Economics Letters 44 (1994) 287–294

Uncertain liquidity and interbank contracting

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Abstract

We study a version of the Diamond and Dybvig (*Journal of Political Economy*, 1983, 91, 401–419) model, where banks would like to obtain insurance against shocks on returns on liquid assets through an interbank borrowing and lending program. We show that if investments in liquid assets and their realized returns are private information to individual banks, the first-best allocation is not incentive-compatible; we then characterize the second-best interbank contract.

JEL classification: G21

1. Introduction

In the past decade or so, much work has been done on micro-theoretic modelling of financial intermediaries (banks), using the paradigm of optimal contracting given uncertain liquidity needs (intertemporal preference shocks) affecting liability-holders' (depositors') demands for early withdrawal of their invested funds. Beginning with the pioneering papers of Bryant (1980) and Diamond and Dybvig (1983), this literature has progressed to analyses of: (i) the superiority of deposit contracts over traded 'mutual fund' contracts for interim withdrawal [Jacklin (1987)]; (ii) the optimal choice between the above two mechanisms, given information about asset returns and the possibility of bank runs [Jacklin and Bhattacharya (1988)]; (iii) interventions such as suspension of convertibility and deposit insurance [Gorton (1985), Chari and Jagannathan (1988)], deposit interest rate controls [Smith (1984)], etc.

A somewhat different line of investigation was pursued by Bhattacharya and Gale (1987). In that paper, the authors investigated a situation in which insurance for their depositors' intertemporal preference shocks could only be imperfectly provided at each individual bank, since

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† Work for this paper was initiated while both authors were visiting the Innocenzo Gasparini Institute for Economic Research, Milano, Italy. Bhattacharya thanks the Institute for financial support through a Senior Fellowship, and both authors are grateful to the members of the Institute for their hospitality, and to Riccardo Rovelli and Paolo Siconolfi for many helpful discussions. All errors are our responsibility.

SSDI 0165-1765(93)00350-W

'local' shocks could systematically increase or decrease on the proportion of depositors seeking early withdrawal at each bank. Given the assumption in their model of lower rates of return on short-term ('liquid') versus long-term ('illiquid') investments in banks' portfolios, such withdrawal shocks are optimally coped with by sharing of liquid resource across banks, through a borrowing and lending program.

Bhattacharya and Gale (1987) showed that a competitive (Walrasian) market for inter-bank loans at an interim date would be an *inefficient* solution to the above-mentioned coordination problem, in that (given know free access to such a market) individual banks would underinvest in their liquid asset holdings. They then analyzed a model of optimal second-best contract design, with constrained amounts of borrowing and lending at the interim date, which leads to amelioration of the problems with second-best underinvestment.

A weakness of the analysis in Bhattacharya and Gale (1987) is the following: optimal second-best (borrowing and lending) contracts are *not* allowed to respond to ex post information about the realized extent of early withdrawals by depositors at each bank. This feature exacerbates the incentive-compatibility (representation of true state of withdrawal demand) problem faced by each bank, and makes for a second-best outcome. As an alternative, they suggested looking at a model in which the uncertainty faced by each bank pertained not to its depositors' demands for interim liquidity/withdrawals, but to the timing of the return from its supposedly liquid (short-term) investments. Specifically, one may allow for the possibility that, with statistical independence across banks, (some) investments that were thought to be short-term time are *not* to yield their returns until a later period. For example, 'short-term' inventory loans may not be repaid in time because the borrowing firm was into business problems in selling its goods, owing to variations in local (commodity-specific) demand conditions.

In this paper we analyze a model of optimal interbank coordination through borrowing and lending when each of a large number of banks faces *timing uncertainty* in the return on its short-term or liquid investments. Since the extent of its ex ante investment in such 'liquid' assets is assumed to be privately observed by each bank, it is internally consistent to also assume that conditioning interbank contracts on such investments, or their time of realized return, is not feasible. Hence, as we show, the problem of interbank coordination with such private information is inherently a second-best one, which leads to distortions in the pattern of choice over short-term and long-term investments at each bank.

