

# Why Include Warrants in New Equity Issues? A Theory of Unit IPOs

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## Abstract

We develop a theory of unit IPOs in which the firm going public issues a package of equity with warrants. We model an equity market where insiders have private information about the riskiness as well as the expected value of their firm's future cash flows. We demonstrate that, in equilibrium, high risk firms issue underpriced "units" of equity and warrants; lower risk firms, on the other hand, issue underpriced equity alone. In contrast to the existing literature, underpricing arises as a signal in our model in the context of a *one-shot* equity offering. Though developed in the context of IPOs, our model can also explain the issuance of seasoned equity offerings packaged with warrants. Further, the intuition behind the model generalizes readily to provide a new rationale for packaging call-option-like claims with risky securities other than equity, including convertible debt and debt with warrants.

## I. Introduction

The use of packaged offerings in security issues has been the subject of considerable debate in corporate finance. One such instance of packaged security issues is the use of warrants in the initial public offerings of certain companies, popularly referred to as "unit offerings." In a unit offering, the firm going public issues a package of shares of equity and warrants (commonly referred to as a "unit") rather than equity alone. Often, the companies issuing units are high risk firms, and are marketed by less reputable underwriters. Further, a significantly large proportion of these firms tends to be concentrated in certain high risk industries (e.g., the bio-tech, computer software, and health services industries).

As an illustration, consider the case of Microprobe Corporation, a biomedical company that went public on September 29, 1993. The firm issued 2,500,000

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units, each consisting of one share of common stock and one warrant, priced at \$6.60 per unit (with the warrants having an exercise price of \$6.60 and a time to expiration of five years). The number of warrants and shares of equity in each unit vary considerably across IPOs: in an empirical study of unit IPOs, Schultz (1993) documents that in his sample, the ratio of shares to warrants varied from  $\frac{1}{3}$  to 4, with the median offering containing twice as many shares as warrants. In a unit IPO, investors are allowed to buy equity prior to the IPO only as part of such a package, with the stock and warrants starting to trade separately soon after the IPO. Further, such units of equity and warrants are sold to *all* investors: thus, unit IPOs should not be confused with “underwriter warrants,” the practice of issuing a limited number of warrants to the underwriters of many IPOs as a part of their compensation (see, e.g., Barry, Muscarella, and Vetsuypens (1991) for a discussion of underwriter warrants).

Unit offerings confront financial economists with a puzzle: in a full information setting, issuing such securities with contingent claims attached to them provides no special advantage to the firm. The justification given by practitioners for issuing units is that they act as “sweeteners” to otherwise unattractive, high risk equity issues. However, an article in *Business Week* (May 13, 1991) advises investors to keep away from unit offerings, since “quality deals don’t need such warrants as sweeteners.” Such practitioner notions deepen the unit offering puzzle, since, far from “sweetening” the pot, issuing warrants along with the equity seems to convey to investors the information that these are indeed high risk equity offerings. In any case, such casual explanations do not shed much light on how attaching warrants to equity creates value: after all, firms can make any new equity issue more attractive simply by setting a lower share price. Further, a theory of unit equity offerings should explain why only relatively high risk firms choose to make such offerings, with less risky firms issuing equity alone in their IPO.

In this paper, we develop a theory of unit offerings in a simple asymmetric information set-up, which sheds some light on the above issues. We develop a two-type model in which type G firms (defined as those with a higher expected value of future cash flow) may have greater, equal, or lower variance of future cash flow (riskiness) compared to type B firms (which have a lower expected value of future cash flow). Firm insiders, who are risk averse, have private information about the mean and the riskiness of their firm’s future cash flows (they know the type of their own firm), while outsiders cannot distinguish between the two types. In this setting, the firm approaches the equity market to raise capital in an initial public offering. When making the stock issue, insiders optimally choose the fraction of the equity to be sold, the share price, the number of warrants (if any) to be issued, and the exercise price of these warrants.

To understand the broad intuition leading to our results, it is useful to compare it to the seminal model of Leland and Pyle (1977). Leland and Pyle also models a one-shot equity offering, where risk-averse insiders have private information about the mean of their firm’s future cash flow distribution. In this setting, insiders of the better type firm(s) signal true firm value to the equity market by retaining a larger fraction of their firm’s equity than the poorer type firm. Such a signaling equilibrium exists, because the cost of retaining a larger fraction of the firm’s

equity than would be optimal from purely risk-sharing considerations is lower for the better type firm.

In contrast to the Leland and Pyle (1977) model, we allow the two types of firms to differ in their riskiness as well as in the mean of their future cash flows. It is this difference in riskiness that allows for underpricing as well the issue of warrants to act as signals in our model. To see the intuition behind underpricing as a signal in our setting, consider the case where the type G firm is significantly riskier than the type B firm. Now, starting from the single signal equilibrium point (where the type G firm is constrained to use only the fraction of equity retained as a signal), allow the firm to set the price below the full information value as well. It now becomes optimal, in terms of maximizing insiders' expected utility or equivalently, minimizing signaling costs, for the type G insiders to cut back on the fraction of equity they retain, simultaneously underpricing their firm's equity to deter mimicking by the type B firm. Since the marginal benefit to cutting back on equity retained while simultaneously underpricing is greater for the insiders of the (riskier) type G firm, the equilibrium involves the use of both underpricing and the fraction of equity retained as signals.

Now allow the type G firm to use a third signal, namely the issue of warrants along with equity in the IPO. To see the intuition behind how warrants may be used as part of the equilibrium signaling mix by the type G firm, it is useful to compare the cost of issuing a warrant for a type G firm and for a type B firm. Assume as before that the type G firm is riskier than the type B firm, and assume, for concreteness, that the exercise price of each warrant is set at the mean of the type G firm's cash flow distribution. On the one hand, each warrant issued by a type G firm is more valuable in terms of the true dollar value of the expected cash flow yielded to the holders of the warrant than a warrant issued by a type B firm. On the other hand, given that the firm's insiders are risk averse (and noting that a type G firm has a greater mean and riskiness than type B firms), the marginal utility of each dollar of cash flow that they have to yield to outsiders as part of the payoff when the warrants are in-the-money will be lower for insiders of type G firms. Warrants will be part of the equilibrium signaling mix if the latter *private value effect* dominates the former *dollar value effect* since, in this case, issuing warrants will be less costly for insiders of a type G firm than those of a type B firm.

Issuing warrants differs as a signaling device from underpricing equity. Both signaling devices impose dissipative costs on the firm. Warrants, however, provide a way to incur some of these dissipative costs only selectively, in the higher realizations of the firm's future value. Since insiders are risk averse, this ability to dissipate value selectively in only the higher states is important, since it is precisely in these states that insiders' valuation of the firm's cash flows is the lowest. Alternatives to selling warrants, such as selling underpriced equity, do not have this ability to yield value only in specified states of the world; embedding call-option-like claims in securities offerings thus helps to minimize the dissipative costs associated with asymmetric information, leaving the firm's insiders better off in situations where the firm's cash flow stream is highly risky.

This leads to the testable prediction that, in an ex ante indistinguishable pool of firms, firms that have made unit IPOs should be associated with greater ex post

variance compared to those that have made IPOs with equity alone, a prediction borne out by the evidence in Schultz (1993). A second testable implication of our model is that for firms that have made unit IPOs, the fraction of firm value sold as warrants will be increasing in firm riskiness.

Unit IPOs have been a relatively unexplored area in both the theoretical and the empirical literature. In the only empirical study on unit offerings that we are aware of, Schultz (1993) documented that a significant proportion of IPOs (167 of the 797 IPOs that took place during 1986–1988) were unit offerings. Motivated by Jensen's (1986) free-cash flow hypothesis, Schultz conjectures that perhaps these are multi-stage equity financing arrangements that minimize agency costs. The argument is that managers/insiders may squander any cash raised in excess of the firm's immediate investment requirements by investing in negative NPV projects, and warrants provide a means by which the firm may raise new financing later on, if required. One problem with such an explanation is that the possibility of squandering value arises in almost all firms, and if this were the sole driving factor, unit offerings should be the financing vehicle of choice for most firms rather than being confined to high risk firms in speculative industries. This suggests that agency cost based arguments *alone* cannot explain the use of unit IPOs.

Our paper is related to the theoretical literature that explains the well-documented "underpricing" of initial public offerings.<sup>1</sup> Grinblatt and Hwang (1989) also uses a framework similar to that of Leland and Pyle (1977), along with the assumption of differences in riskiness across firm types. Unlike our paper, however, it also assumes that the firm makes a second, seasoned offering soon after the IPO with some probability of the revelation of true firm type between the initial and seasoned offerings. Thus, while Grinblatt and Hwang (1989) also shows that underpricing, along with the fraction of firm value retained by firm insiders, can serve to signal true firm value, our paper is the first to demonstrate that underpricing may serve as a signal in the context of a *one-shot equity* offering. A second, seasoned equity offering therefore is not a necessary ingredient to generate underpricing as a signal in IPOs. Two other papers that have the extent of underpricing as a signal are Welch (1989) and Allen and Faulhaber (1989). However, in these papers, insiders are risk neutral, and the riskiness of the firm's cash flows is irrelevant. The driving force that generates underpricing as a signal in these models is a second equity offering, with some probability of an exogenous release of additional information about true firm value between the IPO and the seasoned offering. The intuition driving these papers is not therefore directly related to the one in this paper.

This is the first paper to analyze the optimality of packaging warrants along with *equity* offerings, thus making a contribution to the security design literature as well. Further, while our model is developed in the context of new issues of equity, it is also applicable with minor modifications to seasoned issues of equity packaged with warrants. Therefore, our paper also has implications for such equity

<sup>1</sup>The underpricing of IPOs has been documented by a number of papers (see Ibbotson, Sindelar, and Ritter (1988) for a survey of the empirical evidence). Theoretical models of IPO underpricing are presented in Allen and Faulhaber (1989), Benveniste and Spindt (1989), Chemmanur (1993), Grinblatt and Hwang (1989), Hughes and Thakor (1992), Mauer and Senbet (1992), Rock (1986), Spatt and Srivastava (1991), and Welch (1989). Unlike these papers, our primary objective here is to analyze the optimality of unit IPOs (though underpricing arises in equilibrium here as well).

issues.<sup>2</sup> Further, our intuition generalizes readily to provide a new rationale for packaging call-option-like claims with risky securities other than equity as well (e.g., debt with warrants, convertible debt). A well-known explanation for issuing debt with warrants or convertible debt is the “hidden-action” argument provided by Green (1983), which argues that such claims are a way of controlling firm insiders’ incentives to overinvest in riskier projects, which arise from the presence of debt in the firm’s capital structure. Of course, such arguments would not provide a rationale for issuing warrants along with *equity*: the risk-shifting incentive driving Green’s and other related analyses will clearly not be present when there is no debt involved.<sup>3</sup>

The rest of the paper is organized as follows. Section II presents the essential features of the model. Section III characterizes the nature of the equilibrium that will obtain under different settings. Section IV develops a detailed analysis of the equilibrium involving unit IPOs. Section V concludes, after summarizing the testable implications of our model. The proofs of all propositions are in the Appendix.

