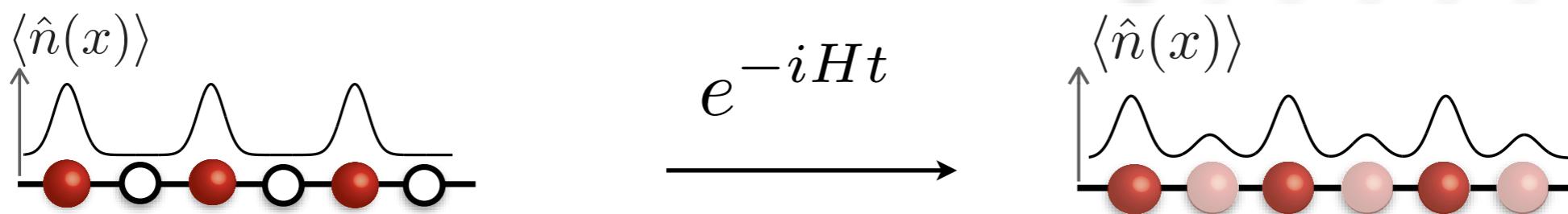
Thermal equilibrium



# Anderson localization: Ergodicity breakdown in disordered quantum systems?

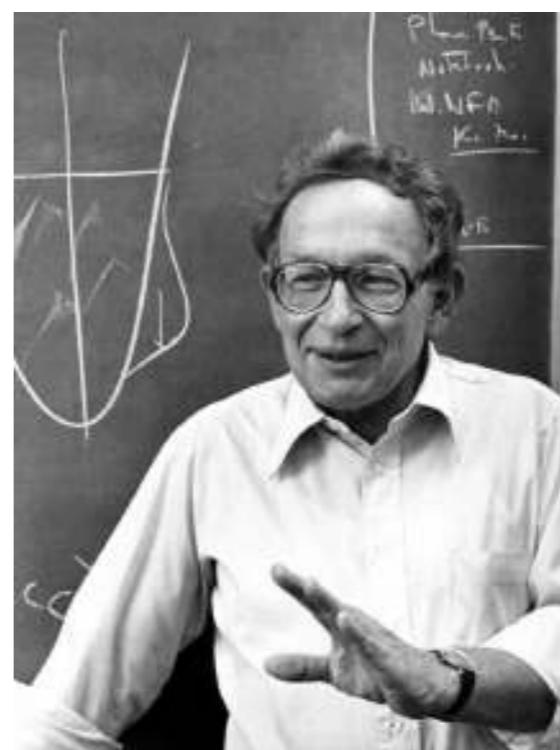


Memory of initial state retained



a system in which an approach  
to equilibrium is impossible!

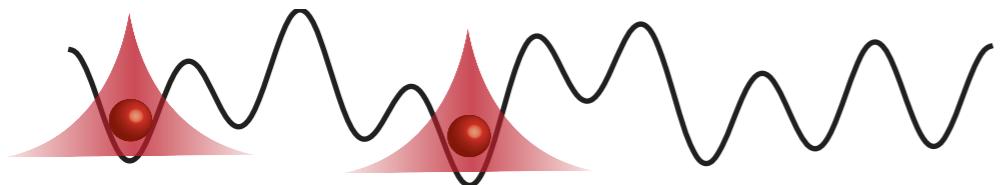
Anderson'58



# New paradigm: many-body localization (MBL)

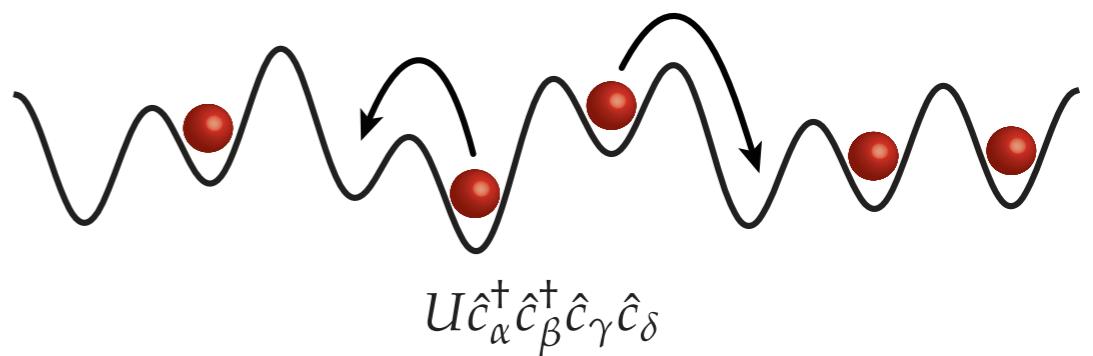
Anderson localization

Single-particle

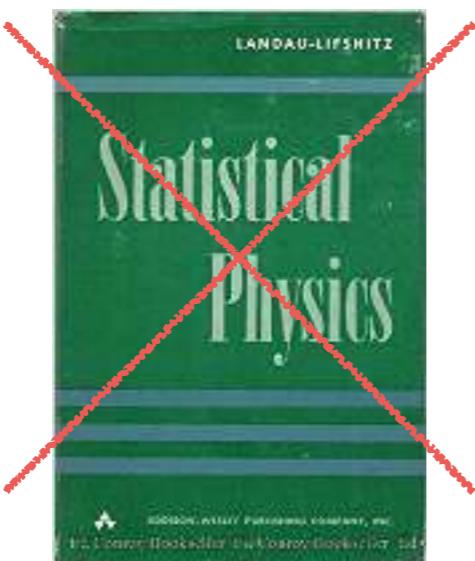


**Many-body localization in interacting systems**

**Generic non-ergodic phase**



Anderson Fleischmann'80; Gornyi et al'05; Barsko et al'05  
Review: DA, Altman, Bloch, Serbyn, Rev. Mod. Phys. 2019



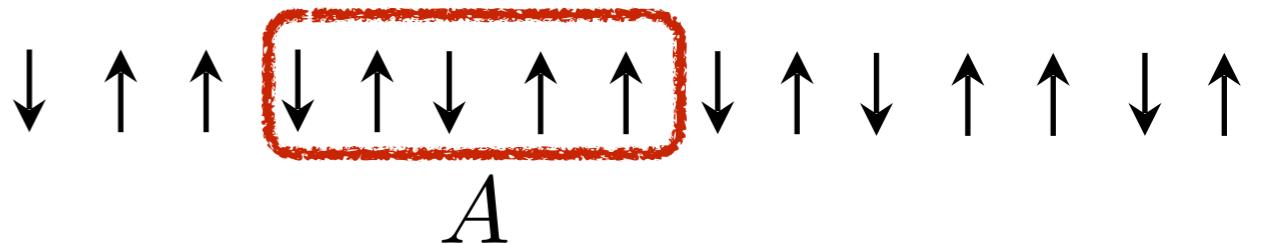
**New opportunities**

Difficult: Interactions + disorder + highly non-equilibrium

# Complexity and entanglement

$N$  spins

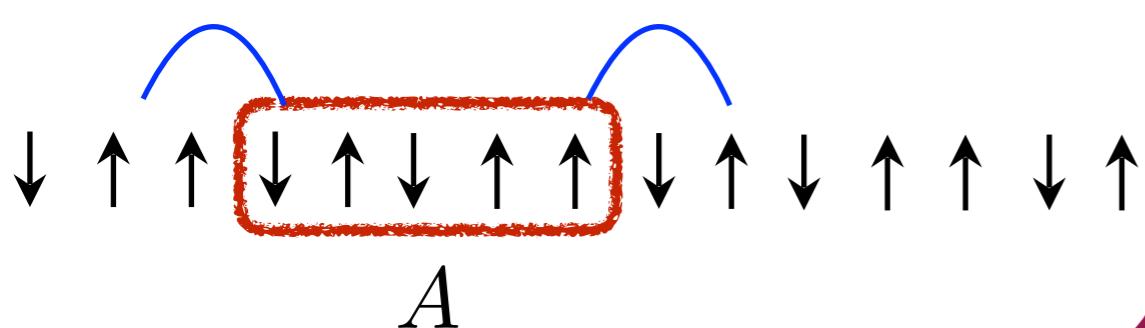
wave function  $|\psi\rangle$



How difficult to represent? Naively  $\sim 2^N$  parameters

**Weakly entangled states can be efficiently “compressed”  $\sim N^a$**

$$S_{\text{ent}}(A) \propto \text{vol}(\partial A) \text{ “area-law”}$$

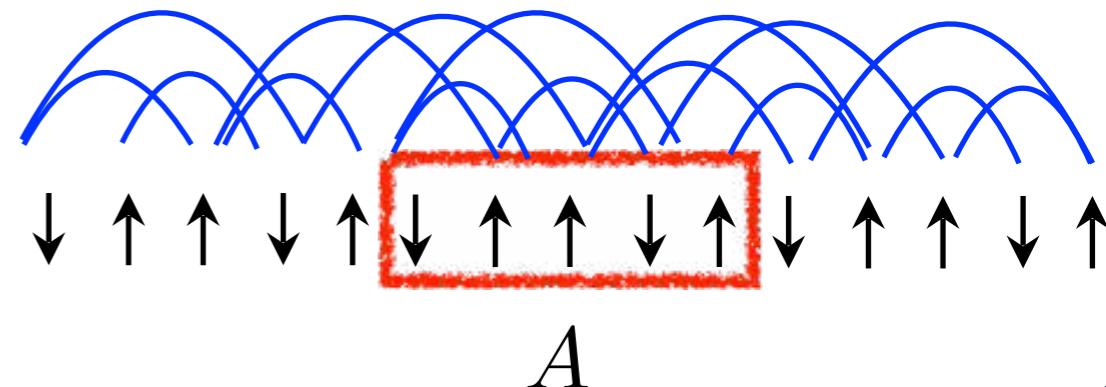


Ground states, “easy”  
Tensor networks methods

(Frédéric Mila talk)

Highly entangled states “hard”

$$S_{\text{ent}}(A) \propto \text{vol}(A) \text{ “volume-law”}$$

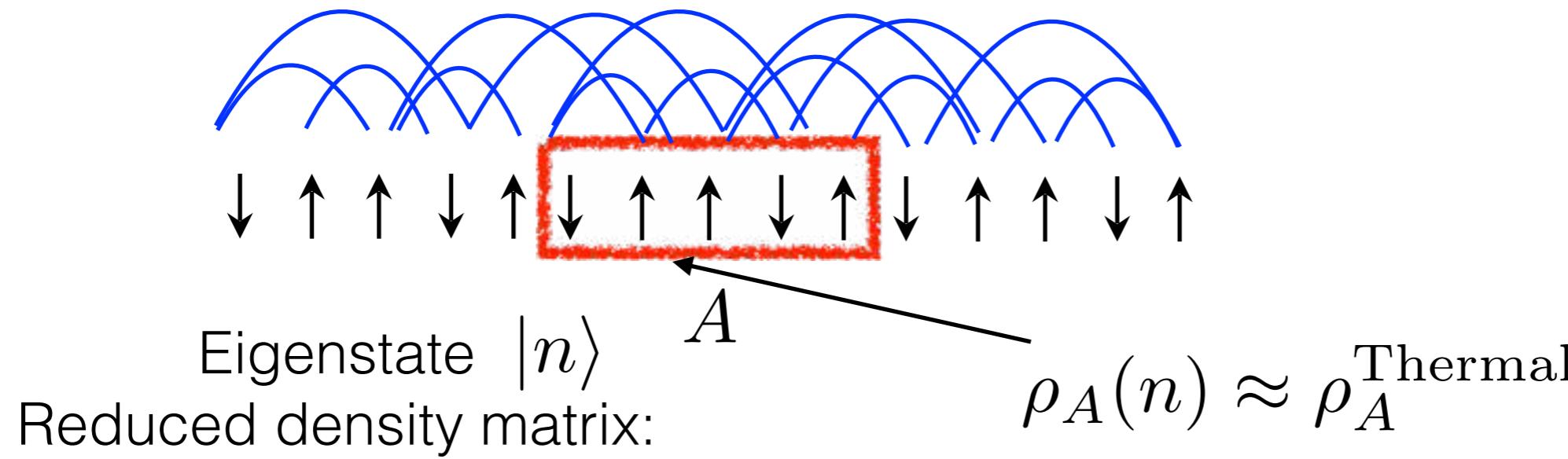


Non-equilibrium states, “hard”

# Eigenstate thermalisation hypothesis (ETH)

Deutch'91, Srednicki'94

-Individual **excited** eigenstates of a quantum-ergodic Hamiltonian  $H$  have thermal observables



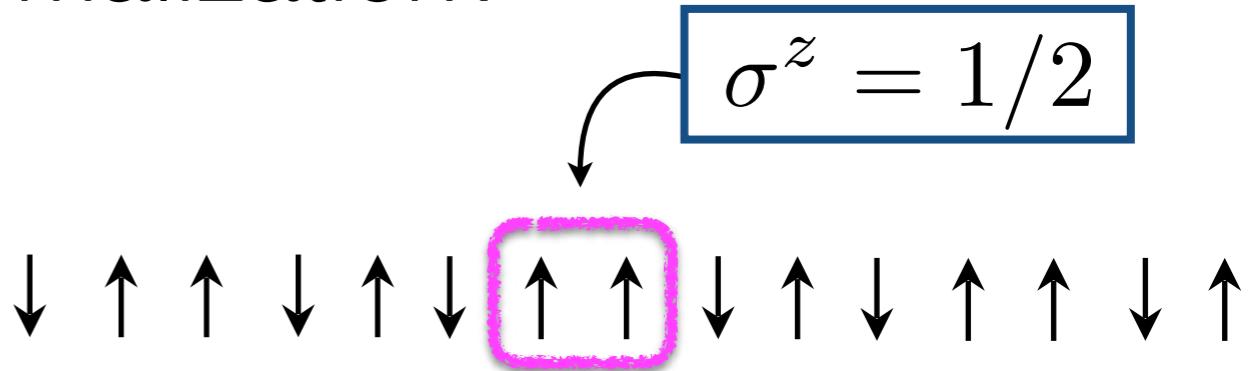
-System acts as a heat bath for its subsystems

- “Volume-law entanglement”

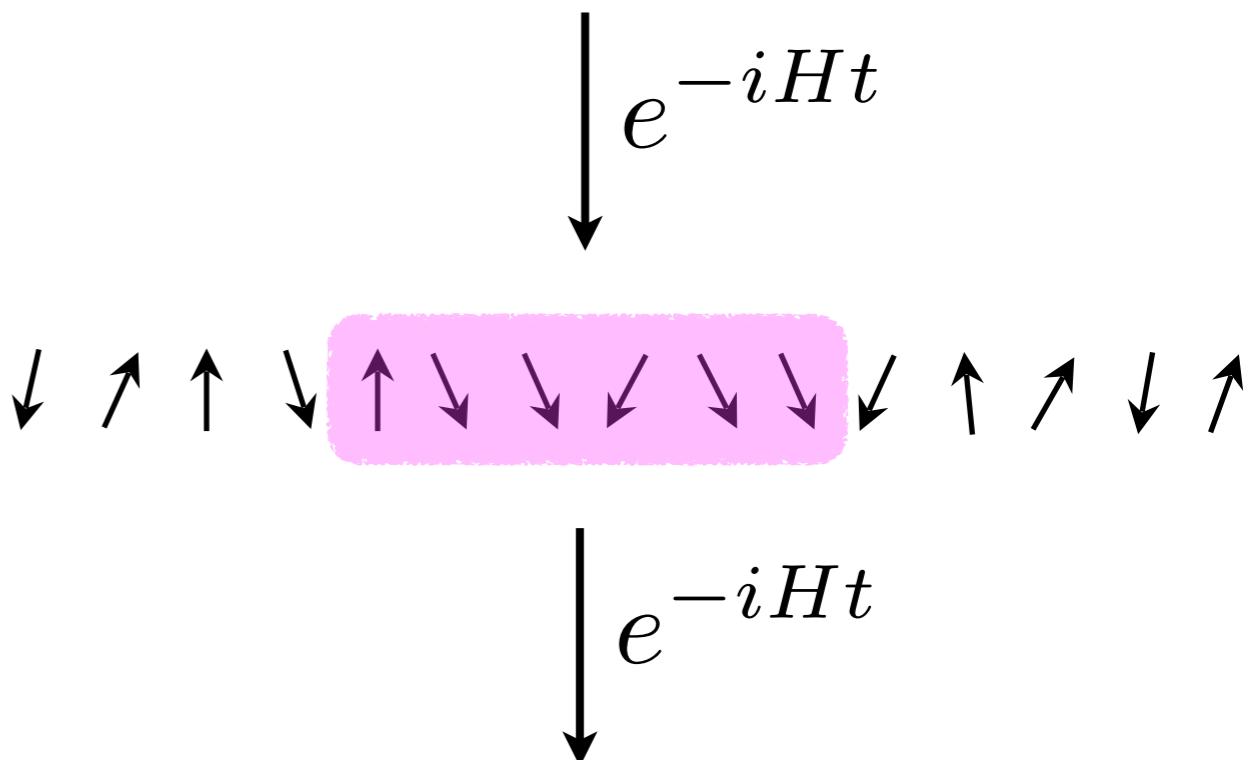
$$S_A(n) \approx S_A^{\text{Thermal}} \propto \text{Vol}(A)$$