2. A model of banking

There are a countable infinity of banks, each with a countable infinity of ex ante identical depositors with one unit of initial endowment in total. There are three time periods, $t = 0, 1, 2$. For each bank, depositors' intertemporal preferences for consumption at the three dates are, as in Diamond and Dybvig (1983), subject to a privately observed shock, and they are given by the utility function $U(C_1)$, for a proportion ϵ of depositors, and $U(C_2)$ for a proportion $1-\epsilon$. We assume that $U(C)$ is strictly increasing and concave, with the coefficient of relative risk-aversion everywhere greater than unity, and that $U'(0) = \infty$. These 'preference shocks' are assumed to be i.i.d. across depositors (in each bank).

Each bank's investment opportunity set consists of a short-term and a long-term investment technology. The short-term technology has a stochastic return pattern so that a unit investment at $t = 0$ will yield a unit return either at $t = 1$, with probability $1 - p$, or it will be delayed until $t = 2$, with probability p. A bank with a zero return at $t = 1$ from investment in the short-term technology will be willing to borrow from the market, and it will be denoted as a type-b bank.

Conversely, a bank with a positive return at $t=1$ from investment in the short-term technology will be denoted as a type-a bank. The long-term investment opportunity, instead, offers a safe return $R>1$ at $t=2$, and zero at $t=1$.

Differently from Diamond and Dybvig (1983), we will assume that a bank chooses its allocation $\{\ell,(1-\ell)\}$ of investments in the short- and long-term investment opportunities *irreversibly* at $t=0$. The shocks determining the timing of realized return on the short-term investment opportunity are assumed to be i.i.d. across banks. Hence, in the aggregate across all banks each unit of investment at $t=0$ in the short-term technology yields $(1-p)$ at $t=1$, and p at $t=2$, almost surely.

We now consider the problem of deriving the ex ante optimal first-best investment pattern and deposit withdrawal rights. Letting $\{C_{ta}, C_{tb}\}$ be the deposit withdrawal right per unit of initial deposit in bank of type-a or type-b at time $t=\{1,2\}$, and letting ℓ be the investment proportion of each bank in the short-term technology, we have that the problem is to

$$\max_{\{\ell, C_{a1}, C_{a2}, C_{b1}, C_{b2}\}} p\{\epsilon U(C_{1b})+(1-\epsilon)U(C_{2b})\}+(1-p)\{\epsilon U(C_{1a})+(1-\epsilon)U(C_{2a})\} \quad (1)$$

s.t.

$$\epsilon\{pC_{1b}+(1-p)C_{1a}\}=(1-p)\ell\ , \quad (2)$$

$$(1-\epsilon)\{pC_{2b}+(1-p)C_{2a}\}=R(1-\ell)+p\ell\ . \quad (3)$$

The obvious solution to this problem is to set

$$C_{1b}=C_{1a}=C_1^*\ , \qquad C_{2b}=C_{2a}=C_2^*\ , \quad (4)$$

$$(1-p)U'(C_1^*)=(R-p)U'(C_2^*)\ . \quad (5)$$

Let $\{C_1^*, C_2^*, \ell^*\}$ be the first-best ex ante optimal allocation solving program (1). Note that it is possible to interpret the first-best solution in terms of a borrowing–lending 'contract' across banks, in which (if types and investments were observable) each bank invests ℓ^* in short-term assets and $(1-\ell^*)$ in the long-term assets at $t=0$, and at $t=1$ type-b banks borrow an amount $B^*=\epsilon C_1^*=(1-p)\ell^*$ from some set of type-a banks, each of which lends $L^*=B^*/(1-p)=p\ell^*$. At $t=2$, each borrower repays and each lender gets repaid DB and DL, respectively, where by the fact that $C_{2b}=C_{2a}=C_2^*$, we have that $D^*=1$, i.e. to insure against timing uncertainty on the return from the short-term asset, the net interest rate on interbank loans must be zero.