## II. The Model

The model has two dates (time 0 and time 1). At time 0, entrepreneurs, who are risk averse, take their private firms “public” by selling equity to risk-neutral outsiders.<sup>4</sup> Each firm has a positive NPV project that can be implemented by investing an amount  $K$  at this date. For simplicity, we normalize the total number of shares in each firm to 1; entrepreneurs offer a fraction  $(1 - \alpha)$  of the firm’s equity to the public in the IPO, retaining the remaining fraction  $\alpha$  of equity.

Firms are of two types: type G (“good”) and type B (“bad”) firms. The firms differ in both the mean and the variance of the distribution of future cash flows. At time 0, the equity market is characterized by asymmetric information: while outside investors cannot distinguish between the two types of firms, entrepreneurs (“firm insiders”) know the types of their own firms. However, at time 0, even entrepreneurs are uncertain about their firm’s time 1 value. We assume a simple three-point probability distribution for each firm’s time 1 value: for the type G firm, the value at time 1 will be one of  $\mu^G + \delta/p_H$  (with a probability  $p_H$ ),  $\mu^G$  (with probability  $p_M$ ), or  $\mu^G - \delta/p_L$  (with probability  $p_L$ ). We will often refer to these

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<sup>2</sup>The impact of asymmetric information, though significant, can be expected to be less severe for seasoned equity issues than for IPOs. Therefore the phenomena we model here will also be less pronounced for seasoned issues. However, Jayaraman, Shastri, and Tandon (1991) documents that a significant number of seasoned firms make offerings of equity packaged with warrants. Also, Smith (1977) (among several others) documents that the offering price of seasoned equity is set below the market value of the equity; the study finds a statistically significant abnormal return from the offer price to the closing price of the offer date (though the magnitudes involved are smaller compared to the underpricing of IPOs).

<sup>3</sup>Other rationales for issuing convertible debt, also based on asymmetric information but unrelated to ours, are provided by Brennan and Kraus (1987) and Constantinides and Grundy (1989), which argue that issuing convertible debt may allow the firm to reveal its type costlessly. See also Stein (1993), which assumes the existence of significant deadweight costs of financial distress to drive a signaling equilibrium involving the use of convertible debt.

<sup>4</sup>While the assumption of risk-neutral outsiders serves to simplify our computations significantly, it does not drive any of our results. The intuition behind our results goes through even if outsiders are risk averse.

possible realizations of time 1 firm value as high, medium and low “states.” For type B firms, the time 1 value will be one of  $\mu^B + \epsilon/p_H$ ,  $\mu^B$ , or  $\mu^B - \epsilon/p_L$  (with the same state probabilities  $p_H$ ,  $p_M$ , and  $p_L$ ). Clearly,  $\delta$  is a measure of the variance of the time 1 value for a type G firm (since the variance will be larger for a larger  $\delta$ ); similarly  $\epsilon$  is a measure of the variance of the time 1 value of the type B firm. We will therefore refer to  $\delta$  and  $\epsilon$  as the “risk parameters” or “riskiness” of the two types of firms from now on. We will assume throughout that the type G firm has a greater mean time 1 value than the type B firm (i.e.,  $\mu^G > \mu^B$ ).

At time 0, insiders observe their own firm type and they know both the true mean  $\mu^G$  or  $\mu^B$  and the risk parameter  $\delta$  or  $\epsilon$  of their firm; they do not know, however, which firm value state will occur at time 1. Outsiders, on the other hand, do not know the type of a given firm approaching the equity market; based on publicly available information, they form a prior probability assessment of the firm making the IPO a type G firm. They may revise these prior beliefs after observing the insiders’ actions, namely, the package of securities issued in the IPO, the price of these securities, and the fraction of the firm’s equity  $\alpha$  retained by insiders. The state probabilities  $p_H$ ,  $p_M$ , and  $p_L$ , as well as other model parameters, are common knowledge. At time 1, the firm’s true value is realized, and is observed by both insiders and outsiders, and all asymmetric information is resolved.

Insiders may choose to sell either equity alone, or a combination of equity with a certain number  $\gamma$  of warrants in the IPO. We assume that any warrants sold expire at time 1, after the resolution of information asymmetry and the uncertainty about the state. Investors holding warrants will exercise them if (and only if) these warrants are in-the-money at time 1, so that the number of shares in the firm will then be  $(1 + \gamma)$ . Therefore,  $\theta \equiv \gamma/(1 + \gamma)$  represents the fraction of the firm value surrendered to warrant holders upon exercise. We denote the exercise price of the warrants by  $k$ . To minimize computational complexity, we will restrict the insiders’ choice of warrant exercise price to values greater than  $\mu^G$ , thus ensuring that the warrants are in-the-money only in the high state.<sup>5</sup>

Now, for warrants to play any signaling role in our model, it is important that even the type B firm’s warrants be in-the-money for *some* realization of the firm’s time 1 value. Otherwise, knowing that their warrants will never be in-the-money, the type B firm can always issue warrants whenever the type G firm does so, thus enabling it to mimic the type G firm costlessly. We will therefore impose the parametric restriction that  $\mu^B + \epsilon > \mu^G$ , ensuring that the type G firm can choose an exercise price greater than or equal to  $\mu^G$  for which the type B firm’s warrants are also in-the-money with a positive probability.<sup>6</sup> Thus, if warrants are issued in equilibrium, type G firm insiders will always choose an exercise price in the range  $\mu^G \leq k < \mu^B + \epsilon$ . These warrants will be in-the-money if the time 1 firm value

<sup>5</sup>It is straightforward, in principle, to relax this restriction on the range of possible exercise prices. This would involve solving the type G insiders’ optimization problem in various steps, picking that value of  $k$  that is optimal for each possible range of  $k$  values, and then choosing from among these constrained optimal  $k$  values the one that gives them the greatest expected utility.

<sup>6</sup>Intuitively, warrants can play an important informational role in stock issues even when this parametric restriction is not satisfied, since the only requirement is that there be some overlap in the possible time 1 values of type G and type B firms. Thus, the parametric restriction we impose here is stronger than required for the intuition behind our model to go through. It is needed only because of the assumption made earlier confining the range of exercise prices to values above  $\mu^G$ .

is high, and out-of-the-money otherwise, regardless of whether the issuing firm is of type G or type B. (The amount of the payoff will, of course, be higher for a warrant issued by a type G firm than that issued by a type B firm.)

We assume, without loss of generality, that the risk-free rate is zero; outsiders therefore value the package of securities issued by the firm at the expected value, conditional on their equilibrium beliefs, of the stream of future cash flows that accrues to these securities. This is, therefore, the highest price that any rational investor is willing to pay for the securities issued; a package of securities offered at a higher price will be rejected by the capital market. We assume that no firm would choose to set the price of its securities in the IPO at such a level that the offering is rejected. (This is consistent with any non-zero frictional cost of issuing securities.)

### A. The Insiders' Objective

At time 0, firm insiders choose the fraction  $(1 - \alpha)$  of equity to be sold, the fraction of the firm's value to be surrendered to warrants in the event of exercise,  $\theta$ , the exercise price  $k$  of each warrant, and the price of the package of securities issued,  $V$ . The insiders' objective is to maximize the expected value of their end-of-period (time 1) utility, which in turn, depends on their time 1 wealth. We will assume that insiders invest their net wealth after the IPO (i.e., their initial wealth before the IPO,  $w_0$ , plus the proceeds from the stock issue  $V$ , minus the project investment  $K$ ) in the risk-free asset.<sup>7</sup> Thus, in our model, the insiders' only alternative to investing in the equity of their own firm is to invest in the risk-free asset.<sup>8</sup> We assume that insiders' utility functions are characterized by constant absolute risk aversion,

$$(1) \quad U(\tilde{w}_1) = -e^{-A\tilde{w}_1},$$

where  $A$  is the coefficient of absolute risk aversion, and  $\tilde{w}_1$  is their (uncertain) time 1 wealth level.<sup>9</sup>

The time 1 wealth level of firm insiders clearly depends on the realization of the firm's time 1 value, since it determines the value of the equity they retain in the firm after the IPO. Let  $w_H^G$ ,  $w_M^G$ , and  $w_L^G$ , respectively, denote the three possible

<sup>7</sup>As is standard in this literature (see, e.g., Leland and Pyle (1977)), the investment in the risk-free asset can be positive or negative. Also, setting  $K = 0$  addresses the case where the purpose of the IPO is only to diversify the insiders' portfolio (and not to raise any capital).

<sup>8</sup>Our results go through even if we assume (along the lines of Leland and Pyle (1977)) that insiders may choose to allocate their net wealth after the IPO between the market portfolio and the risk-free asset (in addition to holding equity in their own firm). We choose not to add this unnecessary complication to the model, and focus instead on the effect of issuing warrants in the IPO.