# Why avoid thermalization?

Write information into initial state

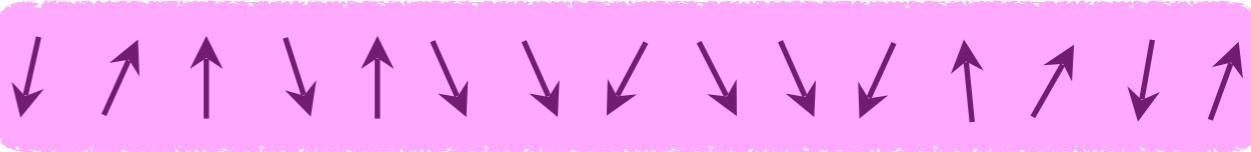


Quantum information spreads over  
many spins  $\longleftrightarrow$  entanglement



Becomes unrecoverable

$$\sigma^z(t) = e^{iHt} \sigma^z e^{-iHt}$$



Thermalisation  $\longrightarrow$  scrambling of quantum information

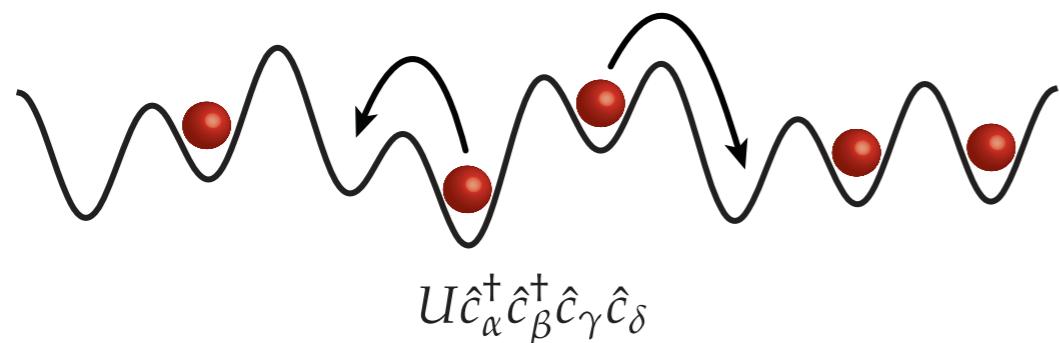
Protect coherence? New phases?

MBL phase: insights from entanglement

# Models of many-body localisation (MBL)

MBL=strong disorder + short-ranged interactions  
+highly non-equilibrium

**Fermions**



**Spins**

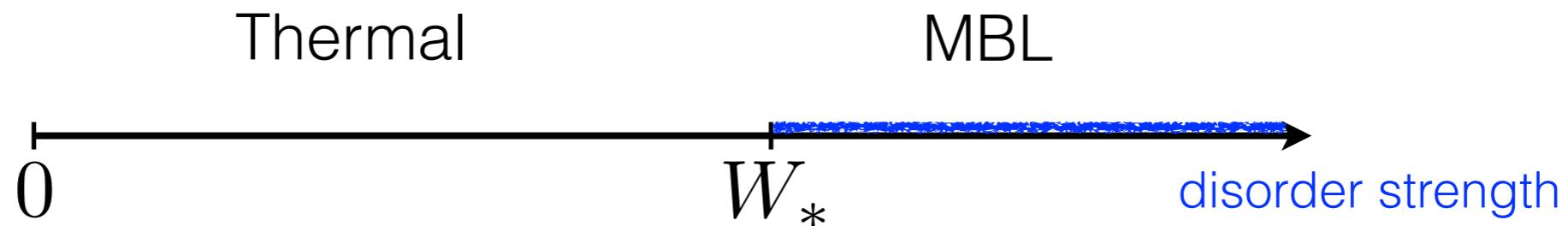
$$H = \sum_i h_i \sigma_i^z + J \sum_{\langle ij \rangle} \sigma_i^+ \sigma_j^- + V \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z$$

disorder

interactions

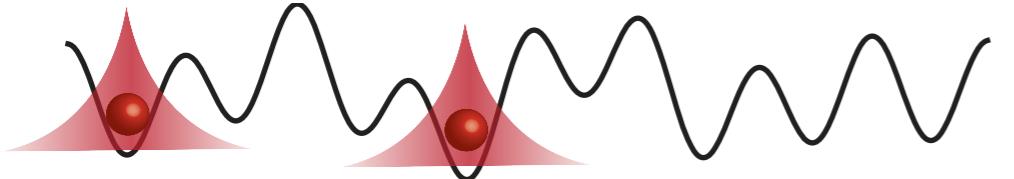
$$h_i \in [-W, W]$$

**Dynamical phase diagram**



# Are interactions important?

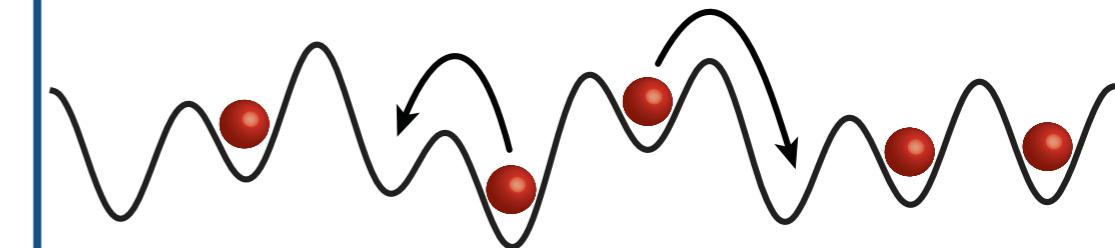
Anderson localization



single-particle

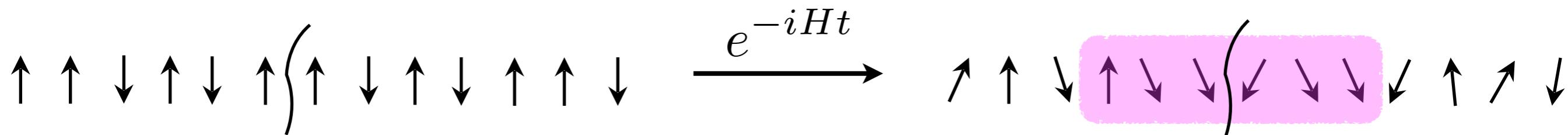
Conductivity  $\sigma = 0$

**Many-body localization**



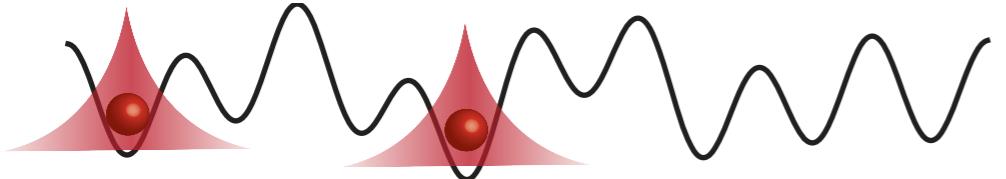
$U \hat{c}_\alpha^\dagger \hat{c}_\beta^\dagger \hat{c}_\gamma \hat{c}_\delta$

Conductivity  $\sigma = 0$



# Are interactions important?

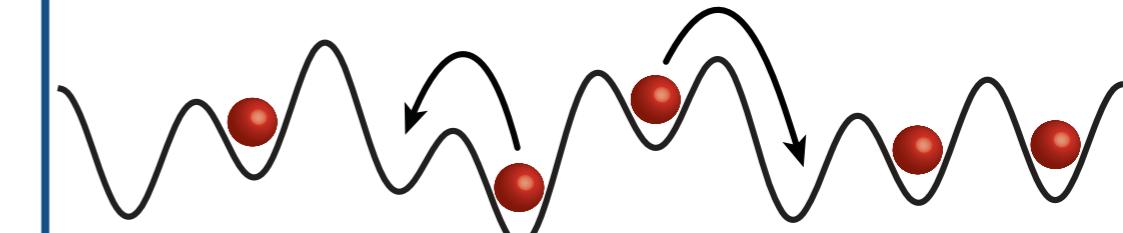
Anderson localization



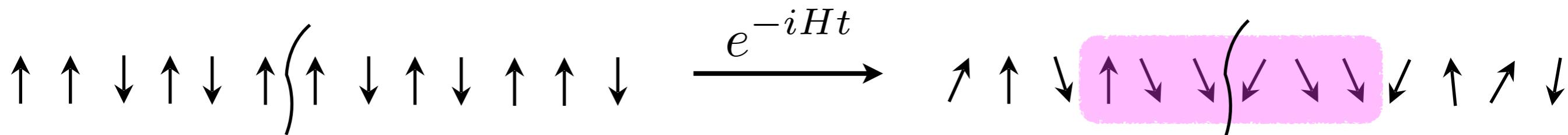
single-particle

Conductivity  $\sigma = 0$

**Many-body localization**



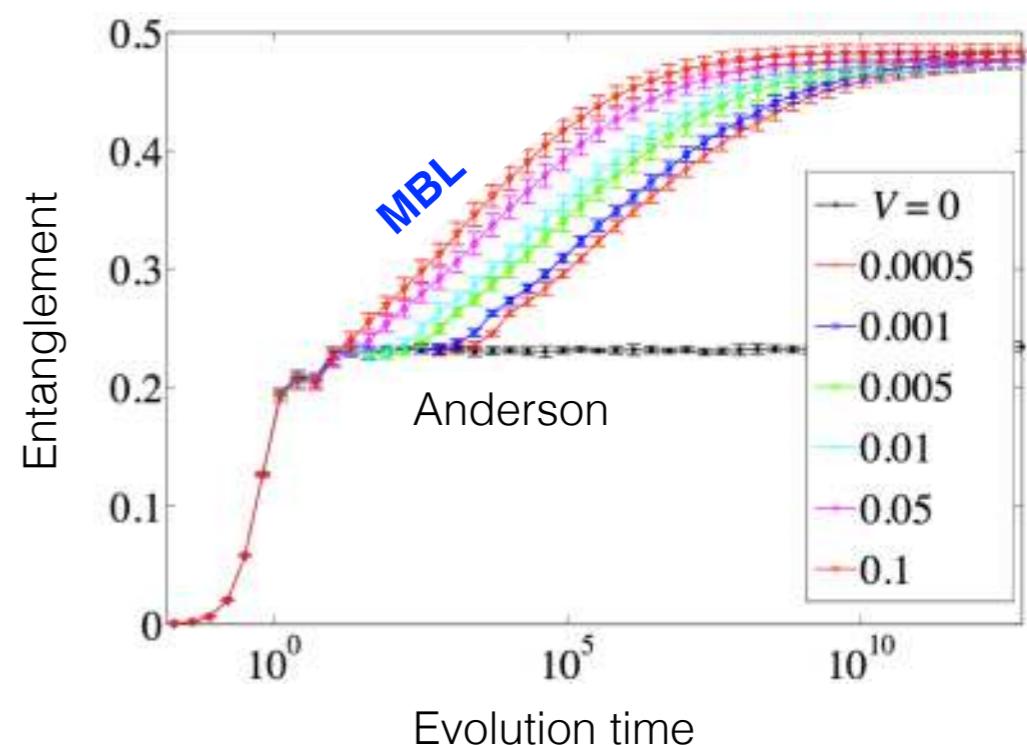
Conductivity  $\sigma = 0$



MBL: no particle transport, but “glassy” spreading of quantum information

$$S_{\text{ent}}(t) \sim \log(t) \quad ???$$

Bardarson, Pollmann, Moore'12, Serbyn, Papić, DA'13



# Emergent integrability

Serbyn, Papić, DA'13; Oganesyan, Huse'14 Imbrie'16

## Infinite-disorder limit

$$H_0 = \sum h_i \sigma_i^z + V \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z$$

Eigenstates: product states



Trivially “integrable”

$$[\sigma_i^z, H_0] = 0$$

# Emergent integrability

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Trivially “integrable”

$$[\sigma_i^z, H_0] = 0$$

## MBL phase, finite disorder

$$H = H_0 + J \sum_{\langle ij \rangle} \sigma_i^+ \sigma_j^-$$

Local integrals of motion

$$[\hat{\tau}_i^z, \hat{H}] = 0$$

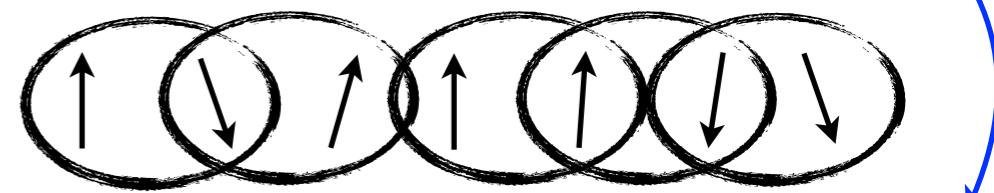
## “Action-angle” variables

- quasi-local
- qubits with  $T_1 = \infty$

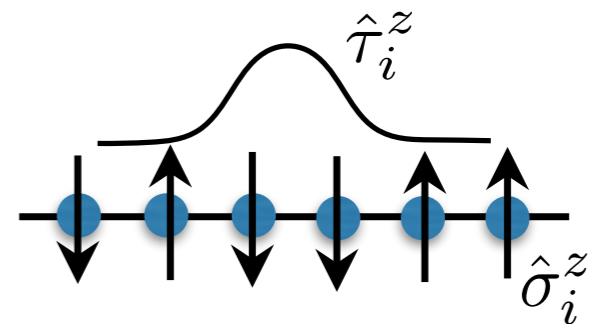
Eigenstates: product states



canonical transformation



Eigenstates: area-law entangled



# Emergent integrability

Serbyn, Papić, DA'13; Oganesyan, Huse'14 Imbrie'16

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Eigenstates: product states



Trivially “integrable”

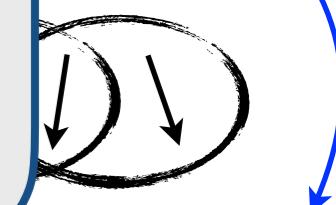
$$[\sigma_i^z, H_0] = 0$$

MBL ph

**Integrability supported by numerics & established rigorously at strong disorder in 1D**

$$H = H$$

canonical transformation

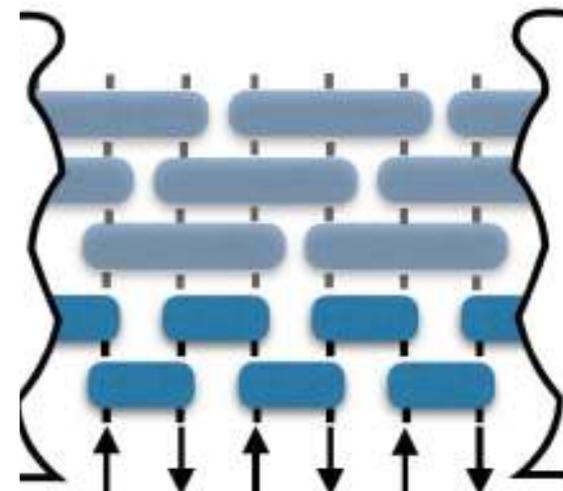


Local integrals of motion

$$[\hat{\tau}_i^z, \hat{H}] = 0$$

Eigenstates: area-law entangled

**Compress efficiently by tensor networks - new algorithms**



## “Action-angle” variables

- quasi-local
- qubits with  $T_1 = \infty$

# Dynamical properties of MBL phase

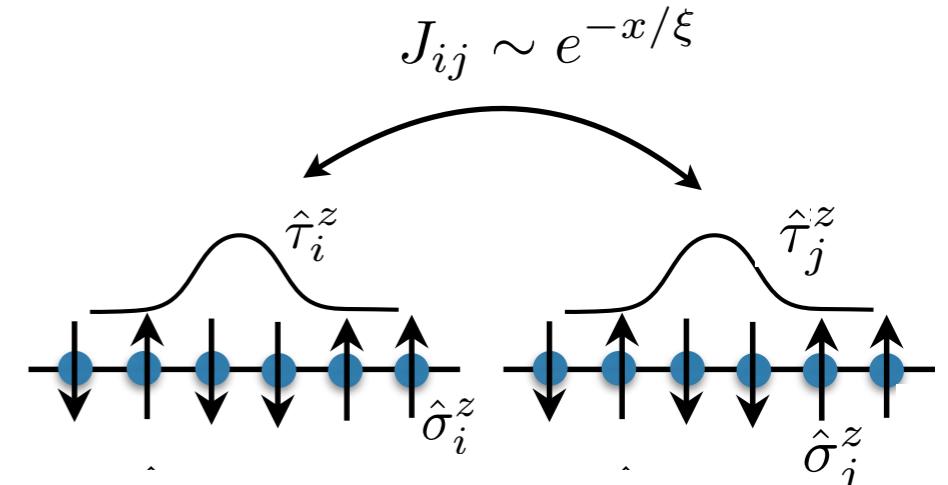
Review: DA, Altman, Bloch, Serbyn, RMP'19