3. Second-best optimal interbank coordination

We now analyze the case in which the choice of the amount ℓ of investment and the realization of the return on the short-term technology are private information to a bank. We consider a contract, which must satisfy incentive-compatibility constraints for each bank, in which banks are 'asked' to invest in proportions $\{\ell,(1-\ell)\}$ in the short- and long-term investments at $t=0$, and type-a banks are asked to each lend an amount L to type-b banks, each of which borrows B, at some gross interest rate D. Since material balance between total borrowing and lending among banks requires that $(1-p)L=pB$, this borrowing and lending program will allow banks to sustain a level of per capita consumption given by

$$C_{1b} = \frac{B}{\epsilon}, \qquad C_{2b} = \frac{\ell + R(1-\ell) - DB}{1-\epsilon}, \tag{6a}$$

$$C_{1a} = \frac{\ell - pB/(1-p)}{\epsilon}, \qquad C_{2a} = \frac{R(1-\ell) + pDB/(1-p)}{1-\epsilon}. \tag{6b}$$

The optimal second-best mechanism design contract may then be formulated as the program:

$$\max_{\{\ell,B,D\}} W(\ell, B, D) \equiv p\{\varepsilon U(C_{1b}) + (1-\epsilon)U(C_{2b})\} + (1-p)\{\epsilon U(C_{1a}) + (1-\epsilon)U(C_{2a})\} \tag{7}$$

$$\text{s.t. eq. (6)}, \quad 0 \le \ell \le 1,$$
$$W(\ell, B, D) \ge V(x, B, D), \quad \forall x \in [0, 1], \tag{8}$$

where $V(x, B, D)$ is the expected payoff from a bank investing in proportions $(x, 1-x)$ in the short- and long-term assets at $t = 0$, and then always borrowing B at $t = 1$ to repay DB at $t = 2$. Thus

$$V(x, B, D) \equiv p\{\epsilon U(\hat{C}_{1b}) + (1-\epsilon)U(\hat{C}_{2b})\} + (1-p)\{\epsilon U(\hat{C}_{1a}) + (1-\epsilon)U(\hat{C}_{2a})\}, \tag{9}$$

where

$$\hat{C}_{1a} \equiv \frac{x + B}{\epsilon}, \qquad \hat{C}_{2a} \equiv \frac{R(1-x) - DB}{1-\epsilon}, \tag{10a}$$

$$\hat{C}_{1b} \equiv \frac{B}{\epsilon}, \qquad \hat{C}_{2b} \equiv \frac{R(1-x) + x - DB}{1-\epsilon}, \tag{10b}$$

are the per capita consumption levels of the depositors of a 'deviant' bank. The reasons that Eq. (8) is the only incentive-compatibility constraint are that: (i) since a type-b bank has no cash available at $t = 1$, type misrepresentation at $t = 1$, which implies lending, is not feasible for this bank type, and (ii) if a bank does *not* plan to misrepresent its type at $t = 1$ in the event that it turns out to be of type-a (and thus borrow rather than lend), then at $t = 0$ it is optimal to choose $x = \ell$, the optimizing level for program (7), subject to Eqs. (6).

Our first observation is that, differently from Bhattacharya and Gale (1987), the first-best ex ante optimal allocation is *never* attainable as an incentive-compatible contract when banks choose their liquid investment levels and representation of interim types (realized returns), in their private interest.

Lemma 1. The first-best allocation $\{C_1^*, C_2^*, \ell^*\}$ is not implementable as an incentive-compatible allocation satisfying Eq. (8).

Proof. By direct substitution, it may be immediately verified that $V(0, B^*, D^*) > W(\ell^*, B^*, D^*)$, so that the first-best allocation is not incentive-compatible. Q.E.D.

In essence, since the first-best loan contract offers a marginal rate of substitution of $D^* = 1$, and the available marginal rate of transformation between short-term and long-term investments is $R > 1$, the (deviant) banks find that investing nothing in the short-term investment dominates the first-best investment plan ℓ^*. Hence, the first-best investment optimum is never incentive-compatible.