<sup>9</sup>The assumption of constant absolute risk aversion simplifies the analysis in general, and in particular, facilitates the development of comparative static results. However, our main results (propositions 1 through 4) go through for any risk-averse utility function. Leland and Pyle (1977) and Grinblatt and Hwang (1989) also assumes that firm insiders have constant absolute risk aversion, and combines this assumption with the additional assumption that stock returns are normally distributed, so that this analysis proceeds in a mean-variance framework. Since, unlike those papers, we allow the firm to issue warrants along with equity in a setting of asymmetric information, assuming normally distributed stock returns would complicate our analysis considerably rather than simplifying it.

realizations of the type G firm insiders' time 1 wealth level  $\tilde{w}_1$  (corresponding to the high, medium, and low firm value states). These are given by

$$(2) \quad w_H^G \equiv w_0 + \alpha \frac{\mu^G + \delta/p_H + \gamma k}{1 + \gamma} + V - K$$

$$= w_0 + \alpha (\mu^G + \delta/p_H) - \alpha \theta (\mu^G + \delta/p_H - k) + V - K,$$

$$(3) \quad w_M^G \equiv w_0 + \alpha \mu^G + V - K,$$

$$(4) \quad w_L^G \equiv w_0 + \alpha (\mu^G - \delta/p_L) + V - K.$$

In each case above, the insiders' time 1 wealth in a particular state is given by the sum of the value of their equity holdings in the firm in that state and the value of their investment in the risk-free asset; notice that the expression for  $w_H^G$  incorporates the effect of warrant conversion, which occurs in the high state, on the firm's equity value. Similarly, the three possible realizations of the time 1 wealth level of insiders of a type B firm, denoted by  $w_H^B$ ,  $w_M^B$ , and  $w_L^B$ , are given by the analogues to equations (2), (3), and (4), respectively, with  $\mu_G$  replaced by  $\mu_B$ , and  $\delta$  replaced by  $\epsilon$ , in each equation.

The time 1 expected utility of insiders of type G and type B firms, denoted by  $\bar{U}^G$  and  $\bar{U}^B$ , respectively, are now given by

$$(5) \quad \bar{U}^G(\alpha, \theta, V, k) \equiv p_H U \left[ w_0 + \alpha \left( \mu^G + \frac{\delta}{p_H} \right) (1 - \theta) + \alpha \theta k + V - K \right]$$

$$+ p_M U \left[ w_0 + \alpha \mu^G + V - K \right]$$

$$+ p_L U \left[ w_0 + \alpha \left( \mu^G - \frac{\delta}{p_L} \right) + V - K \right],$$

$$(6) \quad \bar{U}^B(\alpha, \theta, V, k) \equiv p_H U \left[ w_0 + \alpha \left( \mu^B + \frac{\epsilon}{p_H} \right) (1 - \theta) + \alpha \theta k + V - K \right]$$

$$+ p_M U \left[ w_0 + \alpha \mu^B + V - K \right]$$

$$+ p_L U \left[ w_0 + \alpha \left( \mu^B - \frac{\epsilon}{p_L} \right) + V - K \right].$$

## B. The Full Information Outcome

Before characterizing the solution to the insiders' problem under asymmetric information, it is useful to note the solution under full information (i.e., in the case where outsiders observe firm type at time 0). Denote by  $V^G$ , the proceeds of the type G firm's IPO under full information, which will simply be the expected value of the stream of cash flows that will accrue to the holders of the package of equity and warrants in the type G firm at time 1,

$$(7) \quad V^G(\alpha, \theta, k) \equiv (1 - \alpha) \mu^G + \alpha p_H \theta (\mu^G + \delta/p_H - k).$$

The full information value of any package of securities sold by the type B firm, denoted by  $V^B$ , is given by an expression similar to (7), with  $\mu^G$  replaced by  $\mu^B$ , and  $\delta$  replaced by  $\epsilon$ . Lemma 1 characterizes the full information outcome.



*Lemma 1.* Under full information,  $\alpha^* = 0$ ,  $\theta^* = 0$ , optimizes the objectives of both types of firms. Further, the type G firm prices its equity at  $\mu^G$ , and the type B firm prices its equity at  $\mu^B$ .

In the full information setting, insiders, being risk averse, choose to divest their equity holdings in the firm completely, since the time 1 value of the firm is uncertain. There is no reason for issuing warrants in the absence of asymmetric information. It is the interaction between asymmetric information and risk aversion on the part of insiders that provides a rationale for issuing warrants in our model.

### III. Equilibrium under Asymmetric Information

We now proceed to the analysis of the insiders' problem under asymmetric information. Under asymmetric information, insiders of type G firms may find it optimal to distinguish themselves from type B firms, which have a lower intrinsic value; type B firms, on the other hand, have an incentive to mimic the more valuable type G firms, unless it is too costly for them to do so. We will focus here only on separating equilibria, where the type G firm structures the IPO in such a way that any attempt by the type B firm to mimic it imposes such a high cost on the type B firm that it is deterred from doing so, and instead sells its securities in the IPO at their true full information value. Thus, equilibrium strategies and beliefs in our model are defined as those that constitute a separating sequential equilibrium (see Kreps and Wilson (1982)), and are such that the dissipative costs of separation incurred are the least. In other words, the equilibrium concept used is that of a Pareto dominant or efficient separating sequential equilibrium. Using this equilibrium concept ensures that investors' beliefs in response to out-of-equilibrium moves by firm insiders are explicitly specified, and allows us to rule out equilibria with excessive, inefficient amounts of signaling by the type G firm. It also rules out equilibria where the type B firm, even though revealed as type B, deviates from its full information optimal price for the package of securities sold in the IPO. (See Milgrom and Roberts (1986) and Engers (1987) for a detailed discussion of why this is the appropriate equilibrium concept in signaling games).

Thus, an equilibrium in our model emerges as the solution to the following *non-mimicry problem* faced by type G firm insiders,

$$(8) \quad \max_{\alpha, \theta, V, k} \bar{U}^G(\alpha, \theta, V, k),$$

subject to

$$(9) \quad \bar{U}^B(\alpha, \theta, V, k) \leq U(\mu^B),$$

$$(10) \quad V \leq V^G(\alpha, \theta, k),$$

$$(11) \quad 0 \leq \alpha \leq 1, \quad 0 \leq \theta \leq 1, \quad V \geq 0, \quad k \geq \mu^G.$$

Constraint (9) is the non-mimicry constraint, which ensures that the value of the type B firm's objective if they mimic the type G firm is less than or equal to that obtained by choosing the full information values of  $\alpha$ ,  $\theta$ , and  $V$ . Constraint (10), which we refer to as the competitive rationality constraint, reflects the notion that the maximum price that outsiders will pay for the package of securities issued by

a firm that has been revealed to be of type G is  $V^G$ , the full information value. Finally, the constraints in (11) reflect the range of allowable values of  $\alpha$ ,  $\theta$ ,  $V$ , and  $k$ . The equilibrium that meets our requirements therefore involves the type B firm picking its full information optimum values of  $\alpha$ ,  $\theta$ ,  $V$ , and  $k$  (characterized in lemma 1), while the type G firm does just enough signaling to distinguish itself from the type B firm. Further, given this equilibrium behavior by issuers, the equilibrium beliefs of outside investors are such that they infer true firm type with probability 1. Finally, if investors observe any firm choosing out-of-equilibrium values for  $\alpha$ ,  $V$ , or  $\theta$ , they infer with probability 1 that it is a type B firm, and value its securities accordingly. It is easy to verify that such beliefs by outsiders when confronted with out-of-equilibrium actions by issuing firms can support the equilibria that we will describe in propositions 1 to 4 as sequential equilibria. We now characterize the equilibrium under various alternative settings.

#### A. The No-Underpricing Equilibrium

In this section, we assume, as in Leland and Pyle (1977), that the firm is allowed to sell only equity, and the variability of future cash flows of the type G firm is the same as that for the type B firm ( $\delta = \epsilon$ ). Unlike Leland and Pyle (1977), however, we allow the firm to underprice its equity. In later sections, we will relax these assumptions, allowing for the riskiness of the two types to be different, and for the firm issuing warrants in IPO as well. The equilibrium we characterize in proposition 1 will thus serve as a benchmark of comparison and as a starting point for our discussion of the equilibria that prevail under those more general situations.

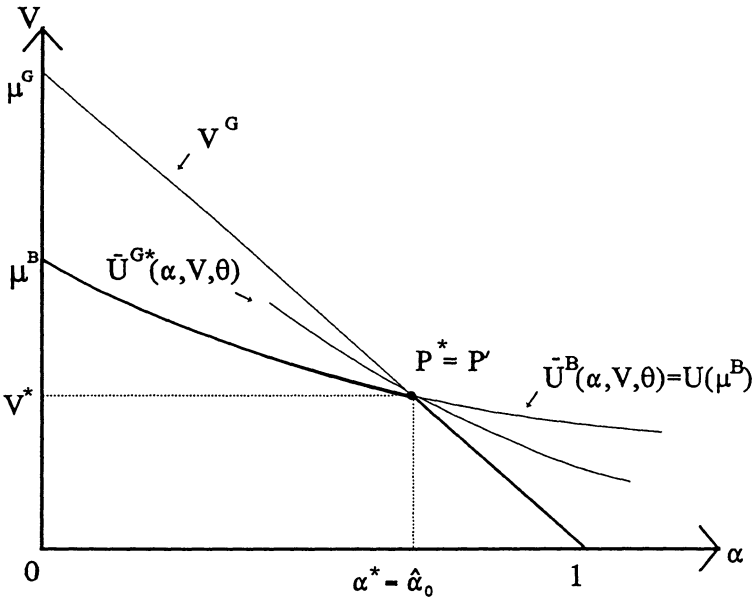
*Proposition 1 (Equilibrium with one signal).* If  $\delta = \epsilon$ , the separating equilibrium involves the type G firm retaining a positive fraction of the firm's equity ( $\alpha^* > 0$ ), and will not involve any equity underpricing (i.e.,  $V^* = V^G$ ).

The equilibrium that emerges here is thus akin to that in Leland and Pyle (1977), where the fraction of equity retained is the only signal. The above proposition demonstrates, however, that if the riskiness of the two firm types is the same, the firm will not underprice its equity even if allowed to do so, in contrast to Leland and Pyle (1977), where underpricing is exogenously ruled out.<sup>10</sup>

The intuition behind proposition 1 can be seen from Figure 1, which depicts the equilibrium choice of the firm between the two signals,  $\alpha$  and  $V$ , when the riskiness of the type G firm equals that of the type B firm ( $\delta = \epsilon$ ). Here, the straight line  $\mu^G - 1$  represents the full information value  $V^G$  of the equity issued by the type G firm as a function of the fraction of equity retained by insiders,  $\alpha$ . This line is downward sloping since, as the firm sells a larger fraction of the equity to outsiders (i.e., as  $\alpha$  falls), the proceeds from the sale of equity increase. The indifference curves of type G as well as the type B firm insiders are also downward sloping, since, as insiders sell a larger fraction of the firm's equity to

<sup>10</sup>For convenience of comparison with Leland and Pyle (1977), we have presented proposition 1 in a setting where equity is the only security issued by the firm. However, it can be shown that even when the firm is allowed to issue a combination of equity and warrants, there will be no underpricing of this package of securities if  $\delta = \epsilon$ . Further, this more general result holds also for the case where  $\delta < \epsilon$ , provided that  $\epsilon \geq \delta \geq \epsilon - p_H(\mu^G - \mu^B)$ . Interested readers are referred to Chemmanur and Fulghieri (1995) for a proof.