## “Action-angle” variables

$$[\hat{\tau}_i^z, \hat{H}] = 0$$

$$H = \sum_i \tilde{h}_i \tau_i^z + \sum_{i,j} J_{ij} \tau_i^z \tau_j^z + \sum J_{ijk} \tau_i^z \tau_j^z \tau_k^z + \dots$$

## Solve for dynamics

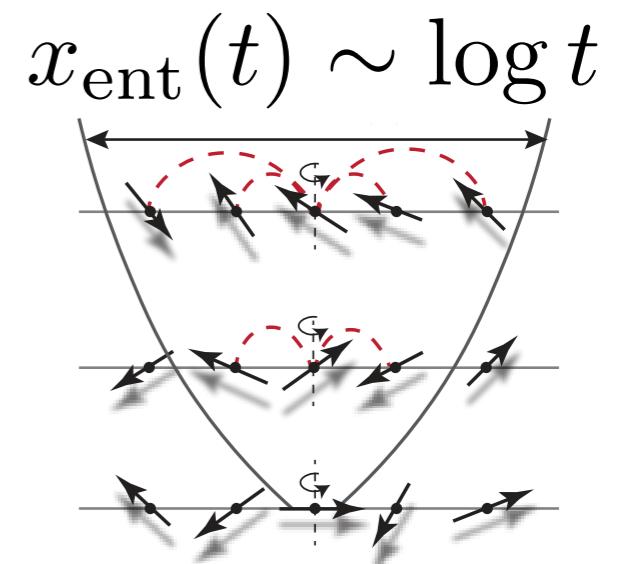


entanglement time

$$t \sim \frac{1}{J_{ij}} \sim e^{x/\xi}$$

## Logarithmic spread of entanglement

ergodic: linear spread of entanglement

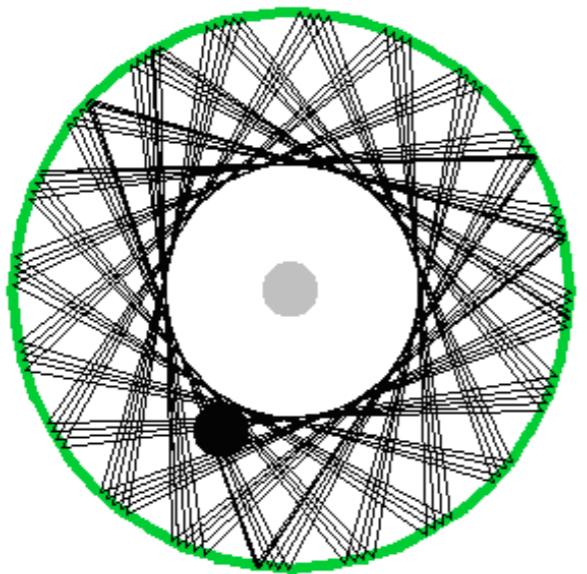


## Equilibration to a highly non-thermal state

$$\langle \tau_i^z \rangle = \text{const}$$

# Ergodicity breaking: Classical vs quantum

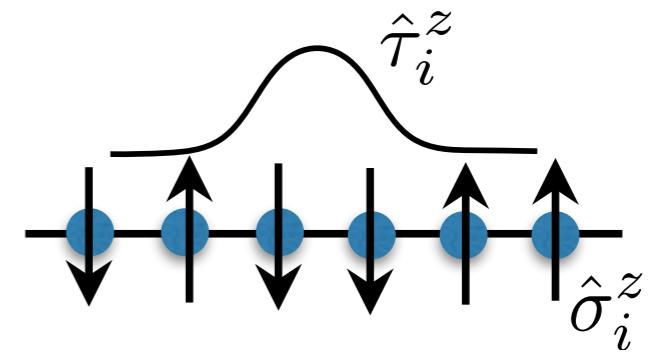
**Classical:** Integrable, regular motion



Kolmogorov-Arnold-Moser (KAM)  
theory

Robustness to weak perturbations  
**few-body only**     $N \sim 1$

**Quantum**



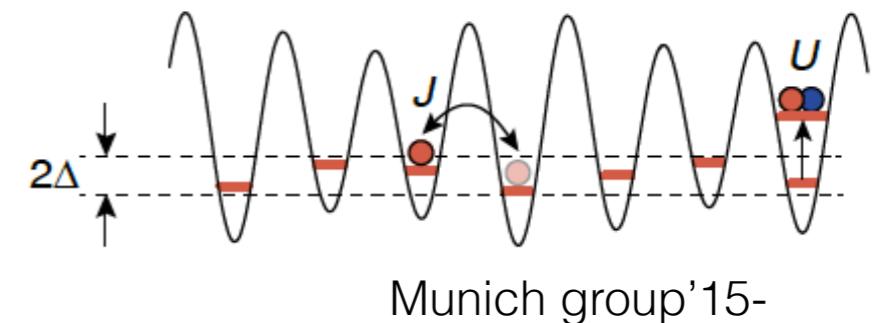
Disorder “protects” integrability

“Quantum KAM”

A stable phase of matter  $N \rightarrow \infty$

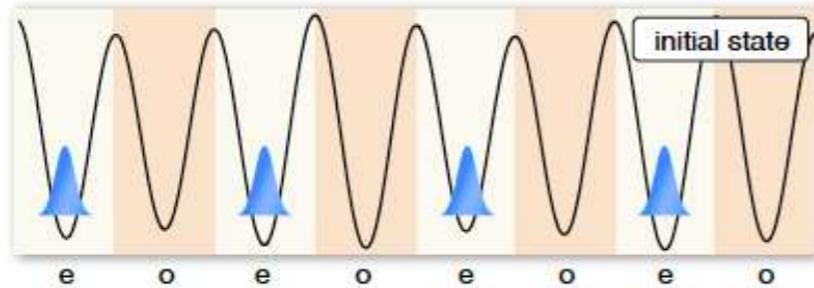
# Experiments

**System:** disorder+short-range interactions

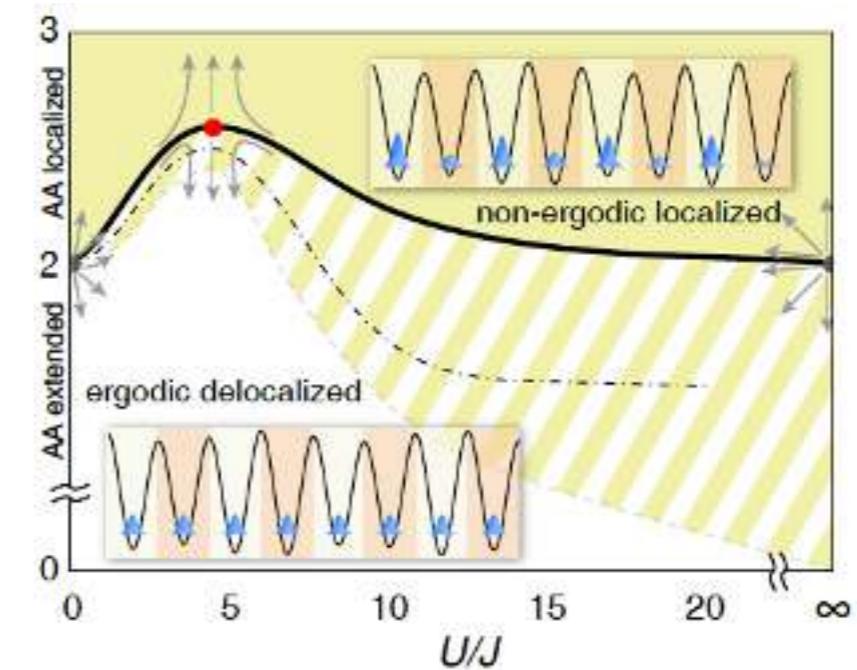


## Quantum quench setup

Lack of relaxation signals MBL  
(exps in 1d, 2d)



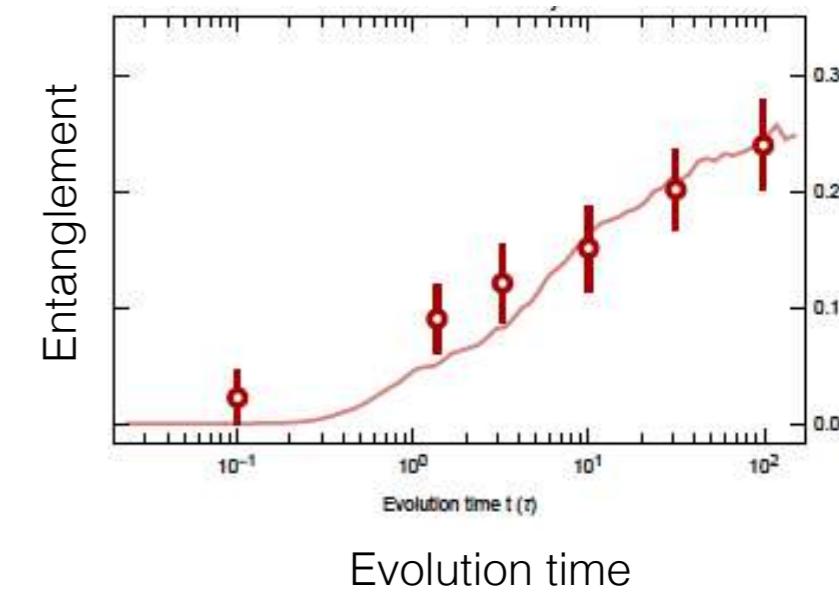
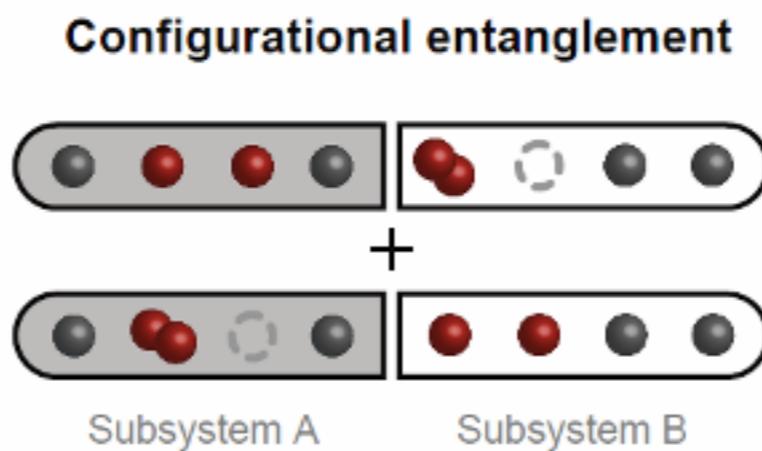
$$e^{-iHt}$$



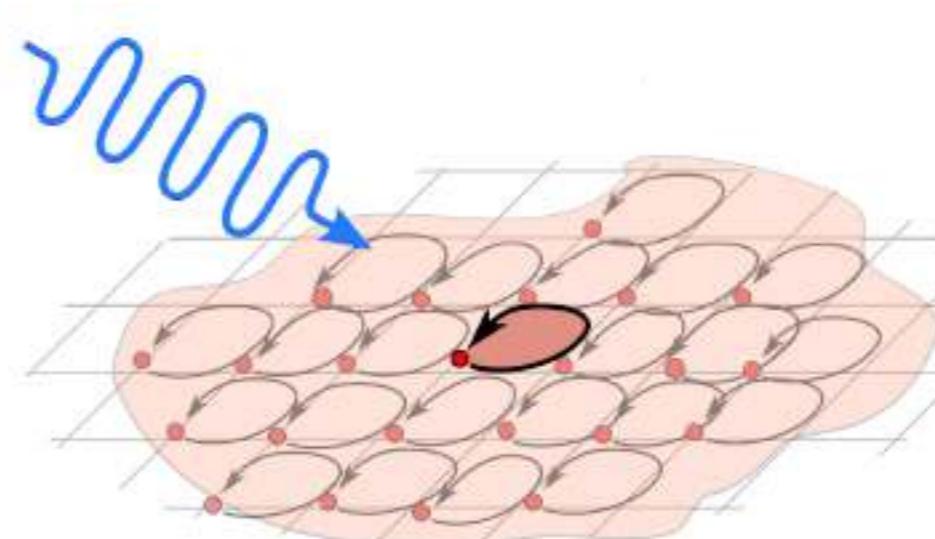
## Slow entanglement growth measured

Measure fluctuations of particle number

## Unique MBL signature



MBL enables new non-equilibrium phases



# Periodically driven systems and the heating problem

$$H(t + T) = H(t)$$

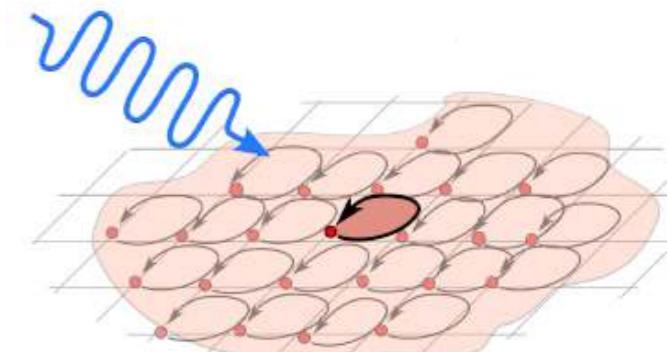
Floquet operator

$$F = \mathcal{T} \exp \left( -i \int_0^T H(t) dt \right)$$

Floquet eigenstates, quasi-energies

Effective Hamiltonian

$$F = e^{-iH_{\text{eff}}T}$$



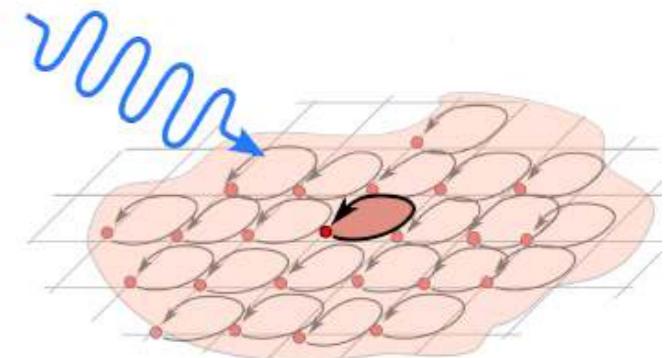
# Periodically driven systems and the heating problem