We may now proceed with the characterization of the second-best, incentive-compatible contract solving program (7). A difficulty here is the particular form of the incentive constraint (8), since the optimal choice of x for a deviant bank may be at a corner, in particular at $x = 0$. To allow for this possibility, we proceed as follows. To simplify the discussion, note first that $U'(0) = \infty$ and Eqs. (6) together imply that an optimum $B > 0$ and $\ell > 0$. Second, note that incentive-compatibility requires that $D > 0$. We may now show that the solution to program (7) is a saddlepoint of a suitably defined Lagrangian expression.

Proposition 1. Let $(\ell^s, D^s, B^s, x^s, \lambda^s, \mu_\ell^s, \mu_x^s, \theta_x^s)$ be a saddlepoint of the Lagrangian

$$\mathscr{L} \equiv W(\ell, D, B) + \lambda[W(\ell, D, B) - V(x, D, B)] + \mu_\ell[1 - \ell] - \mu_x[1 - x] - \theta_x x\,, \tag{11}$$

with non-negative multipliers $(\lambda^s, \mu_\ell^s, \mu_x^s, \theta_x^s) \geq 0$. Then (ℓ^s, D^s, B^s) is a solution to the optimization program (7), subject to the conditions in Eqs. (6) and (8).

Proof. See the appendix.

Note that the above procedure differs from the usual saddlepoint characterization of constrained optima [such as the one in Mangasarian (1969)], in that here the Lagrangian is *minimized* w.r.t. the variable x, and it is *maximized* w.r.t. the multipliers (μ_x, θ_x). This procedure is necessary to guarantee that the value x^s minimizing $\mathscr{L}$ is indeed a *global* optimum for $V(x, D, B)$ for all $x \in [0, 1]$.

Substituting from Eqs. (6) and (10) into $W(\ell, B, D)$ and $V(x, B, D)$, and assuming an interior solution w.r.t. ℓ, that is $\ell < 1$, we have that a saddlepoint for the Lagrangian (11) requires that at an optimum the following conditions must be satisfied:

$$p(R - 1)U'(C_{2b}) = (1 - p)(U'(C_{1a}) - RU'(C_{2a}))\,, \tag{12}$$

$$p(1 + \lambda)\{U'(C_{2b}) - U'(C_{2a})\} = \lambda\{pU'(\hat{C}_{2b}) + (1 - p)U'(\hat{C}_{2a})\}\,, \tag{13}$$

$$p(1 + \lambda)(U'(C_{1b}) - U'(C_{1a})) = \lambda[pU'(\hat{C}_{1b}) + (1 - p)U'(\hat{C}_{1a})]\,. \tag{14}$$

Finally, from $\partial\mathscr{L}/\partial x = 0$, and noting that $\hat{C}_{2a} > 0$ implies that an optimum $x < 1$, we have that

$$V_x = (1 - p)(U'(\hat{C}_{1a}) - RU'(\hat{C}_{2a})) - p(R - 1)U'(\hat{C}_{2b}) = -\frac{\theta_x}{\lambda} \leq 0\,, \tag{15}$$

with, from the complementary slackness conditions, $V_x x = 0$. We may then prove the following.

Proposition 2. A second-best, incentive-compatible contract $\{\ell, D, B\}$ solving program (7) is characterized by the following:

(i) $\lambda > 0$,
(ii) $\{C_{1a}, C_{2a}\} > \{C_{1b}, C_{2b}\}$,
(iii) $\dfrac{B}{\ell} < \dfrac{B^*}{\ell^*}$, $\dfrac{L}{\ell} < \dfrac{L^*}{\ell^*}$ and $\dfrac{DB}{\ell} > \dfrac{D^*B^*}{\ell^*}$, $\dfrac{DL}{\ell} > \dfrac{D^*L^*}{\ell^*}$,
(iv) $D > R$.

Proof. See the appendix.

First, if $\lambda = 0$, the first-best plan would be a solution to problem (7), contradicting Lemma 1. Hence, in a second-best optimum $\lambda > 0$, and the incentive-compatibility constraint must be binding. Second, the second-best consumption plan for type-a banks strictly dominates the one for the type-b banks, and the second-best program offers only *partial insurance* against shocks on the short-term technology. This residual risk, which gives a preferential treatment to type-a banks, is necessary ex ante to induce the desired investment in the short-term technology, and ex post to reward type-a banks for revealing themselves at $t = 1$ by lending in the interim market for interbank loans at $t = 1$.