FIGURE 1



This figure illustrates the equilibrium when the two types of firms have the same riskiness (so that the indifference map of type G insiders is steeper than that of type B insiders) and the firm is exogenously constrained to issue equity alone (no warrants). The feasible region in this case is the area below the thick curve  $0\mu^B P' 1$ , enclosed by the full information value line  $\mu^G 1$  (representing the competitive rationality constraint), the type B indifference curve corresponding to the expected utility level  $U(\mu^B)$  (representing the incentive compatibility constraint), and the two axes (representing the non-negativity constraints). The equilibrium point  $P^*$  in this case is the same as the single signal equilibrium point  $P'$  (i.e., the equilibrium is the same as the one that would have obtained if the firm were exogenously constrained not to set the price of its equity below its full information value). Since the type G insiders' indifference map is steeper than that of type B insiders, the marginal benefit of cutting back on the fraction of equity retained (at the expense of a lower equity price) is greater for type B insiders than for type G insiders; it is, therefore, not possible to find a point in the feasible region that yields type G firm insiders a greater utility level compared to that associated with the point  $P'$  (thus ensuring that the equilibrium remains at  $P'$  even when underpricing is allowed).

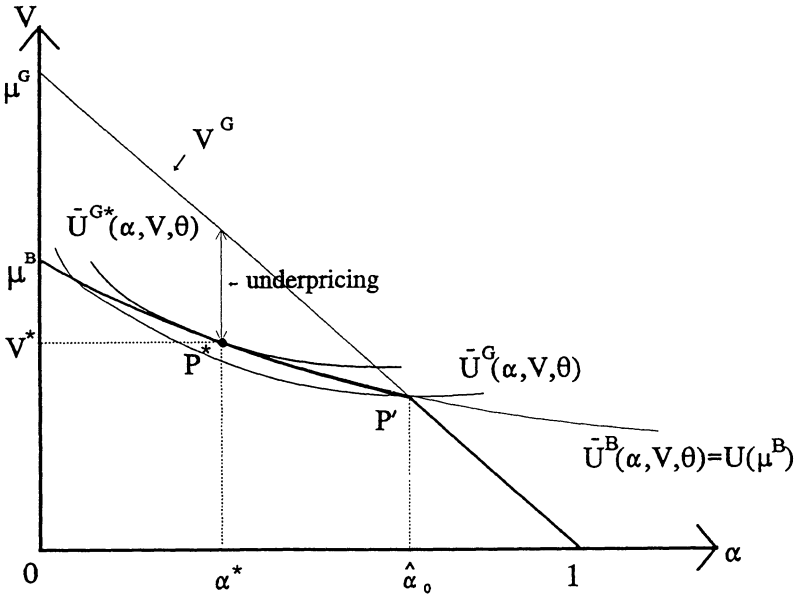
outsiders, the total amount received from the sale,  $V$ , also has to be larger in order to maintain a given expected utility level. For  $\alpha$  close to 1, the insiders' marginal benefit from diversification is very high, so that the price they need to receive for each additional percentage of equity divested in order to maintain a given expected utility level is low (i.e., the magnitude of  $\partial V/\partial\alpha$  is small). As  $\alpha$  falls, however, the marginal benefit from diversifying also falls, so that the price they need to receive, to maintain a given expected utility level, for each additional percentage of equity divested increases (i.e., the magnitude of  $\partial V/\partial\alpha$  increases as  $\alpha$  falls). Thus, the insiders' indifference curves are flattest for  $\alpha$  close to 1, and become steeper for smaller values of  $\alpha$ . Notice also that, for any given  $\alpha$ , the slope of the insiders' indifference curve depends on the riskiness of their firm's future cash

flows. Thus, if  $\delta = \epsilon$  (as in Figure 1), the type B insider's indifference curves are always flatter than that of the type G. Since type G firms have a greater mean future cash flow, insiders of such firms will need to receive a greater price than type B insiders for any given fraction of equity sold in the IPO in order to maintain a given utility level. (In contrast, if  $\delta$  is significantly larger than  $\epsilon$ , then the type G insiders' indifference map will be flatter than that of type B insiders, as depicted in Figure 2. The ordering is reversed because the benefits from diversifying are greater in that case for type G insiders than for type B insiders, thus dominating the effect of the type G firm having a greater mean future cash flow than the type B firm.)

Now, to begin with, assume that the type G firm is exogenously constrained to use only the fraction of equity retained,  $\alpha$ , as a signal: i.e., the firm is constrained to set the price of equity sold,  $V$ , equal to the full information value,  $V^G$  (no underpricing). In this case, the equilibrium will be at the point  $P'$  in Figure 1, which is the intersection of the full information line  $\mu^G - 1$  and the type B insiders' indifference curve at the utility level  $U(\mu^B)$ , which they would receive if they divested the firm's entire equity at its true value  $\mu^B$ . This point  $P'$  satisfies the competitive rationality constraint (10) (as an *equality*, since there is no underpricing here) as well as the incentive compatibility constraint (9) and the constraints in (11). Thus, in this single signal equilibrium, type G insiders retain a fraction of equity just large enough to dissuade the type B insiders from mimicking them (type B insiders have no incentive to mimic the type G firm if they hold the fraction of the equity represented by  $P'$ , since they can receive the same expected utility simply by divesting their firm's entire equity at its full information value  $\mu^B$ ). From now on, we will use  $\hat{\alpha}_0$  to denote this single signal equilibrium fraction of the equity retained by type G insiders. Since, in this case, we have exogenously ruled out any underpricing (in addition to assuming that the firm is allowed to issue only equity, and  $\delta = \epsilon$ ), the single signal equilibrium here is essentially the same as that in Leland and Pyle (1977). Such a separating equilibrium involving the signal  $\alpha$  alone exists because the marginal cost of maintaining a greater  $\alpha$  is greater for type B firm insiders than type G insiders.

Now, suppose we allow the type G firm to underprice its equity as well. The competitive rationality (10) constraint now needs to be satisfied only as a weak inequality. However, the equilibrium would not shift inward from point  $P'$  in Figure 1 even now. To see why, notice that, even when underpricing is allowed, the area below the thick curve  $0\mu^B P' 1$  represents the  $\alpha, V$  combinations satisfying the incentive compatibility, competitive rationality, and other constraints. Given this feasible region, and given that the type G firm's indifference map is steeper than that of the type B firm, it is not possible to find another type G indifference curve that yields insiders a greater utility level compared to that associated with the one passing through  $P'$  while simultaneously deterring the type B firm from mimicking. Thus,  $\alpha^* = \hat{\alpha}_0$ , and the type G firm does not underprice its equity in equilibrium.

FIGURE 2



This figure illustrates the case where the type G firm is significantly riskier than the type B firm (so that type G insiders' indifference map is flatter than that of type B insiders), and the firm is exogenously constrained to issue equity alone (no warrants). To start with, let the firm be allowed to use only the fraction of equity retained,  $\alpha$ , as a signal (no underpricing). Since the firm is constrained in this case to sell its equity at the full information value, the single signal equilibrium point is at  $P'$ , the intersection of the full information value line  $\mu^G 1$  and the type B indifference curve corresponding to the expected utility level  $U(\mu^B)$ . If the type G firm is now allowed to underprice its equity as well, then the feasible region becomes the area enclosed by the thick curve  $0\mu^B P^* P' 1$  (i.e., the entire area enclosed by the full information value line  $\mu^G 1$ , the type B indifference curve corresponding to the expected utility level  $U(\mu^B)$ , and the two axes). Since the type G insiders' indifference map is flatter in this case than that of type B insiders (implying that the marginal benefit of cutting back on the fraction of equity retained, at the expense of a lower equity price, is greater for type G insiders than for type B insiders), it is possible to find a point in the feasible region that yields type G firm insiders a greater expected utility level compared to that associated with the point  $P'$ . The equilibrium point therefore moves to this point of maximum expected utility,  $P^*$ , where the type G indifference map is tangent to the type B indifference curve corresponding to the utility level  $U(\mu^B)$ . The equilibrium fraction of equity retained thus falls to  $\alpha^*$ , and the firm underprices its equity in equilibrium.

### B. One-Shot IPOs with Equity Underpricing

We now study the equilibrium outcome in the case where the riskiness of the type G firm is significantly greater than that of the type B firm (also, we will continue to maintain the assumption that the firm is exogenously constrained not to issue warrants).

*Proposition 2 (Equilibrium with two signals).* If the riskiness of the type G firm is sufficiently greater than that of the type B firm, the equilibrium involves underpricing the firm's equity ( $V^* < V^G$ ). In this case, the equilibrium proportion of

equity retained by insiders will be positive, but lower than the fraction that would be retained if insiders were constrained to use the fraction of equity alone as a signal (i.e.,  $\alpha^* < \hat{\alpha}_0$ ).