$$H(t+T) = H(t)$$

Floquet operator

$$F = \mathcal{T} \exp \left( -i \int_0^T H(t) dt \right)$$

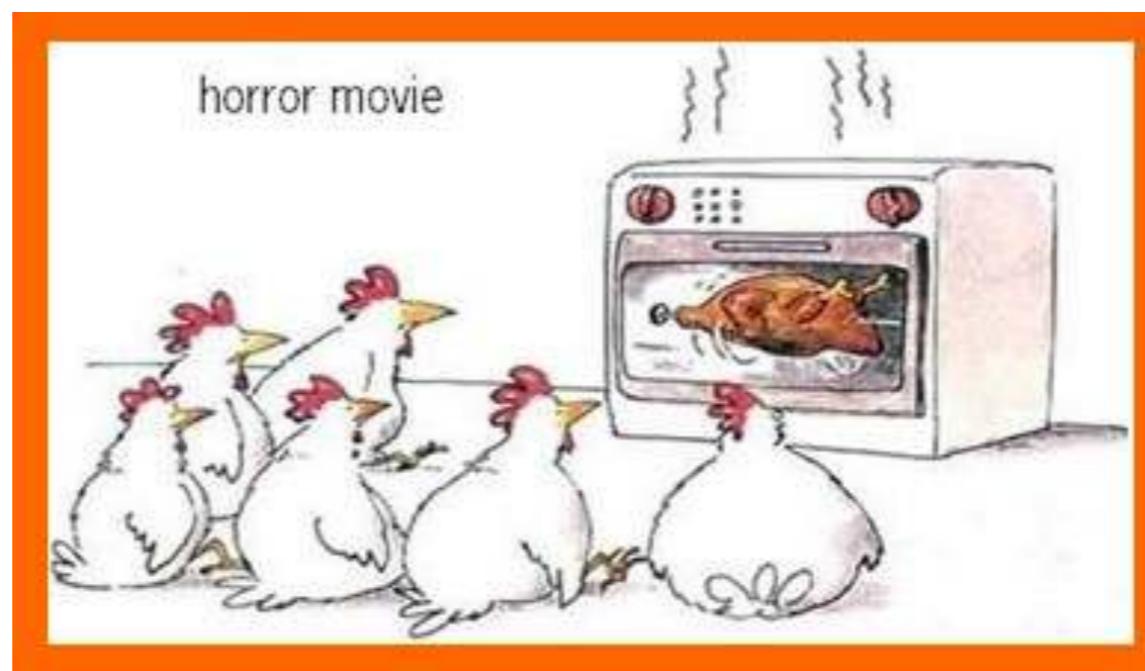
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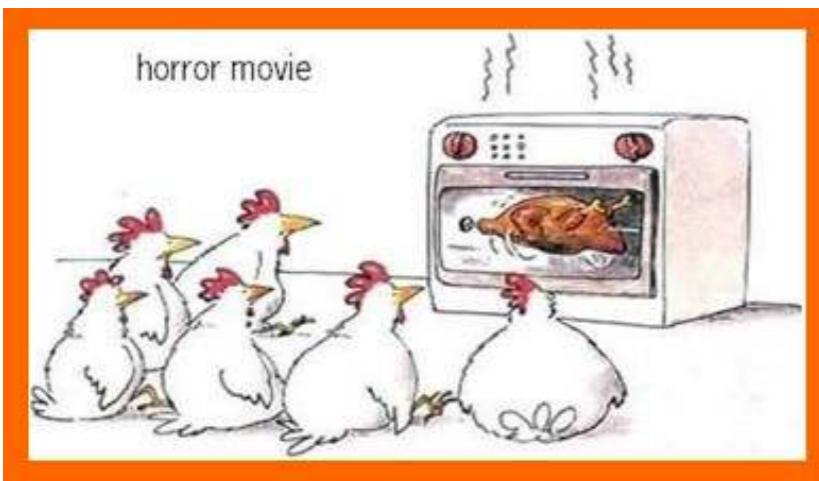
Heating  $\rightarrow H_{\text{eff}}$  nonlocal, non-unique (no energy conservation!)



# New phases enabled by MBL

- **MBL in driven systems**

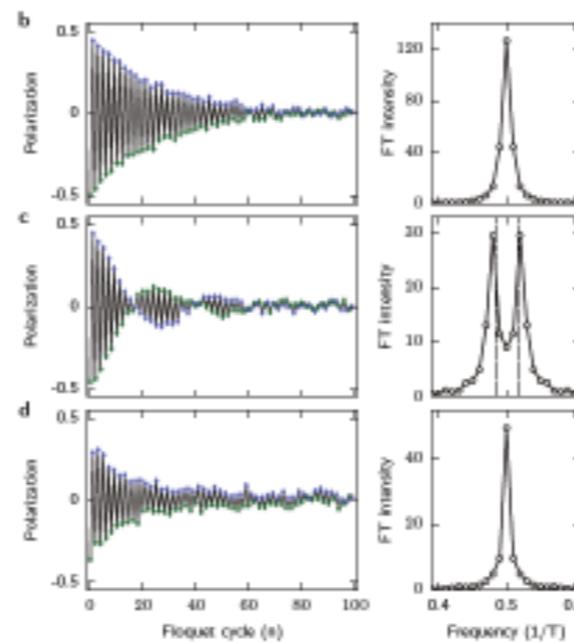
Solves heating problem



Ponte, Papic, Huvaneers DA'15  
Lazarides, Das, Moessner '15  
DA, Huvaneers, Roeck'15

- **Unique non-equilibrium phases**

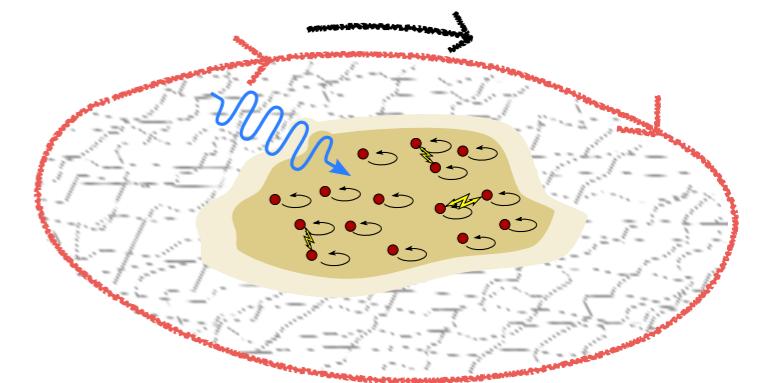
Time crystals



Khemani et al'16, Else et al'16

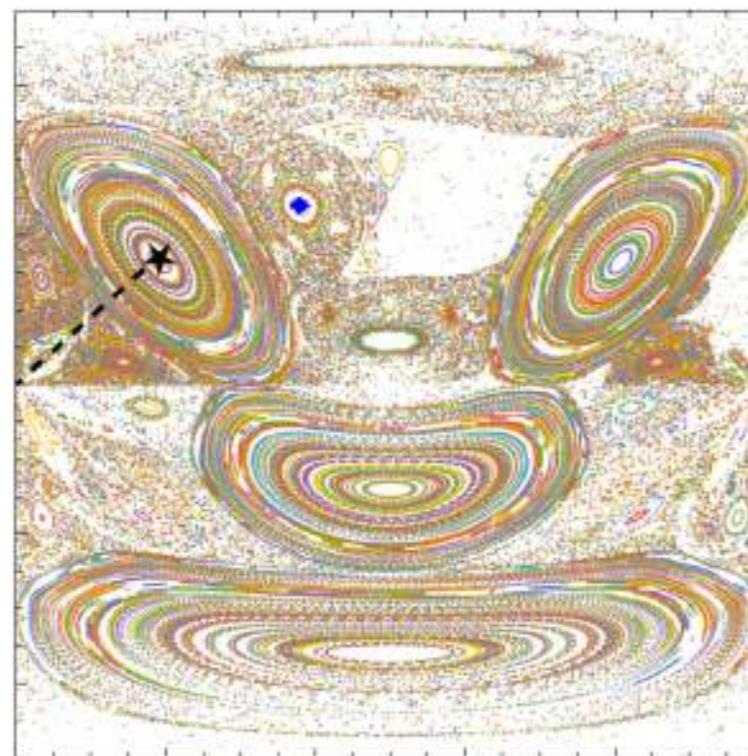
Experiment: Lukin group'17  
Theory: Ho, Choi, Lukin, DA'18

New topological invariants



Rudner et al'13, Potter et al'16, Nathan, DA, Berg, et al'18,...

# Beyond MBL & Outlook



# MBL and thermal systems: comparison

## Ergodic

Eigenstate  
entanglement

Conserved  
quantities

Spreading of  
correlations

Volume-law

Only global  
(e.g. energy)

$$x_{\text{ent}} \sim vt$$

## MBL

Area-law

Local integrals  
of motion

$$x_{\text{ent}}(t) \sim \log t$$

# MBL and thermal systems: comparison

**Ergodic**

Eigenstate  
entanglement

Volume-law

**MBL**

Area-law

Con-  
qua-

**Phases with intermediate properties?**

**Different entanglement scaling?**

Spreading of  
correlations

$$x_{\text{ent}} \sim vt$$

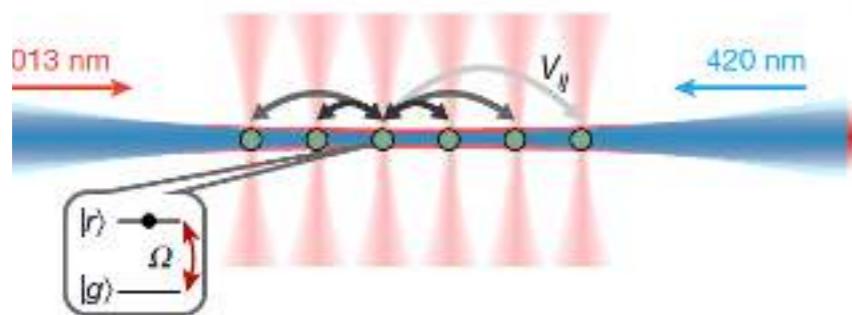
$$x_{\text{ent}}(t) \sim \log t$$

Integrals  
on

# Non-ergodicity beyond MBL

## Non-ergodicity w/o disorder

Bernien et al'17



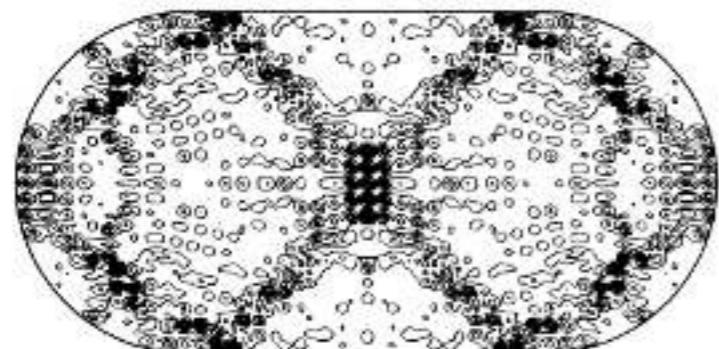
Rydberg atom simulator

- non-thermalising initial states
- wave function revivals



Turner et al'18; Ho et al'18

## Quantum many-body scars



## Non-Abelian symmetries+disorder

New non-ergodic regime

- symmetry prohibits conventional MBL
- different entanglement pattern, scaling

$$S_{\text{ent}}(L) \sim \log(L)$$



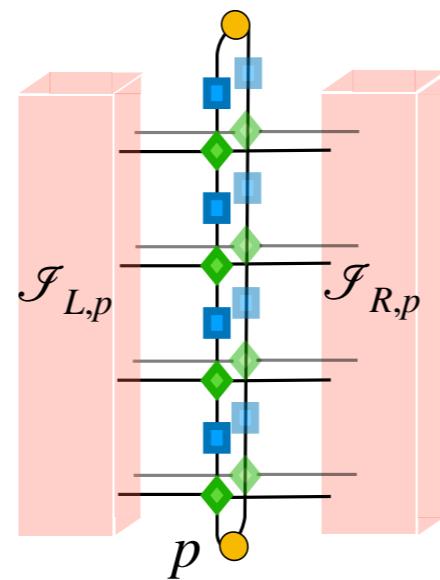
Vasseur et al'17-, Protopopov, DA+Demler,  
Scardicchio group'18-

## Prethermalization

Metastable non-thermal states

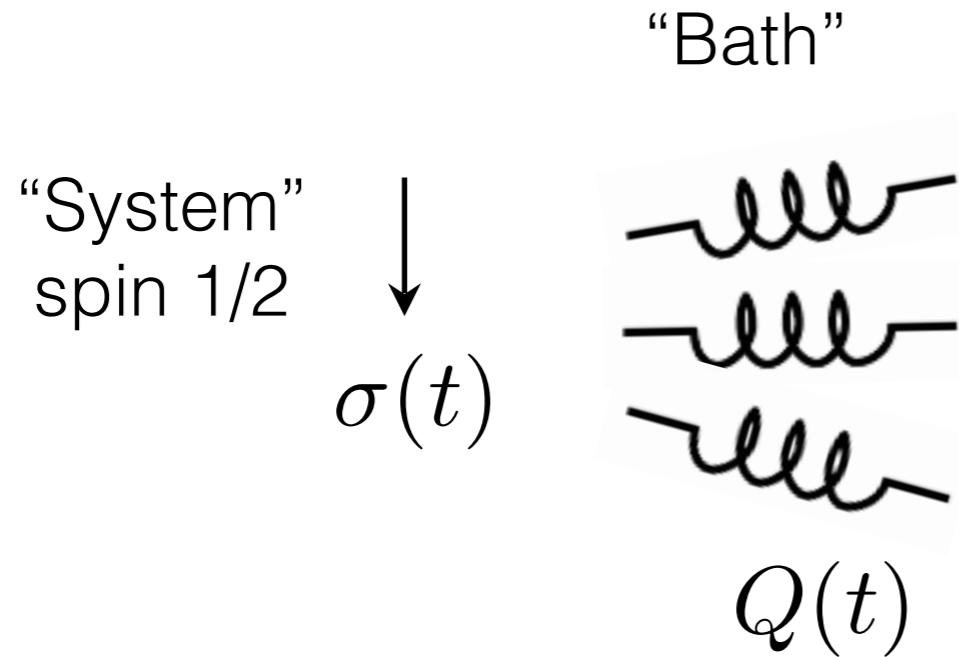
DA, Ho, Roeck, Huvaneers'15-; Else,  
Bauer, Nayak'17-, Fendley et al'16

# New approach to quantum dynamics: Influence Matrix



# Characterising a quantum bath by influence functional

ANNALS OF PHYSICS: 24, 118–173 (1963)



## The Theory of a General Quantum System Interacting with a Linear Dissipative System

R. P. FEYNMAN

California Institute of Technology, Pasadena, California

AND

F. L. VERNON, JR.\*

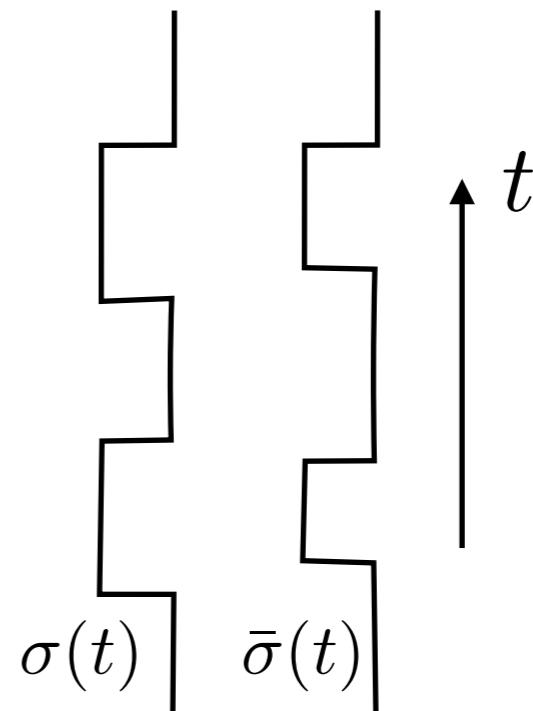


Influence functional acts on system trajectory

$$\mathcal{I}(\sigma(t), \bar{\sigma}(t))$$

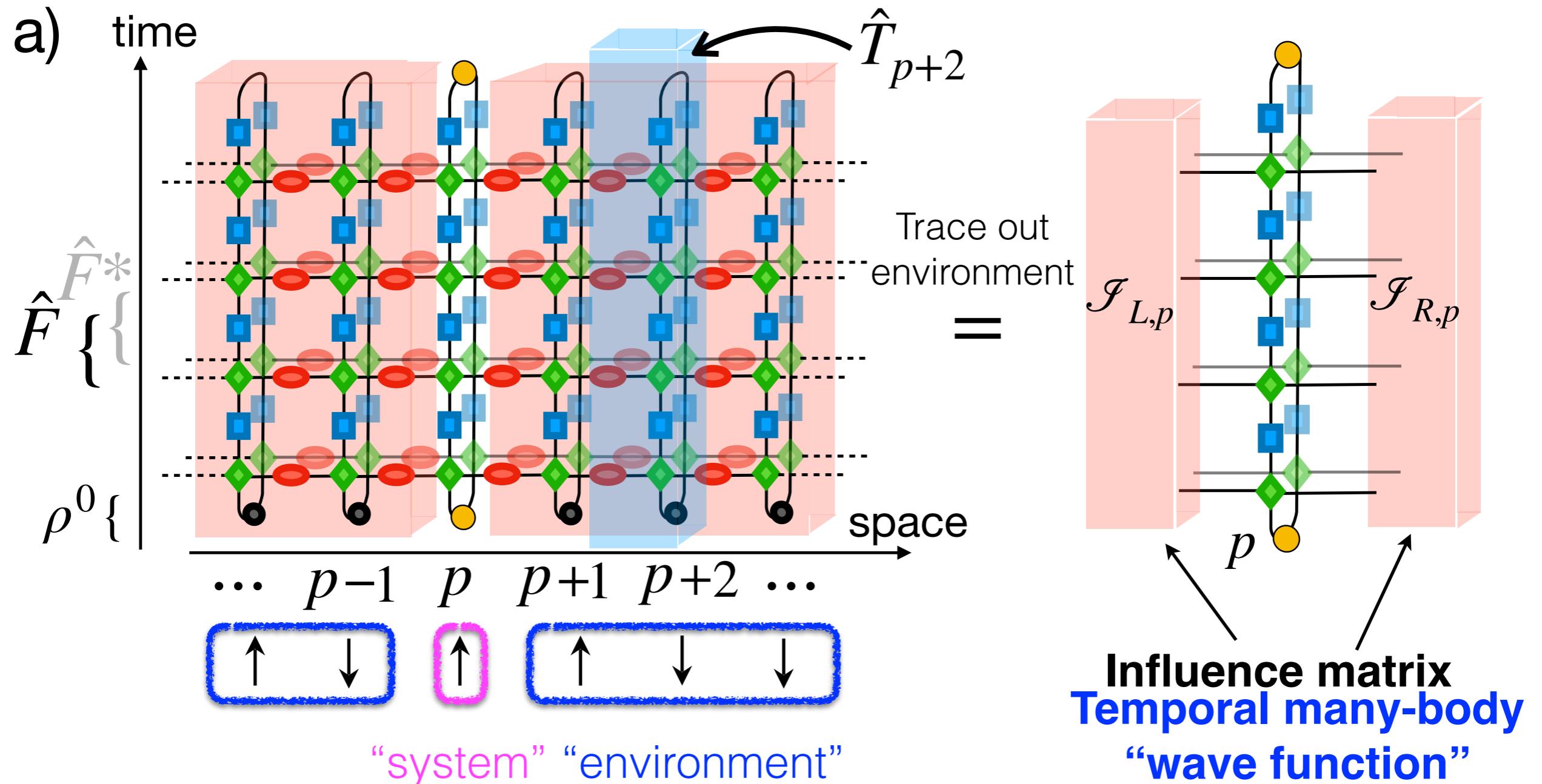
Previously: bath of oscillators

Leggett et al, RevModPhys'89,..



# A new theoretical & numerical tool: influence matrix

Key idea: describe quantum many-body dynamics via **Feynman-Vernon influence functional** Lerose, Sonner, DA, PRX'21, PRB'21, ...



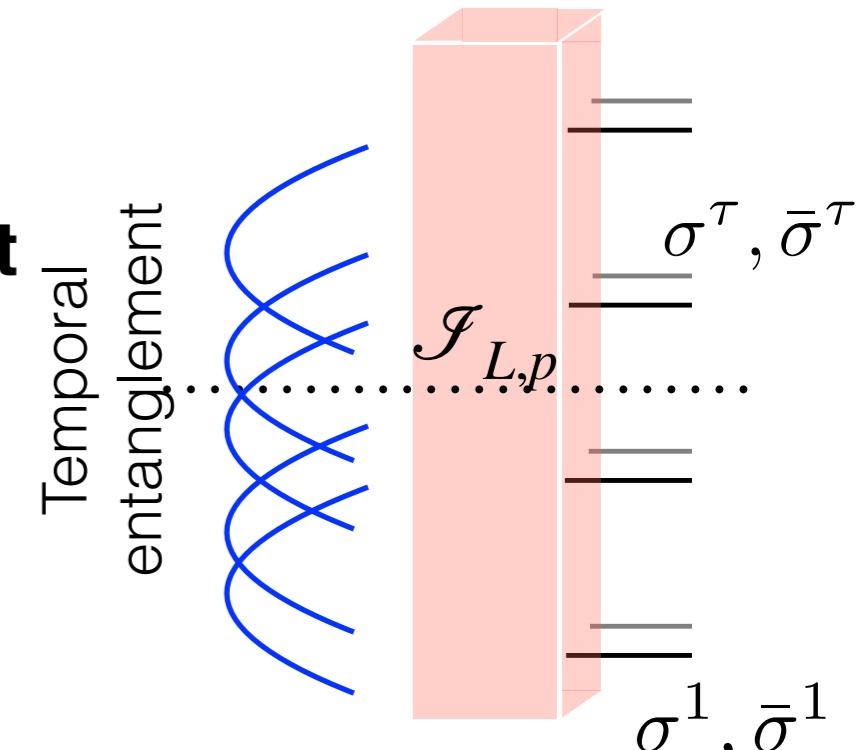
Tensor of dimension  $2^{2T}$   
 $T$  evolution time

# From spatial to temporal entanglement

Complexity of IM: naively  $2^{2T}$        $T$  -evolution time

Complexity characterized by **temporal entanglement**

Lerose, Sonner, DA, PRX'21, PRB'21, ...



**Efficient description** of MBL and thermalization

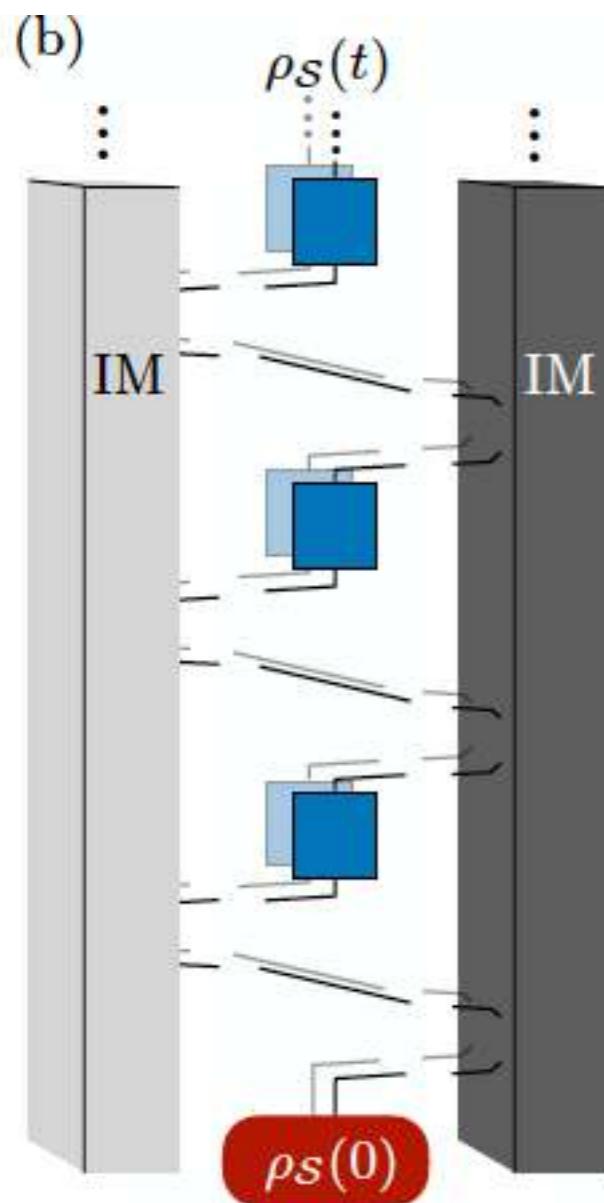
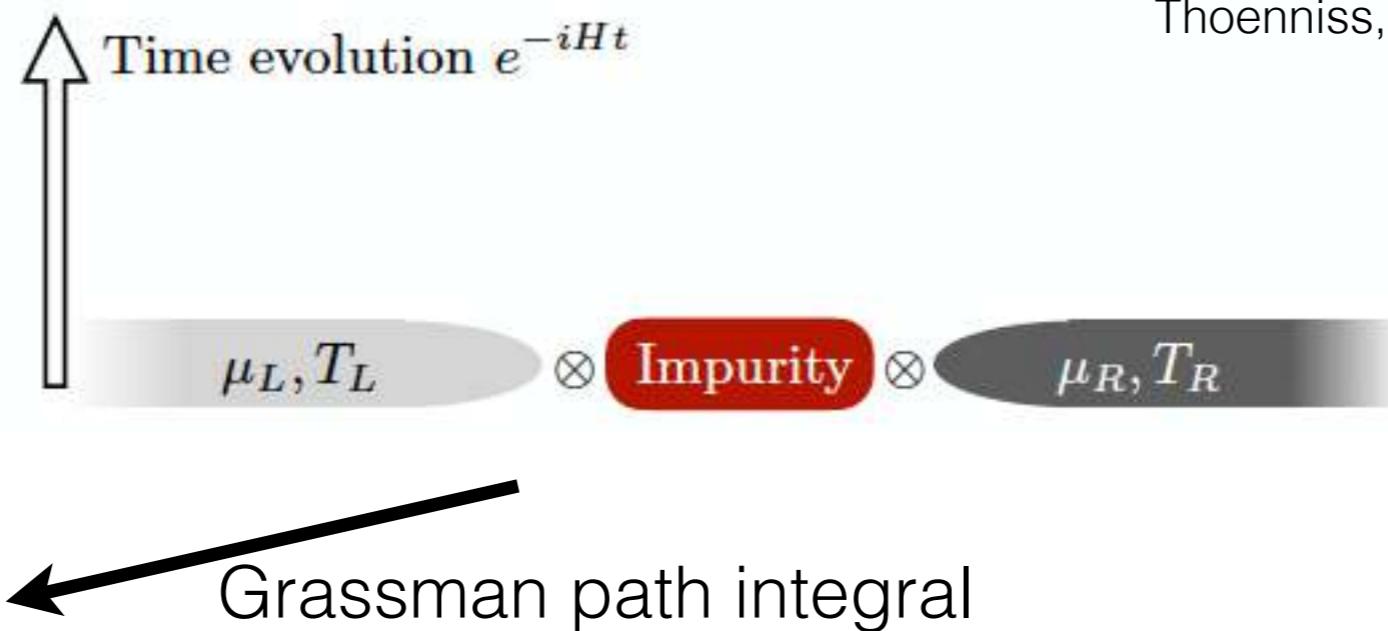
Sonner, Lerose, DA, PRB'22; related work by Garratt, Chalker, PRL'21

Access to **disorder-averaged observables** — resonances included

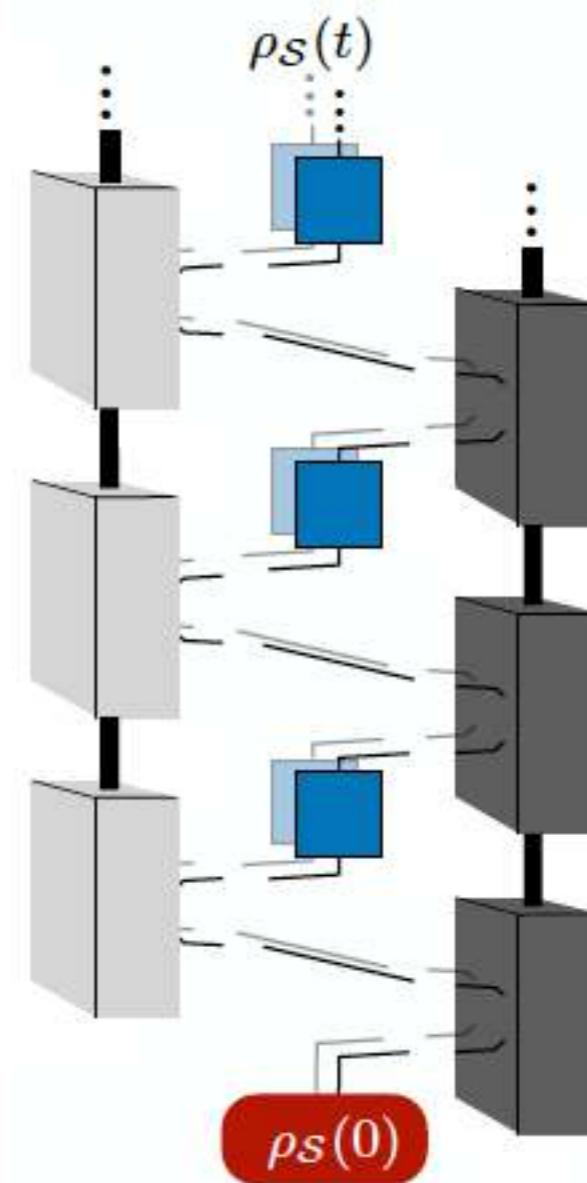
*Direct, complete description of many-body system as bath*