The intuition behind this proposition can be seen from Figure 2, which depicts the equilibrium choice of the firm between the two signals,  $\alpha$  and  $V$ , when the riskiness of the type G firm is significantly greater than that of the type B firm ( $\delta > \epsilon$ ). In this case, the feasible region consisting of the set of  $\alpha, V$  values satisfying the incentive compatibility condition (9), the competitive rationality constraint (10), as well as the other constraints is given by the area below the thick curve  $0\mu^B P^* P' 1$ . To arrive at the equilibrium in this case, we can think of the type G firm as starting out at the single signal equilibrium point  $P'$  in Figure 2 (the intersection of the full information value line  $\mu^G 1$  and the type B indifference curve corresponding to the expected utility level  $U(\mu^B)$ ), and moving inward (along the type B firm's indifference curve through  $P'$ ) by cutting back on the equity retained by insiders and underpricing at the same time to prevent the type B firm from mimicking. Recall that the type G firm is significantly riskier in this case than the type B, resulting in the type G insiders' indifference map being flatter than that of type B insiders. Consequently, the marginal benefit of cutting back on the fraction of equity retained, at the expense of a lower equity price, is greater for type G insiders than for type B insiders. It is therefore possible to find a point in the feasible region that yields type G firm insiders a greater expected utility level compared to that associated with point  $P'$ . The equilibrium moves to the point of maximum expected utility,  $P^*$ , where the type G indifference map is tangent to the type B indifference curve corresponding to the utility level  $U(\mu^B)$ . The equilibrium fraction of equity retained thus falls to  $\alpha^*$ , and the firm underprices its equity in equilibrium.

It is useful to place proposition 2 in the context of previous models that obtain underpricing as a signal (Allen and Faulhaber (1989), Grinblatt and Hwang (1989), and Welch (1989)). In these models, the IPO occurs at time 0, full resolution of information asymmetry occurs at the final date, and there is a second, seasoned offering by the firm after the IPO at an intermediate date, with a positive probability of additional information about true firm type arriving between the two equity issues. In contrast to these models, proposition 2 demonstrates that underpricing may arise as part of the efficient signaling mix even in the context of a one-shot equity offering. A seasoned equity offering is not a necessary ingredient required to generate the use of underpricing as a signal.<sup>11</sup>

<sup>11</sup>To make a closer comparison between proposition 2 and the results of the above models, it is useful to match-up the time line of our model with those of the above models (after ruling out the issuing of warrants by the firm, to facilitate such a comparison). If we do so, time 0 in our paper coincides with time 0 in those models, with the final date (time 1) in our model coinciding with the final date (time 2) in those models (i.e., we can think of time 1 in our model as being far into the future, so that firm insiders hold a fraction  $(1 - \alpha)$  of the equity in the firm indefinitely). The essential difference, then, between our time line and those of the above models is that we have no intermediate date (at which revelation of additional information about firm type, and a subsequent seasoned offering is assumed to take place in the above models). It thus becomes evident that, if insiders are risk averse, and the type G firm is riskier than the type B firm, a separating equilibrium with underpricing as a signal arises in a *one-shot equity offering* as well.

### C. Unit IPOs and IPOs without Warrants

This section reintroduces the possibility of the firm issuing warrants. Firm insiders may use any combination of the three signals,  $\alpha$ ,  $V$ , and  $\theta$ , to distinguish the type G firm from the type B firm. In this case, the type G firm's problem is to choose that triplet  $\{\alpha^*, V^*, \theta^*\}$ , which maximizes the expected utility of firm insiders, while ensuring that the type B firm does not mimic.

*Proposition 3 (Unit IPOs with underpricing and a positive fraction of equity retained).* There exists a certain value  $\widehat{\delta}$  (where  $\widehat{\delta} > \epsilon$ , and is defined after (A-7)) such that if the riskiness  $\delta$  of the type G firm exceeds  $\widehat{\delta}$ , then the equilibrium strategy of the firm involves the following:<sup>12</sup>

- (a) the firm issues equity with warrants (i.e.,  $\theta^* > 0$ );
- (b) the strike price of the warrants,  $k$ , is set equal to  $\mu^G$ ;
- (c) the package of equity and warrants is underpriced (i.e.,  $V^* < V^G$ ).
- (d) firm insiders retain a positive fraction of the firm's equity ( $1 > \alpha^* > 0$ ).

Such a separating equilibrium will always exist, provided that  $\mu^G - \mu^B > (\delta - \epsilon)/(1 - p_H)$ .<sup>13</sup>

In order to see how issuing warrants along with equity allows the type G firm to signal firm value more efficiently, it is useful to study how issuing warrants differs as a signaling device from underpricing equity. We saw before that underpricing may be part of the optimal combination of signals because giving away value by underpricing allows the type G firm to cut back on  $\alpha$ , the fraction they retain in the firm, while deterring the type B firm from mimicking. Now, underpricing *equity* amounts to dissipating value in *all* states. In contrast, issuing warrants allows firm insiders to yield a part of the firm's time 1 value to outsiders *in the high state alone*. This ability to yield firm value only in the high state becomes attractive when the insiders' assessment of firm riskiness is very large. This is because insiders' private valuation of each dollar of firm value is lowest in the high state, and consequently, packaging warrants along with equity in the IPO allows high risk type G firms to satisfy the incentive compatibility condition more efficiently.

Two broad factors determine whether warrants are part of the equilibrium package of securities issued in the IPO. First, for  $\delta > \epsilon$ , the marginal utility of income in the high state is lower for type G firm insiders than for type B firm insiders. However, the marginal utility of income in the low state is higher for type G insiders than for type B insiders. In other words, the *private valuation* of every dollar of cash flow that they have to yield to outsiders in the high state is lower for type G insiders than for type B insiders, while the inequality is reversed for the low state. This factor, taken alone, would indicate that warrants, which allow the type G firm to sell cash flows in the high state alone, should always be part of the efficient signaling package. Because of the difference in private valuations, packaging warrants along with equity in the IPO would allow the type G firm to

<sup>12</sup>The condition that  $\delta > \widehat{\delta}$  is a sufficient (but not necessary) condition: if this condition is met, the separating equilibrium always involves the issue of warrants along with equity in the IPO; however, such an equilibrium involving warrants *may* exist even when this condition is not met.

<sup>13</sup>This is a sufficient (but *not necessary*) condition, required only to ensure that the solution to the type G insiders' optimization problem leads to their setting  $\alpha^* > 0$ ; such a separating equilibrium *may* exist even when this condition is not satisfied.

cut back on one or both of the other two signals: namely, the fraction of firm value retained by insiders, and the underpricing of the package of securities sold required to deter mimicking by the type B firm.

There is, however, a second factor that determines whether warrants will be a part of the equilibrium package of securities issued in the IPO. The type G firm's expected time 1 value is greater than that of the type B firm ( $\mu^G > \mu^B$ ). Further, since  $\delta > \epsilon$ , type G cash flows are also riskier than type B cash flows. This means that the true magnitude of time 1 cash flows that each warrant is entitled to if the high state is realized is greater for type G warrants than for type B warrants. This second factor, taken alone, would indicate that warrants should not be part of the efficient signaling package, since the *dollar value* of cash flows that they have to yield to each warrant is always higher for the type G firm than for the type B firm. Thus, warrants are issued in equilibrium by type G firms if, and only if, the first factor ("private value effect") dominates the second ("dollar value effect"), which is indeed the case if the riskiness  $\delta$  of the type G firm is higher than that of the type B firm  $\epsilon$ , and exceeds the threshold value  $\hat{\delta}$ .<sup>14</sup> The number of warrants,  $\theta^*$ , is also determined from the tension between these two opposing effects.

In equilibrium, the type G firm sets the exercise price of any warrants issued equal to  $\mu^G$ . The intuition here is that warrants will be most effective as a separating device if the type B firm has to yield as much value as possible for each warrant it issues as part of any attempt to mimic the type G firm. Setting  $k = \mu^G$  accomplishes this, since doing so maximizes the private value effect discussed above, while the dollar value effect is not sensitive to the exercise price of the warrants issued (it can be easily verified that the *difference* in the cash flow yield between type G and type B warrants is independent of  $k$ ).

We have seen so far that issuing warrants allows the type G firm to signal true value more efficiently when the type G firm is riskier than the type B firm. This naturally raises the question: What are the conditions under which the type G firm chooses *not* to include warrants in the equilibrium package of securities issued (thus using only  $\alpha$  and  $V$  as signals)?<sup>15</sup> We provide the answer to this question in proposition 4.

*Proposition 4 (Underpriced IPOs without warrants).* There exists an interval  $(\delta_1, \delta_2)$  lying between  $\hat{\delta}$  and  $\epsilon$  (with  $\delta_1$  and  $\delta_2$  as defined after (A-11) and (A-8), respectively), and a value  $C > 0$ , such that if the riskiness  $\delta$  of the type G firm falls within this interval, and if  $\mu^G - \mu^B > C$ , then the equilibrium strategy of the type G firm involves the following:

<sup>14</sup>Unit IPOs are puzzling to those who try to analyze them based on the intuition that the warrants of riskier, higher value firms are more valuable than those of less risky, less valuable firms: such a casual intuition might suggest (in opposition to the stylized facts about unit IPOs) that it is the *less risky* firm that would issue warrants in a world where outsiders are uncertain about firm riskiness (since, if the market price they obtain from selling warrants is the same, insiders of truly less risky firms would be selling an intrinsically less valuable security than those of more risky firms). We demonstrate here that such an argument is based on only half the story, since the marginal value of each dollar yielded to warrants by the risk-averse insiders of higher risk, higher value firms is also lower than that yielded by the insiders of lower risk, lower value firms.

<sup>15</sup>This is clearly a question different from that addressed by proposition 2, which characterizes the conditions under which the type G firm uses underpricing as a signal in addition to the fraction of equity retained, in a situation where issuing warrants was exogenously ruled out.



- (a) the firm issues equity alone in the IPO ( $\theta^* = 0$ );
- (b) the equity is underpriced ( $V^* < V^G$ ); and
- (c) insiders retain a positive fraction of the firm's equity ( $\alpha^* > 0$ ). Further, the equilibrium fraction retained,  $\alpha^*$ , is always smaller here compared to the case where  $\alpha$  alone is used as a signal ( $\alpha^* < \hat{\alpha}_0$ ).

The intuition behind this proposition is as follows. For given values of  $\mu^G$  and  $\mu^B$ , if the type G firm's riskiness is greater than that of the type B firm but falls in an interval less than the threshold value  $\hat{\delta}$ , the dollar value effect dominates the private value effect so that the efficient signaling mix of the type G firm does not involve issuing warrants. However, the signaling mix does involve the firm selling underpriced equity in addition to insiders retaining a fraction of the equity  $\alpha^*$  in the firm (for the reasons discussed under proposition 2), and consequently, the equilibrium fraction of equity retained,  $\alpha^*$ , will be less than the single signal equilibrium fraction  $\hat{\alpha}_0$ .