# Non-equilibrium quantum impurity problems

Thoenniss, Lerose, DA, arXiv:2205.04995



Extend Fishman-White  
algorithm (2015) to convert  
Gaussian IM to MPS



# Quench in an Anderson impurity model

Thoenniss, Sonner, Lerose, DA, arXiv:2205.04995+2210.xxxx

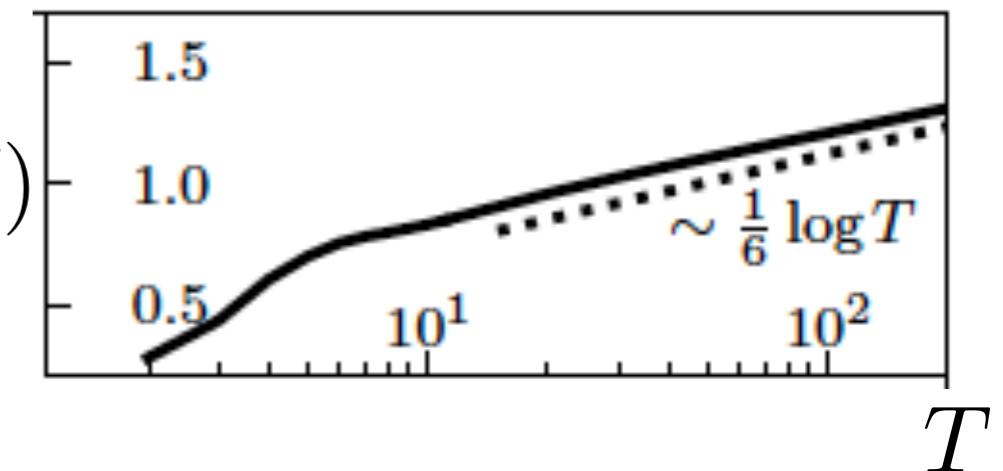
For Fermi-sea initial states

$$S_{\text{TE}}(T) \sim \log t$$

For finite-temperature initial states

$$S_{\text{TE}}(T) \sim C$$

1d nearest-neighbour fermions,  
Fermi sea state



(Most) non-equilibrium impurity problems can be efficiently solved

# Quench in an Anderson impurity model

Thoenniss, Sonner, Lerose, DA, arXiv:2205.04995+2210.xxxx

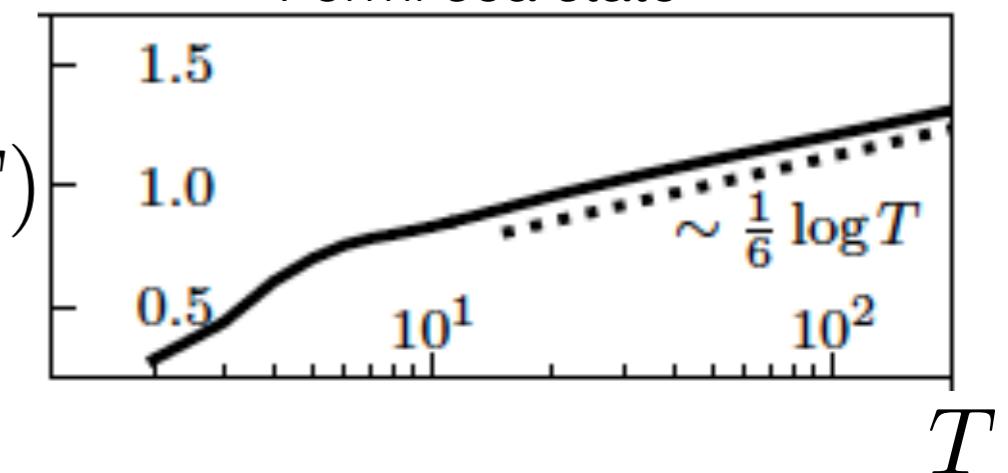
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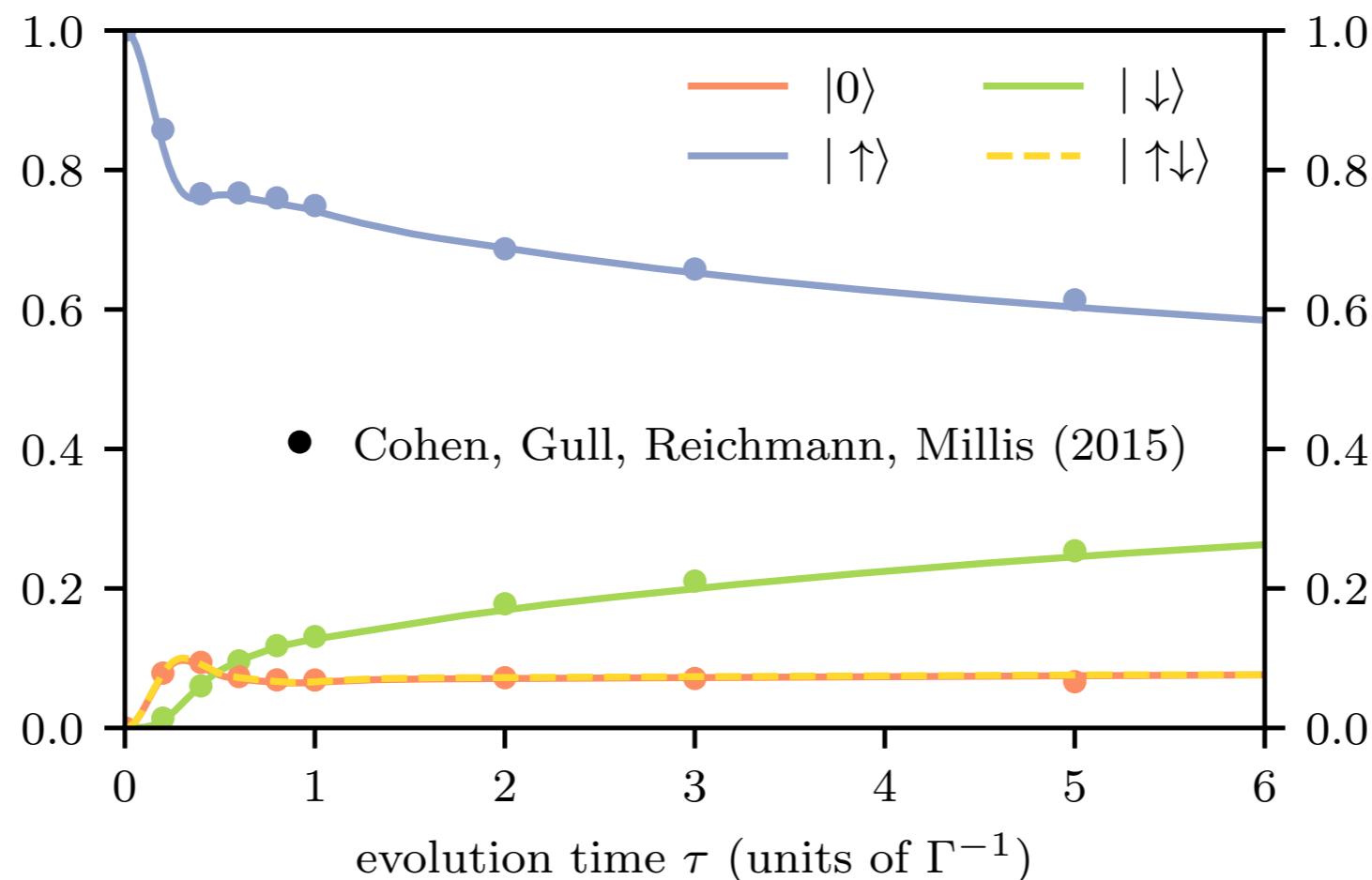
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1d nearest-neighbour fermions,  
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(Most) non-equilibrium impurity problems can be efficiently solved

**Benchmark:** influence matrix vs inchworm quantum Monte Carlo



# Summary

Many-body localisation: ergodicity breakdown, integrability

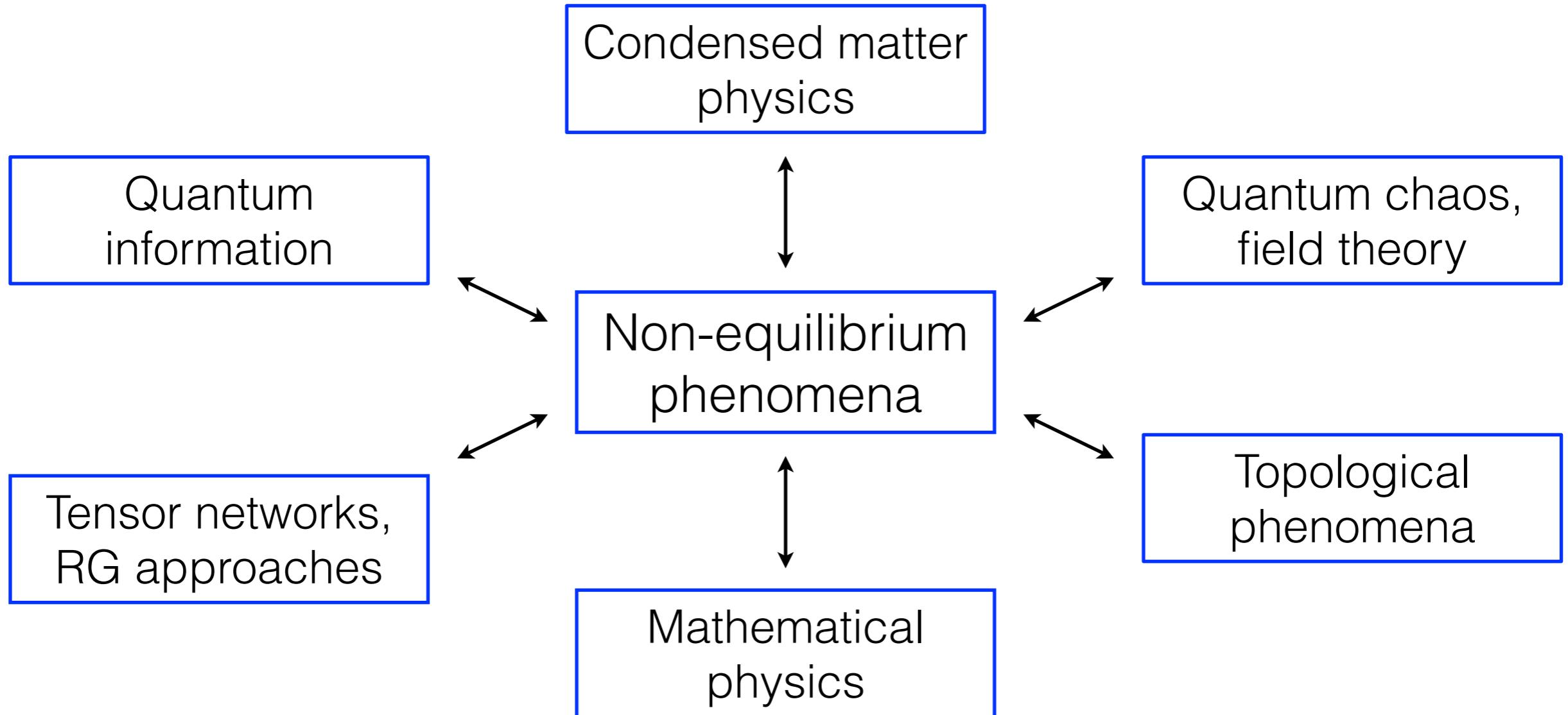
MBL protects coherence, enables new driven phases of matter

Influence matrix approach for many-body dynamics & quantum impurity models

Entanglement: a tool for non-equilibrium states

*No thermalisation → new dynamical phases  
New ways to control quantum matter*

# Outlook



*Classify non-ergodic phases? Entanglement patterns?*

*Practical ways to control synthetic matter*

*Control correlated materials by driving?*

# Acknowledgements



Maks Serbyn  
(IST Vienna)



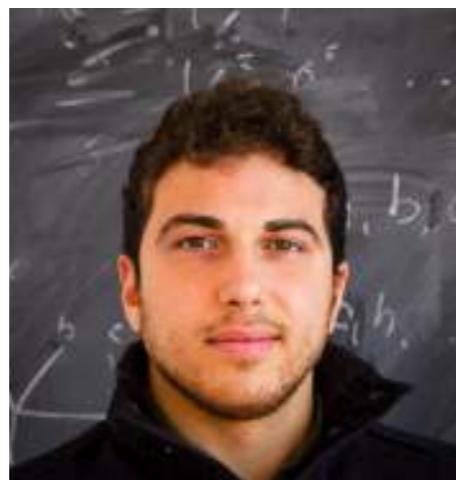
Zlatko Papić  
(Leeds)



François Huvaneers  
(Paris)



W. De Roeck  
(Leuven)



Alessio Lerose



Michael Sonner



Julian Thoenness

Thanks: Wen Wei Ho (Stanford), M. Filippone (Grenoble), A. Michailidis (Geneva), P. Brighi (IST), M. Lukin (Harvard), E. Altman (Berkeley), I. Bloch (Munich), A. Scardicchio (Trieste), E. Demler (Zurich), Google Quantum Team

Thank you!

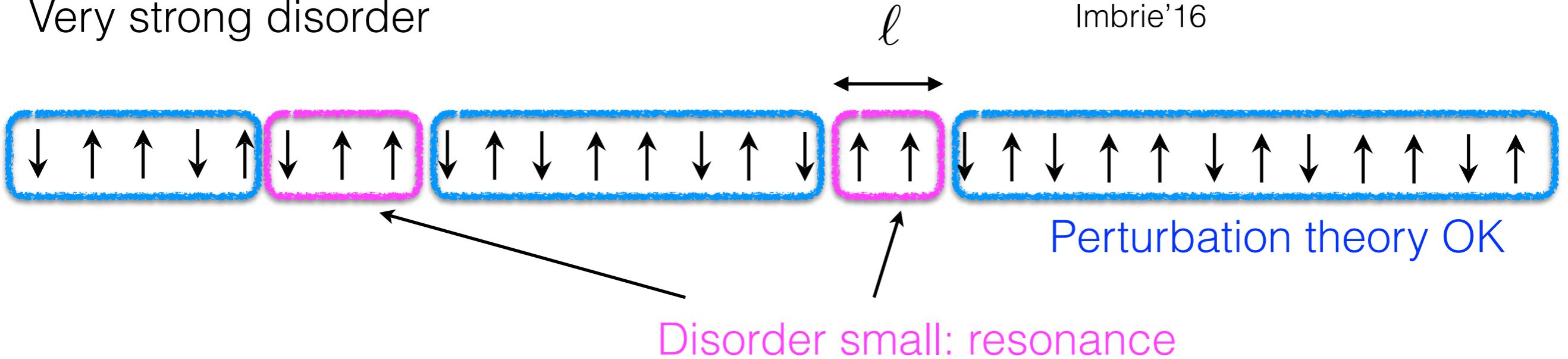


# **Open questions**

MBL in 2d and “thermalisation avalanches”

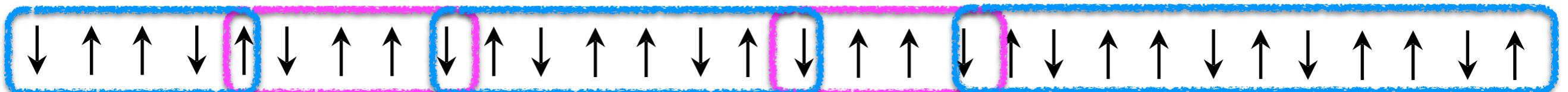
# Rare regions and resonances

Very strong disorder



Resonant regions are rare  $P(\ell) \sim e^{-\ell/a(W)}$

But: they act as a **small thermal bath**, can thermalise nearby spins



**Challenge:** describe the growth of small bath

In 2D such growth may be unbounded: **“thermalisation avalanche”**

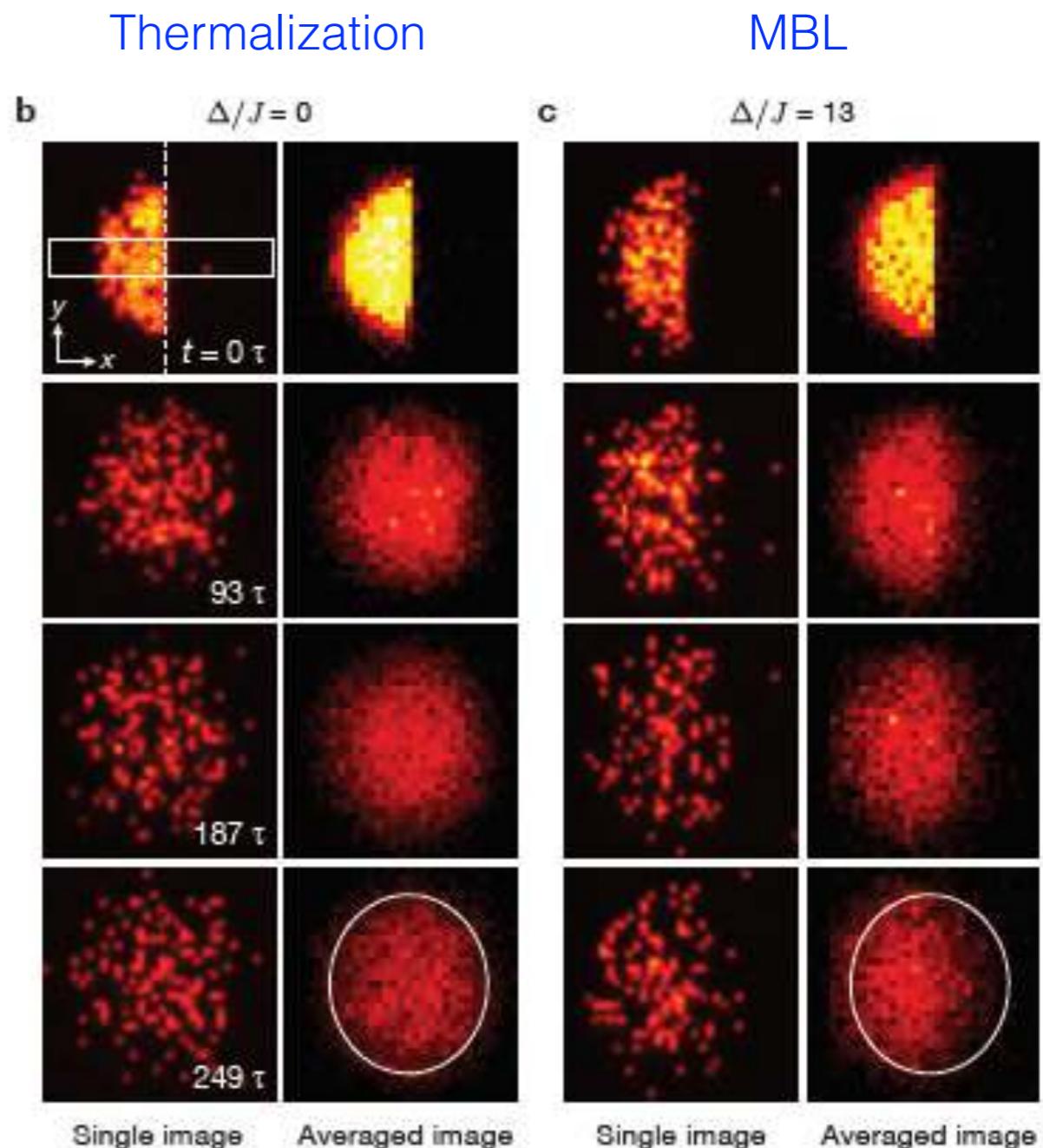
# MBL in 2D

Experiment: signatures of MBL regime      Bloch group'16

Numerics: -tensor-network simulations favour MBL in 2D      Wahl et al'18

-Time-dependent variational principle: critical disorder strength drifts with system size      Doggen et al'21