#### IV. Analysis of Unit IPOs

We now develop the implications of our model for IPOs in general and unit IPOs in particular. In further discussion, we will use  $u^*$  to denote the proportion of equilibrium underpricing of the package of securities issued, whether equity alone or equity with warrants. Clearly,  $u^*$  is given by the *amount* of underpricing divided by the full information value of the securities issued in the IPO,

$$(12) \quad u^* \equiv \frac{V^G - V^*}{V^G} = 1 - \frac{V^*}{(1 - \alpha^*)\mu^G + \alpha^*p_H\theta^* (\mu^G + \delta/p_H - k^*)}.$$

Propositions 5, 6, and 7 below will present the comparative static results on the equilibrium with unit IPOs (proposition 3). We will then briefly discuss the comparative static results of the equilibrium characterized in proposition 4, where the firm sells underpriced equity alone, even though free to issue warrants as well.

*Proposition 5 (Comparative statics on  $\theta^*$ ).* Holding  $\alpha$  constant, the equilibrium amount of warrants issued  $\theta^*$  is:

- (a) positively related to firm riskiness  $\delta$ , provided that it is greater than a certain value  $\delta_3$  and  $\alpha^* > \underline{\alpha}$  (both values are defined after (A-12)); and
- (b) negatively related to the difference in intrinsic values between the type G and the type B firm, ( $\mu^G - \mu^B$ ).

*Proposition 6 (Comparative statics on  $\alpha^*$ ).* Holding  $\theta$  constant, the equilibrium fraction of equity  $\alpha^*$  retained by insiders is:

- (a) negatively related to firm riskiness  $\delta$ ; and
- (b) positively related to the difference in intrinsic values between the type G and the type B firms ( $\mu^G - \mu^B$ ), provided that  $\mu^G - \mu^B > 1/p_H$ .

*Proposition 7 (Comparative statics on underpricing).* The equilibrium amount of underpricing ( $V^G - V^*$ ) is:

- (a) positively related to firm riskiness  $\delta$ , holding  $\theta$  constant; and

- (b) negatively related to the difference in intrinsic values between the type G and the type B firm,  $(\mu^G - \mu^B)$ , holding  $\theta$  constant, and provided that  $\mu^G - \mu^B > 1/p_H$ .

In addition, for  $u^*$  less than a certain value  $\bar{u}$ , the equilibrium percentage of underpricing is:

- (c) positively related to firm riskiness  $\delta$ , holding  $\theta$  constant;  
 (d) positively related to the amount of warrants issued  $\theta^*$ , holding  $\alpha$  constant; and  
 (e) negatively related to the fraction of equity  $\alpha^*$  retained by insiders, holding  $\theta$  constant.

The intuition behind the above comparative statics on the equilibrium where the type G firm uses unit IPOs is as follows. As firm riskiness  $\delta$  increases, it becomes relatively more expensive (in terms of expected utility) for type G firm insiders to retain equity in the firm, so that they cut back on  $\alpha$ , at the same time increasing the extent of underpricing and warrants in the equilibrium signaling mix, thus ensuring that incentive compatibility is maintained. Similarly, when  $(\mu^G - \mu^B)$  is increased, selling underpriced equity or issuing warrants becomes more expensive for the type G firm relative to the other signal  $\alpha$ , so that the firm cuts back on warrants and underpricing while retaining more of the equity in the firm.

The comparative static results for IPOs without warrants, on the equilibrium characterized in proposition 4, are broadly similar to that presented above. In the case of all equity IPOs also, the extent of underpricing is positive related to firm riskiness, and negatively related to the fraction of equity retained by firm insiders. Further, the fraction of equity retained by insiders is declining in firm riskiness.<sup>16</sup> The intuition driving these results is also quite similar to that underlying the comparative static results discussed for unit IPOs: as firm riskiness  $\delta$  increases, employing the signal  $\alpha$  becomes relatively more expensive, thus leading to a reduction in  $\alpha$  and an increase in underpricing in the equilibrium signaling mix.

## V. Empirical Implications and Conclusion

This paper has analyzed the optimality of unit IPOs, in which the firm going public issues a package of equity with warrants. We showed that, in an equity market characterized by asymmetric information, where insiders have private information about the riskiness as well as the expected value of their firm's future cash flows, high risk firms package their equity with warrants, and the package of equity and warrants is underpriced. Lower risk firms, on the other hand, issue underpriced equity alone.

Our model provides several testable predictions. First, it predicts that in a group of firms that are indistinguishable prior to the IPO, the subset of firms employing unit IPOs will have greater variability of future cash flows compared

<sup>16</sup>Details of these results and their derivations are available in Chemmanur and Fulghieri (1995).

to those that employ share IPOs.<sup>17</sup> Strong empirical support for this prediction is provided by Schultz (1993)). First, he provides evidence regarding the probability of a firm surviving one, two, or three years *after* the IPO. Schultz (1993) defines failure as being delisted from the NASDAQ for various reasons; 88.9% of the firms in his sample that had IPOs of equity alone were still listed three years after the IPO, while only 58.8% of the companies that had unit IPOs were listed. Further, when he estimated the probability of failure using a logistic model, the likelihood of failing was significantly higher for firms that used unit offerings. Finally, Schultz (1993) documents that unit IPOs are associated with smaller firms, tend to be concentrated in speculative industries, have shorter operating histories, and have lower values of sales divided by IPO proceeds and firm assets divided by IPO proceeds.

A second prediction of our model is that, in unit IPOs, the proportion of firm value sold as warrants (i.e., the ratio of the number of shares that warrant holders will receive upon exercise to the total number of shares in the firm) is increasing in firm riskiness, holding constant the fraction of equity retained by insiders. We believe that there is as yet no empirical work testing this hypothesis, so that this prediction can perhaps serve as a test of our model. A third prediction of our model is that in unit IPOs, as well as in IPOs without warrants, the percentage of underpricing is increasing in firm riskiness. Evidence consistent with this implication is provided by Schultz (1992) for unit IPOs; Beatty and Ritter (1986) and Ritter (1991) provide such evidence for IPOs without warrants. A fourth prediction is that, in unit IPOs, the fraction of equity retained by firm insiders is decreasing in firm riskiness, keeping the proportion of firm value sold as warrants constant. Our model implies a negative relationship between firm riskiness and the fraction of equity retained by insiders for IPOs with equity alone as well.

The predictions of our model are consistent with the widely observed fact that unit IPOs tend to be marketed by less prestigious underwriters (see, e.g., Schulz (1993)). Assuming that the economic role of an investment bank underwriting an IPO is that of an information producing intermediary, we know from the existing theoretical and empirical literature on investment bank reputation that more prestigious underwriters will prefer to underwrite the IPOs of less risky firms.<sup>18</sup> Therefore, given that the model here demonstrates that it is the intrinsically more risky firms that choose unit IPOs in equilibrium, it is not surprising to find that such unit offerings are associated with less reputable underwriters.

A secondary contribution made by the paper lies in demonstrating, in contrast to the existing literature, that underpricing may emerge as an equilibrium strategy even if the firm going public does not plan a seasoned equity offering, regardless of whether the IPO involves warrants or not. When the firm making the IPO is very

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<sup>17</sup>Applying our model to seasoned equity issues yields a similar implication: firms characterized by greater uncertainty about future cash flows use packaged offerings of equity and warrants, while less risky firms issue equity alone.

<sup>18</sup>Chemmanur and Fulghieri (1994) develops a model of reputation acquisition by investment banks, who act as information producing intermediaries in an equity market characterized by asymmetric information. Their model demonstrates why more reputable investment banks tend to set stricter standards in terms of the kind of firms for which they underwrite an equity issue. They also demonstrate that the variance in the true value of the equity marketed by an investment bank will be decreasing in its reputation. (Carter and Manaster (1990) provide evidence documenting such a relationship.)

risky, underpricing allows firm insiders to cut back on the fraction of equity that they have to retain in the firm after the IPO, thus increasing insiders' equilibrium welfare relative to the case where the firm uses the fraction of equity alone as a signal.

## Appendix

Consider the non-mimicry problem (11) and the corresponding Lagrangean expression,

$$(A-1) \quad \mathcal{L} = \bar{U}^G + \lambda_1 [U(\mu^B) - \bar{U}^B] + \lambda_2 [V^G - V] + \lambda_3 [1 - \alpha] + \lambda_4 [1 - \theta].$$

The Kuhn-Tucker conditions for that problem are:

$$(A-2) \quad \begin{aligned} \mathcal{L}_\alpha &= \bar{U}_\alpha^G - \lambda_1 \bar{U}_\alpha^B + \lambda_2 V_\alpha^G - \lambda_3 \leq 0, \\ \mathcal{L}_\theta &= \bar{U}_\theta^G - \lambda_1 \bar{U}_\theta^B + \lambda_2 V_\theta^G - \lambda_4 \leq 0, \\ \mathcal{L}_V &= \bar{U}_V^G - \lambda_1 \bar{U}_V^B - \lambda_2 \leq 0, \\ \mathcal{L}_q &= \bar{U}_k^G - \lambda_1 \bar{U}_k^B + \lambda_2 V_k^G \leq 0, \end{aligned}$$

$$(A-3) \quad \begin{aligned} \alpha \mathcal{L}_\alpha + \theta \mathcal{L}_\theta + V \mathcal{L}_V + q \mathcal{L}_q &= 0, \\ \lambda_1 [U(\mu^B) - \bar{U}_B] + \lambda_2 [V^G - V] + \lambda_3 [1 - \alpha] + \lambda_4 [1 - \theta] &= 0, \\ \bar{U}^B \leq U(\mu^B), \quad V \leq V^G, \quad 0 \leq \alpha \leq 1, \quad 0 \leq \theta \leq 1, \\ V \geq 0, \quad q \geq 0, \quad \lambda_i \geq 0, \quad i = 1, 2, 3, 4. \end{aligned}$$

For all  $(\theta, k) \geq 0$ , define  $\alpha_0(\theta, k) > 0$  as the (unique) solution to  $\bar{U}^B[\alpha_0, \theta, V^G(\alpha_0, \theta, k), k] = U(\mu^B)$ . Let  $a(\theta, k) \equiv \min\{\alpha_0(\theta, k); 1\}$  and  $\hat{\alpha}_0 \equiv \alpha_0(0, k) < 1$ . Note that, by definition, a triplet  $\{\theta, k, \alpha_0(\theta, k)\}$  has the property that *both* the incentive-compatibility constraint (9) and the competitive rationality constraint (10) hold as equalities. In the proofs below, we will use the fact that, if  $\alpha < a(\theta, k)$ , constraint (9) is binding while constraint (10) is not. The reverse is true for  $\alpha > a(\theta, k)$ . Hence, choosing  $\alpha > a(\theta, k)$  is dominated by setting  $\alpha = a(\theta, k)$ , since the latter is incentive-compatible, and a type G may sell a greater fraction of the firm at  $V^G$ . Hence, all solutions  $(\alpha, \theta, k, V)$  must satisfy  $\alpha \leq a(\theta, k)$ .

*Proof of Proposition 1.* Consider the reduced version of the non-mimicry problem, where  $\theta \equiv k \equiv 0$ , and  $\delta = \epsilon$ . A solution  $(\alpha^*, V^*)$  with  $\alpha^* > 0$  is an optimal strategy for a type G firm in a separating equilibrium. Note first that at an optimum,  $\lambda_1 > 0$ ; if, on the contrary,  $\lambda_1 = 0$ , the unique solution to the non-mimicry problem is the full information optimum, which is not incentive-compatible. Hence,  $\lambda_1 > 0$  and the incentive-compatibility constraint (9) must be binding. Since  $\hat{\alpha}_0$  is incentive-compatible and dominates  $\alpha = 1$ , a type G prefers to sell at least a fraction  $(1 - \hat{\alpha}_0)$  of the firm at  $V = (1 - \hat{\alpha}_0)\mu^G$ , which implies that  $V^* > 0$  and  $\alpha^* < 1$ . Hence, at an optimum  $\mathcal{L}_V = \lambda_3 = 0$ . From  $\mathcal{L}_V = 0$ , we have  $\lambda_1 = [\bar{U}_V^G - \lambda_2]/\bar{U}_V^B$ ; substituting this value into  $\mathcal{L}_\alpha$ , we obtain,

$$(A-4) \quad \begin{aligned} \frac{\bar{U}_\alpha^G}{\bar{U}_V^G} - \frac{\bar{U}_\alpha^B}{\bar{U}_V^B} (\equiv |\text{MRS}^G(V, \alpha)| - |\text{MRS}^B(V, \alpha)|) &\leq \\ &- \frac{\lambda_2}{\bar{U}_V^G \bar{U}_V^B} [\bar{U}_\alpha^B + \bar{U}_V^B V_\alpha^G], \end{aligned}$$

where  $|\text{MRS}^T(V, \alpha)| = \mu^T - (\rho_L^T - 1)\epsilon / (p_H + p_M \rho_M^T + p_L \rho_L^T)$  is the marginal rate of substitution between  $V$  and  $\alpha$  for  $T = G, B$  and  $\rho_S^T \equiv U'(w_S^T)/U'(w_H^T) > 1$ , for  $S = M, L$ . Uniqueness of  $\alpha_0(\theta, k)$  implies that type B utility decreases with  $\alpha$  along the  $V^G$  line (see Figure 2). Hence,  $\bar{U}_\alpha^B + \bar{U}_V^B V_\alpha^G \leq 0$ , and the RHS of (A-4) is non-negative. Suppose now that  $\alpha^* < \hat{\alpha}_0$  so that  $V^* < V^G$  and  $\lambda_2 = 0$ .  $\delta = \epsilon$  implies that  $\bar{U}_\alpha^G/\bar{U}_V^G - \bar{U}_\alpha^B/\bar{U}_V^B = \mu^G - \mu^B > 0$ , violating (A-4). Hence,  $\alpha^* = \hat{\alpha}_0$  and  $V^* = V^G$ .  $\square$

*Proof of Proposition 2.* Following a procedure similar to the one used in the proof of proposition 1, we obtain again that  $V^* > 0$ ,  $\lambda_1 > 0$ , and  $\mathcal{L}_V = \lambda_3 = 0$ . If  $\hat{\alpha}_0$  is an optimum, (A-4) must be satisfied as an equality and the LHS must be non-negative. Since it may be verified that  $|\text{MRS}^G(V, \alpha)|$  is strictly decreasing in  $\delta$ , there is  $\underline{\delta} > \epsilon$  such that for  $\delta > \underline{\delta}$ , (A-4) is violated at  $\hat{\alpha}_0$ . This implies that  $\alpha^* < \hat{\alpha}_0$  and  $V^* < V^G$  and  $\lambda_2 = 0$ . Finally, at  $\alpha^* = 0$ , we have that  $\rho_L^G = \rho_L^B = 1$  and  $\bar{U}_\alpha^G/\bar{U}_V^G - \bar{U}_\alpha^B/\bar{U}_V^B = \mu^G - \mu^B > 0$ , violating (A-4). Hence,  $\alpha^* > 0$ .  $\square$

*Proof of Proposition 3.* Following a procedure similar to the one used in the proof of proposition 1, we obtain again that  $V^* > 0$ ,  $\lambda_1 > 0$ , and  $\mathcal{L}_V = 0$ . The proof proceeds now in four steps: first, we show that  $k^* = \mu^G$ . Second, we establish that  $V^* < V^G$  and  $\alpha^* < 1$ . Next, we show that  $\theta^* > 0$ . Finally, we prove that  $\alpha^* > 0$  for  $\mu^G - \mu^B > (\delta - \epsilon)/(1 - p_H)$ .

Step 1:  $k^* = \mu^G$ . If at an optimum  $\theta^* > 0$ ,  $\theta \mathcal{L}_\theta = 0$  implies that  $\mathcal{L}_\theta = 0$ . By direct calculation, this implies that  $\mathcal{L}_q < 0$ . Hence,  $q = k - \mu^G = 0$ , and  $k^* = \mu^G$ .

Step 2:  $V^* < V^G$  and  $\alpha^* < 1$ . We now show that  $\alpha^* < a(\theta^*, k^*)$ .  $\alpha^* = a(\theta^*, k^*) > 0$  requires that  $\mathcal{L}_\alpha = 0$ . From  $\mathcal{L}_V = 0$ , substituting  $\lambda_1 = [\bar{U}_V^G - \lambda_2]/\bar{U}_V^B$  into  $\mathcal{L}_\alpha = 0$ , we obtain

$$(A-5) \quad \frac{\bar{U}_\alpha^G}{\bar{U}_V^G} - \frac{\bar{U}_\alpha^B}{\bar{U}_V^B} \left( \equiv |\text{MRS}^G(V, \alpha)| - |\text{MRS}^B(V, \alpha)| \right) = \\ - \frac{\lambda_2}{\bar{U}_V^G \bar{U}_V^B} [\bar{U}_\alpha^B + \bar{U}_V^B V_\alpha^G] + \frac{\lambda_3}{\bar{U}_V^G}.$$

From  $\bar{U}_\alpha^B + \bar{U}_V^B V_\alpha^G \leq 0$  and  $\lambda_3 \geq 0$ , the RHS of (A-5) is again non-negative. We now show that there is a  $\delta_a \geq \epsilon$  such that for  $\delta > \delta_a$  the LHS of (A-5) is negative at  $a(\theta, k^*)$ , for all  $\theta$ . Note that  $|\text{MRS}^B(V, \alpha)|$  does not depend on  $\delta$  and  $\mu^G$ . As in the proof of proposition 2,  $|\text{MRS}^G(V, \alpha)|$  is strictly decreasing in  $\delta$ . For any given  $\theta$ , there is a  $d_a(\theta, k^*) \geq \epsilon$  such that for  $\delta = d_a(\theta, k^*)$  it is  $|\text{MRS}^G(V, \alpha)| = |\text{MRS}^B(V, \alpha)|$  at  $a(\theta, k^*)$ , and  $|\text{MRS}^G(V, \alpha)| < |\text{MRS}^B(V, \alpha)|$  for all  $\delta > d_a(\theta, k^*)$ . Let  $\delta_a = \max_\theta d_a(\theta, k^*)$ . Hence, for  $\delta > \delta_a$ , (A-5) is violated at  $a(\theta, k^*)$  for all  $\theta$ , and  $\alpha^* < a(\theta^*, k^*)$ ,  $V^* < V^G$  and  $\lambda_2 = 0$ .

Step 3:  $\theta^* > 0$ . We now show that there is a  $\bar{\delta}$  such that  $\theta^* > 0$  for  $\delta > \bar{\delta}$ . From  $\mathcal{L}_\theta \leq 0$ , and substituting for  $\lambda_1 = [\bar{U}_V^G - \lambda_2]/\bar{U}_V^B$ , the first order condition for  $\theta$  is

$$(A-6) \quad \frac{\bar{U}_\theta^G}{\bar{U}_V^G} - \frac{\bar{U}_\theta^B}{\bar{U}_V^B} \left( \equiv \text{MRS}^B(V, \theta) - \text{MRS}^G(V, \theta) \right) \leq \\ - \frac{\lambda_2}{\bar{U}_V^G \bar{U}_V^B} [\bar{U}_\theta^B + \bar{U}_V^B V_\theta^G] + \frac{\lambda_4}{\bar{U}_V^G},$$

where  $\text{MRS}^T(V, \theta) = p_H \alpha (\mu^T + \tau^T / p_H - k) / (p_H + p_M \rho_M^T + p_L \rho_L^T)$  is the MRS between  $V$  and  $\theta$  for  $T = G, B$ , and  $\tau^G = \delta$  and  $\tau^B = \epsilon$ . In step 2, we showed that  $V^* < V^G$  and  $\lambda_2 = 0$ . A solution at  $\theta = 0$  requires that  $\lambda_4 = 0$  so that the LHS of (A-6) must be non-positive. Let now  $\phi \equiv (p_H + p_M \rho_M^G + p_L \rho_L^G) / (p_H + p_M \rho_M^B + p_L \rho_L^B)$  and  $\psi \equiv (\mu^G + \delta / p_H - k) / (\mu^B + \epsilon / p_H - k)$ ; (A-6) may be restated as

$$(A-7) \quad \text{MRS}^B(V, \theta) - \text{MRS}^G(V, \theta) = \frac{\alpha p_H (\mu^B + \epsilon / p_L - k)}{p_H + p_L \rho_L^G + p_L \rho_L^G} [\phi - \psi] \leq 0.$$

Note that for  $\delta = \epsilon$ , we have that at  $\theta = 0$  it is  $\rho_M^G = \rho_M^B$  and  $\rho_L^G = \rho_L^B$ . Hence,  $\phi = 1 < \psi$ , and  $\text{MRS}^G(V, \theta) > \text{MRS}^B(V, \theta)$ . Note now that  $\psi$  is increasing and linear in  $\delta$ , while  $\phi$  is increasing and convex in  $\delta$ . Hence, for any  $\alpha$ , there is a  $\delta > \epsilon$  for which  $\phi = \psi$  at  $\theta = 0$ . Denote this value of  $\delta$  by  $d(\alpha)$ . Letting  $\bar{\delta} \equiv \max_{\alpha} d(\alpha)$ , we have that at  $\theta = 0$  it is  $\text{MRS}^B(V, \theta) > \text{MRS}^G(V, \theta)$  for all  $\delta > \bar{\delta}$ , violating (A-7). Letting  $\hat{\delta} \equiv \max\{\bar{\delta}, \delta_a\}$ , we have that  $\theta^* > 0$  and  $V < V^*$  for  $\delta > \hat{\delta}$ .