-MBL in 2d dimer models via exact diagonalization      Theveniaut et al'19



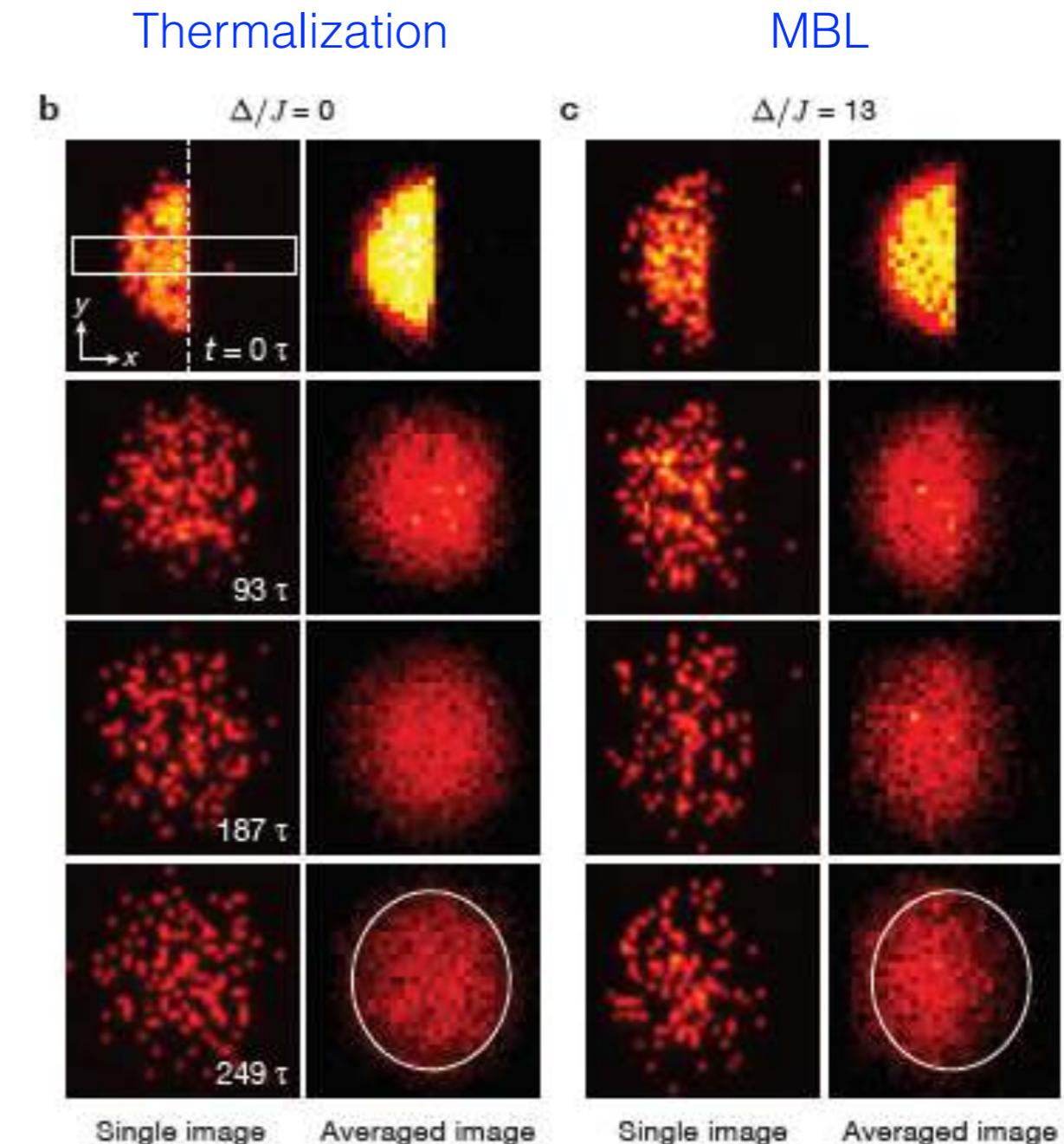
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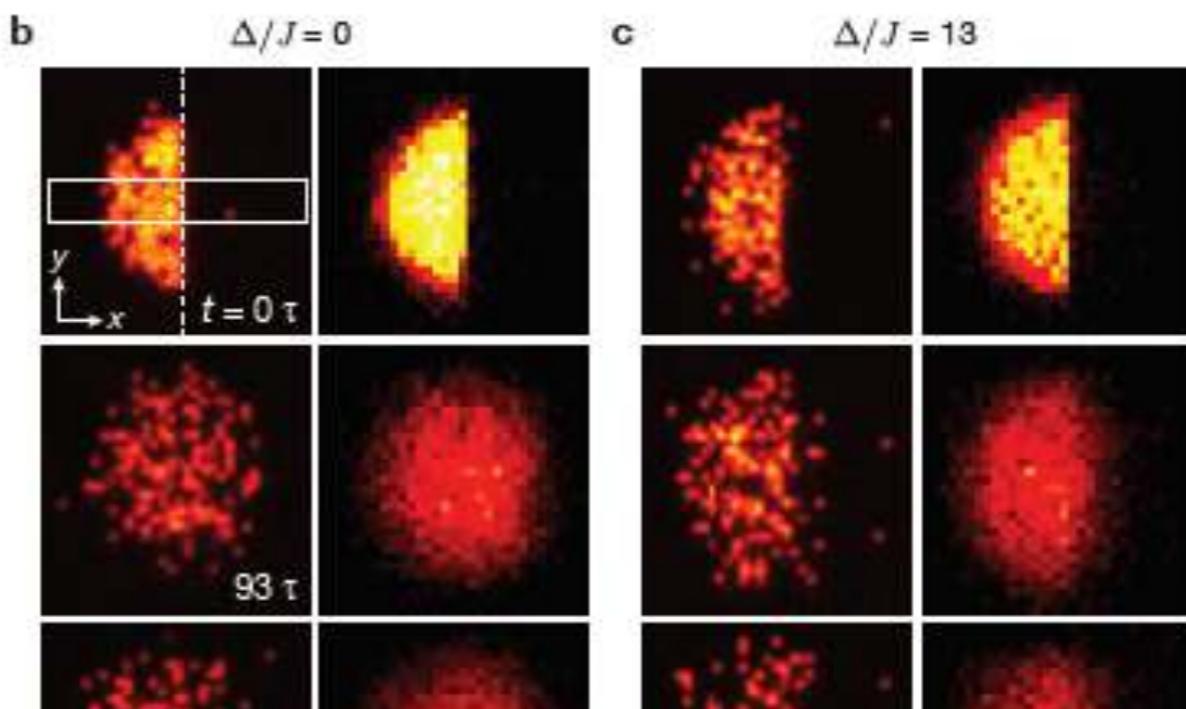
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Thermalization

MBL



Numerics: -tensor-network simulations favour MBL in 2D Wahl et al'18

YES

-Time-dependent variational principle: critical disorder strength

NO

**Challenge: how to access the limit  $L \rightarrow \infty, t \rightarrow \infty$  ?**

Experiment/numerics limited to  $L \sim 100, T \sim 100$

Avalanche mechanism  $\rightarrow$  extremely slow thermalization

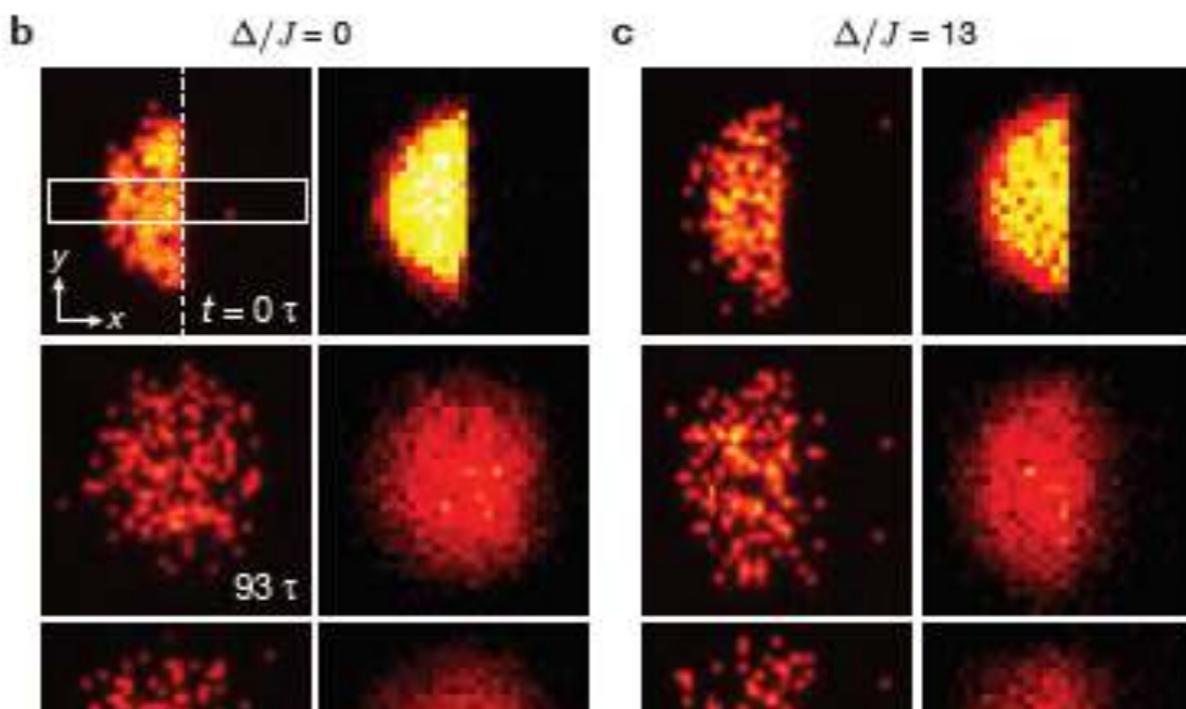
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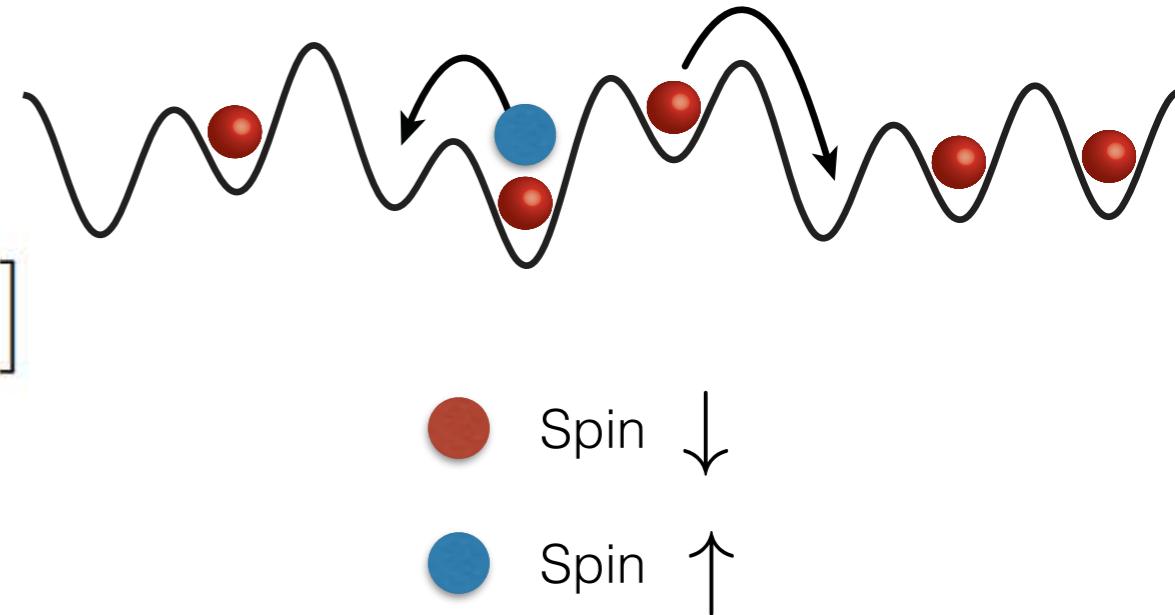
**Faster, accessible avalanches? New methods?**

# Delocalisation in a “supercooled insulator”

Setup: Disordered Fermi-Hubbard model

$$\mathcal{H}_{\text{FH}} = \sum_{j,\sigma} \varepsilon_j n_{j,\sigma} + U \sum_j n_{j,\uparrow} n_{j,\downarrow} + t \sum_{j,\sigma} [c_{j,\sigma}^\dagger c_{j+1,\sigma} + \text{h.c.}]$$

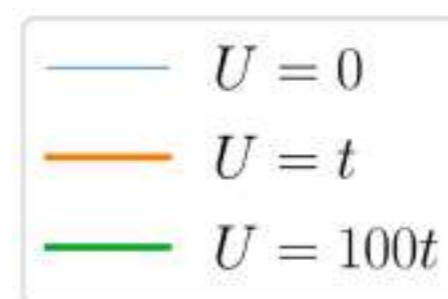
$$\epsilon_j \in [-W; W] \quad \text{localization length } \xi(W)$$



Finite density of spins-down  $\rho_\downarrow$  & single spin-up “impurity”

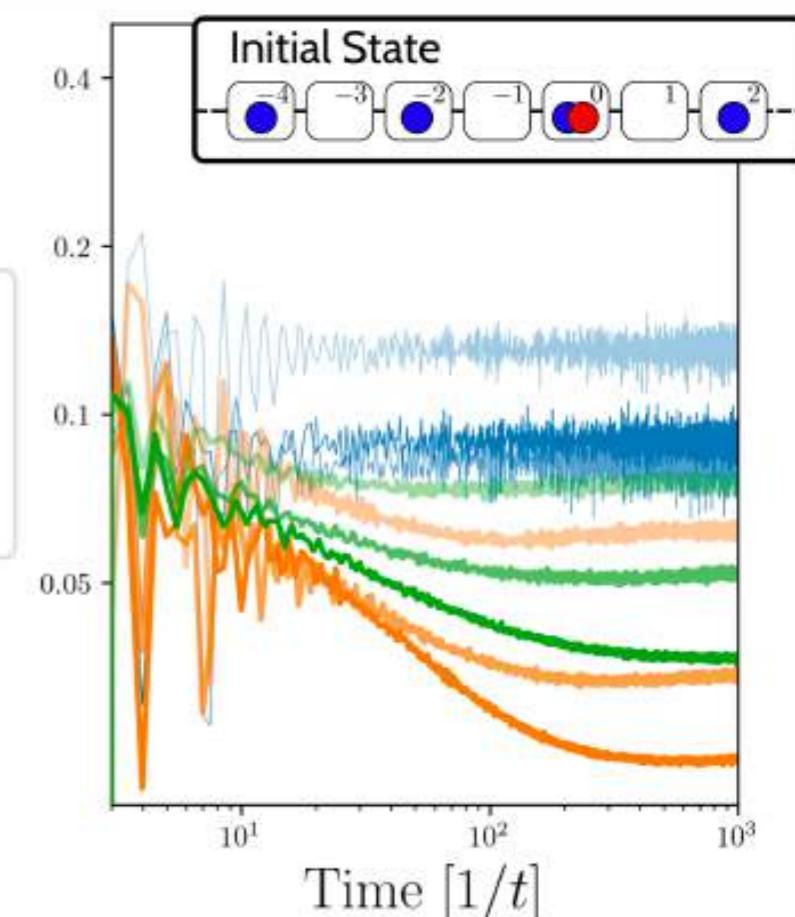
Krause, Brouwer, DA, Filippone, PRL'21

For sufficiently large  $\xi(W) > \xi_c(W)$  **a single impurity nucleates delocalisation**



Shorter thermalisation timescales → accessibility of critical properties in experiment and simulations