Step 4:  $\alpha^* > 0$ .  $\alpha^* = 0$  requires  $\lambda_2 = 0$ ,  $\mathcal{L}_{\alpha} \leq 0$  and  $\rho_M^G = \rho_L^G = \rho_M^B = \rho_L^B = 1$ . Also,  $k^* = \mu^G$  and  $\mu^G - \mu^B > (\delta - \epsilon) / (1 - p_H)$  give  $|\text{MRS}^G(V, \alpha)| = \mu^G - \theta \delta > \mu^B - \theta p_H (\mu^B + \epsilon / p_H - \mu^G) = |\text{MRS}^B(V, \alpha)|$ , violating (A-4). Hence,  $\alpha^* > 0$ , concluding the proof.  $\square$

*Proof of Proposition 4.* We will show that there is a pair  $\{\delta_1, \delta_2\}$  such that, if  $\delta_2 > \delta_1$ , then we have that for  $\delta_1 < \delta < \delta_2$ , the solution for  $\theta$  occurs only at a corner, that is  $\theta^* = 0$  and  $V^* < V^G$ . From the proof of proposition 3, we know that there is a  $\delta_a$  such that for  $\delta > \delta_a$ , it is  $\alpha^* < \alpha(\theta^*)$ ,  $\lambda_2 = 0$ , and  $V^* < V^G$ . Set  $\delta \geq \delta_a$ . (A-6) implies that an interior solution for  $\theta$  must satisfy,

$$(A-8) \quad \begin{aligned} \text{MRS}^B(V, \theta) - \text{MRS}^G(V, \theta) &= \frac{\alpha p_H (\mu^B + \epsilon / p_H - k)}{p_H + p_L \rho_L^G + p_L \rho_L^B} [\phi - \psi] \\ &= \frac{\lambda_4}{U_V^G} \geq 0. \end{aligned}$$

There, we have also shown that at  $\theta = 0$ ,  $\delta = \epsilon$  implies that  $\phi = 1 < \psi$  and  $\text{MRS}^G(V, \theta) > \text{MRS}^B(V, \theta)$ . Define now  $\delta_2 \equiv \min_{\alpha} d(\alpha)$ , with  $\epsilon < \delta_2 < \bar{\delta}$ . Hence, for  $\epsilon < \delta < \delta_2$ , we have that at  $\theta = 0$  it is  $\text{MRS}^G(V, \theta) > \text{MRS}^B(V, \theta)$  so that (A-8) is violated. The proof is concluded by showing that if at  $\theta = 0$ , it is  $\text{MRS}^G(V, \theta) > \text{MRS}^B(V, \theta)$ , then there cannot be another  $0 < \theta < 1$  for which  $\text{MRS}^G(V, \theta) = \text{MRS}^B(V, \theta)$ . Note that  $k^* = \mu^G$  implies that  $\text{MRS}^G(V, \theta) > \text{MRS}^B(V, \theta)$  at  $\theta = 1$ , if

$$(A-9) \quad Q(m) \equiv \frac{p_H + p_M e^{\alpha A m} + p_L e^{\alpha A (m + \epsilon / p_L)}}{p_H + p_M + p_L e^{\alpha A \delta / p_L}} - \frac{\epsilon / p_H - m}{\delta / p_H} > 0,$$

where  $m \equiv \mu^G - \mu^B$ . Let  $c$  be such that  $Q(c) = 0$ , with  $c < (\delta - \epsilon) / p_L$ . Since  $Q'(m) > 0$ , we have that if  $\mu^G - \mu^B > c$  then  $Q(m) > 0$  and  $\text{MRS}^G(V, \theta) > \text{MRS}^B(V, \theta)$ , also at  $\theta = 1$ . Differentiating  $\text{MRS}^G(V, \theta)$  and  $\text{MRS}^B(V, \theta)$  with respect to  $\theta$ , we obtain (after some algebra) that

$$(A-10) \quad \frac{\partial \text{MRS}^T(V, \theta)}{\partial \theta} = \frac{[\text{MRS}^T(V, \theta)]^2}{p_H} A [p_M \rho_M^T + p_L \rho_L^T] > 0,$$

for  $T = G, B$ . Consider now a  $\theta$  for which  $MRS^G(V, \theta) = MRS^B(V, \theta)$ . In this case,

$$(A-11) \quad \frac{\partial MRS^G(V, \theta)}{\partial \theta} - \frac{\partial MRS^B(V, \theta)}{\partial \theta} = \frac{[MRS^T(V, \theta)]^2}{p_H} A \Delta(\theta),$$

where  $\Delta(\theta) \equiv p_M(\rho_M^G(\theta) - \rho_M^B(\theta)) + p_L(\rho_L^G(\theta) - \rho_L^B(\theta))$ . Also,  $\partial \rho_M^G / \partial \theta - \partial \rho_M^B / \partial \theta < 0$  if  $J(\delta) \equiv \delta(p_L - m) / \epsilon p_H - e^{\alpha A m} > 0$ . Let  $\delta_b$  be such that  $J(\delta_b) = 0$ . Then  $J'(\delta) > 0$  implies that  $J(\delta) > 0$  for  $\delta > \delta_b$  so that  $\partial \rho_M^G / \partial \theta - \partial \rho_M^B / \partial \theta < 0$ . A similar argument shows that for  $\delta > \delta_b$ , also  $\partial \rho_L^G / \partial \theta - \partial \rho_L^B / \partial \theta < 0$ . Hence,  $\Delta'(\theta) < 0$  and  $MRS^G(V, \theta)$  and  $MRS^B(V, \theta)$  may intersect at most once. This, and the fact that at  $\theta = 0, 1$ ,  $MRS^G(V, \theta) > MRS^B(V, \theta)$ , gives that  $MRS^G(V, \theta) > MRS^B(V, \theta)$  for all  $\theta \in (0, 1)$ . Hence,  $\theta^* = 0$ . Setting  $\delta_1 \equiv \max\{\delta_a, \delta_b\}$  and  $C \equiv \max\{c; (\delta - \epsilon) / (1 - p_H)\}$  concludes the proof.  $\square$

*Proof of Proposition 5.* (a) In an optimum with underpriced units, the optimal choice for  $\theta$  is given by equating the LHS of (A-6) to zero. It may immediately be verified that

$$(A-12) \quad \frac{\partial MRS^G(V, \theta)}{\partial \delta} < \alpha \frac{p_H + p_M \rho_M^G + p_L \rho_L^G (1 - \delta \alpha A / p_L)}{(p_H + p_M \rho_M^G + p_L \rho_L^G)^2}.$$

Let  $\underline{\alpha} \equiv \min_{\theta} \alpha_0(\theta, \mu^G) - \kappa$ , for some  $\kappa > 0$ , and let  $\delta^c$  satisfy:  $p_H + p_M \rho_M^G(\delta^c) + p_L \rho_L^G(\delta^c)[1 - \delta^c \alpha A] = 0$ . Setting  $\delta_3 \equiv \max\{\delta, \delta^c\}$ , we have that for  $\alpha^* \geq \underline{\alpha}$  and  $\delta > \delta_3$ , the RHS of (A-12) is non-positive and  $MRS^G(V, \theta)$  is a decreasing function of  $\delta$ . By implicit function differentiation of (A-6), this and the second order conditions imply (a). (b): Similarly, from (A-6), it may be verified that  $\partial MRS^B(V, \theta) / \partial \mu^B > 0$ . By implicit function differentiation of (A-6), this, and the second order conditions, give (b).  $\square$

*Proof of Proposition 6.* As in the proof of proposition 5, part (a) is implied by the fact that in the proof of proposition 3, step 2, we have shown that  $\partial |MRS^G(V, \alpha)| / \partial \delta < 0$ . Similarly, part (b) derives from the fact that, by direct calculation, it is  $\partial |MRS^B(V, \alpha)| / \partial \mu^B > 0$  for  $\mu^G - \mu^B > 1/p_H$ .  $\square$

*Proof of Proposition 7.* (a) In an optimum with warrants and underpricing,  $V^*$ , and the optimal amount of underpricing,  $(V^G - V^*)$  are determined by the incentive-compatibility constraint (12). Since it may be shown that the  $MRS^B(V, \alpha)$  is increasing in  $\alpha$ ,  $V^*$  is convex in  $\alpha^*$ . Also,  $\mu^G - \mu^B > (\delta - \epsilon) / (1 - p_H)$  and  $k = \mu^G$  imply that, at  $\alpha = 0$ ,  $|V_{\alpha}^G| > |MRS^B(V, \alpha)|$ . Hence,  $(V^G - V^*)$  is a decreasing function of  $\alpha^*$ . This and proposition 6(a) imply part (a), and that there is a  $\bar{u}$  such that for  $u^* < \bar{u}$ , part (c) obtains. Part (a) of this proposition together with proposition 6(b) imply part (b). Finally, part (c) with  $MRS^B(V, \theta) > 0$  and  $MRS^B(V, \alpha) < 0$  together imply (d) and (e), respectively.  $\square$

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