Bright, Michailidis, DA, Serbyn'22



# New kind of non-thermalizing dynamics in Rydberg arrays

Two states per atom:  $|o\rangle$  and  $|\bullet\rangle$

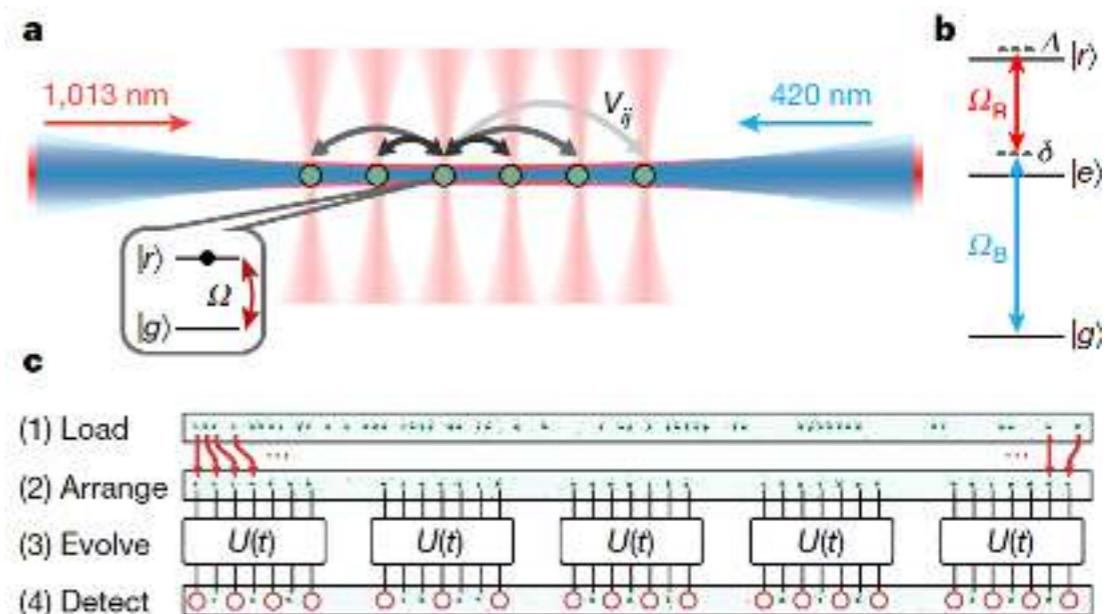
Rydberg blockade:



Initial state:

$$|\mathbb{Z}_2\rangle = o \bullet o \bullet o \bullet o \bullet o \bullet o \bullet$$

Bernien et al, Nature 2017 (Harvard-MIT)



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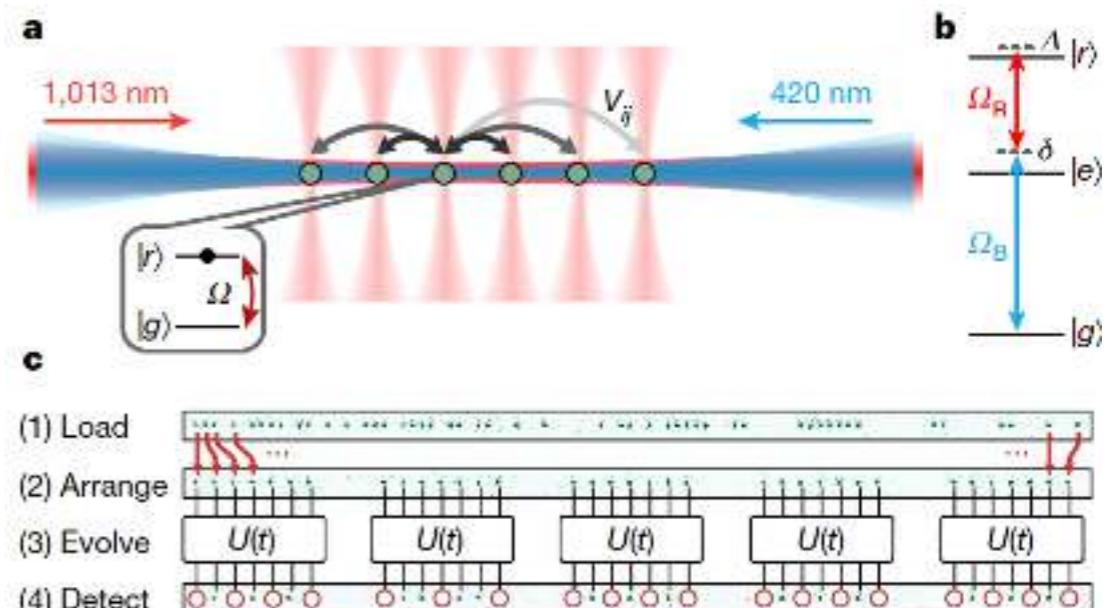
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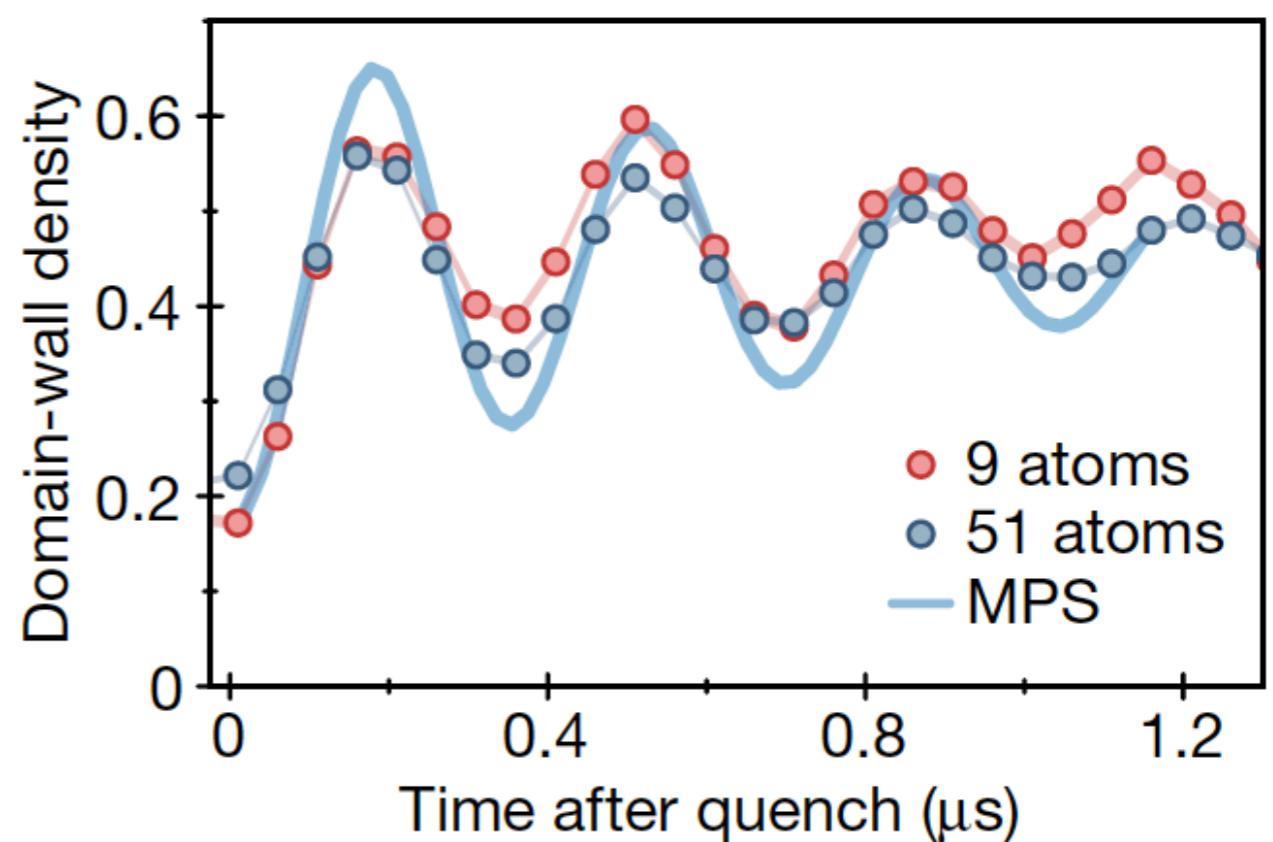
Bernien et al, Nature 2017 (Harvard-MIT)



Many-body revivals, new non-thermal behavior

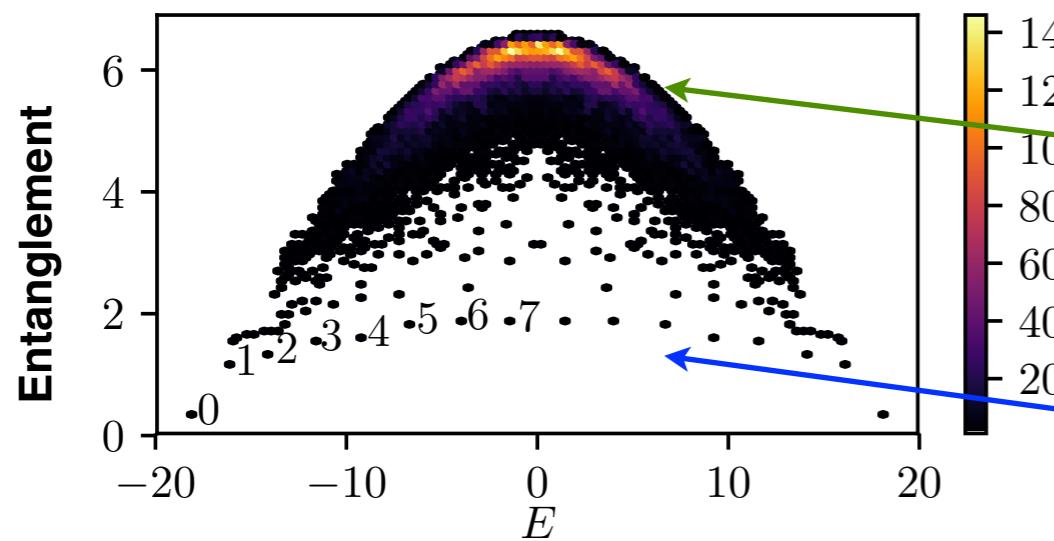
remarkable:  $\dim(H) \sim 2 \times 10^{15}$

No disorder, unlike in MBL



# Quantum Many-Body Scars

Papic, Serbyn group+DA, Nature Physics'18-, Ho et al'19,  
Bernevig et al'18-



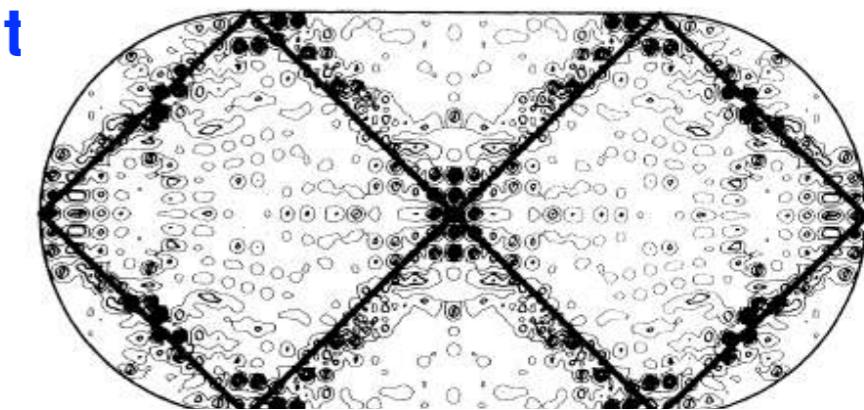
thermal  
eigenstates

special non-thermal  
states

**low entanglement**

**Anomalous eigenstates**

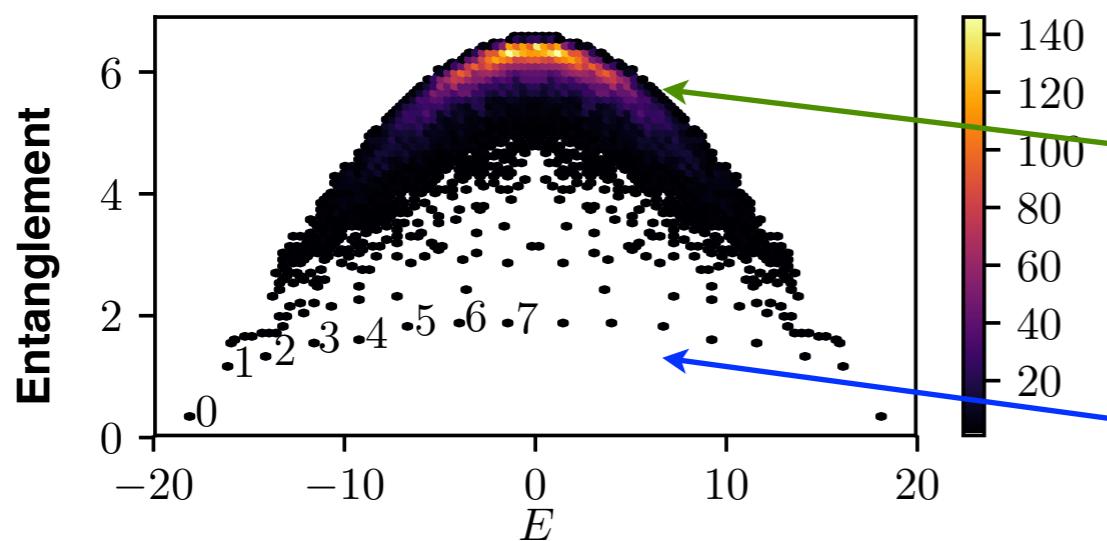
Weak ergodicity breaking



cf quantum scars in billiards

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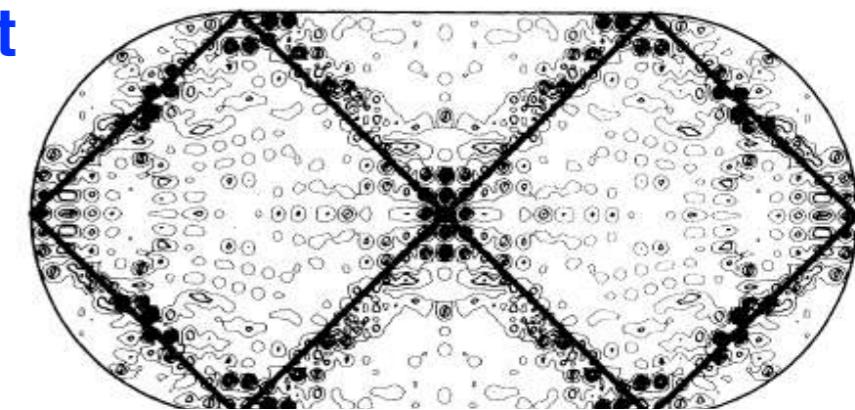
Papic, Serbyn group+DA, Nature Physics'18-, Ho et al'19,  
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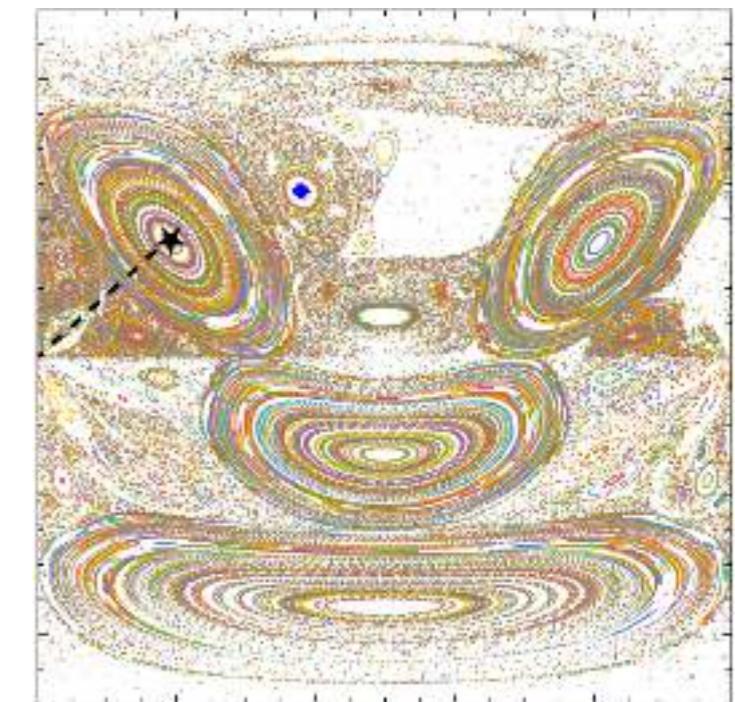


cf quantum scars in billiards

Variational dynamics with tensor networks

Haegeman et al'11

Scars & mixed phase space



**A new connection b/w quantum many-body and classical chaos**

Michailidis, Serbyn, DA et al, PRX'20