The preferential treatment of type-a banks in a second-best program is achieved in two ways: by restricting the size of loans, as a proportion of investment in the short-term technology, and by raising the interest rate on these loans. In the second best, the amount loaned by a type-a bank to a type-b bank is proportionally smaller than in the first best. However, despite the smaller loan size, the total repayment at $t = 2$ from a type-b bank to a type-a bank is proportionally greater than in the first best. Furthermore, the second-best optimal borrowing–lending rate, D, is strictly greater than R, and hence strictly higher than the first-best level. In order to induce a bank to invest the desired amount of resources in the short-term technology, the chance of being a lender in the interim loan market must be sufficiently attractive. This will be the case *only if* the potential return from investment in the short-term technology is greater than the rate of return, R, on the long-term one. The unobservability of bank assets allocation and of the realization of the liquidity shocks on assets has the effect of sharply increasing the interest rates on the interbank loan market (with respect to the first-best level).

Finally, the second-best level of short-term investment, ℓ, may be either greater or smaller than the first-best level, ℓ^*. The choice of ℓ, privately done by the bank, is subject to two opposing incentives: $D > R$ creates a reward for a bank from being a lender in the interim interbank market. The size of the loan that the bank is allowed to make at $t = 1$, however, is smaller than in the first best, per unit of investment in the short-term technology. The net effect on the incentives to invest in the short-term technology, therefore, is in general ambiguous.

Appendix

Proof of Proposition 1. Let $(\ell^s, D^s, B^s, x^s, \lambda^s, \mu_\ell^s, \mu_x^s, \theta_x^s)$ be a saddlepoint for the Lagranian $\mathscr{L}$, and let W^s and V^s be the corresponding values for the functions W and V, so that

$$\mathscr{L}(\ell^s, D^s, B^s, x^s, \lambda^s, \mu_\ell^s, \mu_x^s, \theta_x^s) \geq \mathscr{L}(\ell, D, B, x^s, \lambda^s, \mu_\ell^s, \mu_x^s, \theta_x^s) \quad \text{for all } \ell, D, B\,; \tag{A1}$$

$$\mathscr{L}(\ell^s, D^s, B^s, x^s, \lambda^s, \mu_\ell^s, \mu_x^s, \theta_x^s) \leq \mathscr{L}(\ell^s, D^s, B^s, x^s, \lambda, \mu_\ell, \mu_x^s, \theta_x^s) \quad \text{for all } (\lambda, \mu_\ell) \geq 0\,; \tag{A2}$$

$$\mathscr{L}(\ell^s, D^s, B^s, x^s, \lambda^s, \mu_\ell^s, \mu_x^s, \theta_x^s) \leq \mathscr{L}(\ell^s, D^s, B^s, x, \lambda^s, \mu_\ell^s, \mu_x^s, \theta_x^s) \quad \text{for all } x\,; \tag{A3}$$

$$\mathscr{L}(\ell^s, D^s, B^s, x^s, \lambda^s, \mu_\ell^s, \mu_x^s, \theta_x^s) \geq \mathscr{L}(\ell^s, D^s, B^s, x^s, \lambda^s, \mu_\ell^s, \mu_x, \theta_x) \quad \text{for all } (\mu_x, \theta_x) \geq 0\,. \tag{A4}$$

Consider (A2). This implies that for all $(\lambda, \mu_\ell) \geq 0$, it is

$$\lambda(W^s - V^s) + \mu_\ell(1 - \ell^s) \geq \lambda^s(W^s - V^s) + \mu_\ell^s(1 - \ell^s)\,. \tag{A5}$$

Letting $\mu_\ell = \mu_\ell^s$, (A5) gives that for all $\lambda \geq 0$ it must be

$$(\lambda - \lambda^s)(W^s - V^s) \geq 0 . \tag{A6}$$

Setting $\lambda = \lambda^s + \delta$, with $\delta > 0$, (A6) implies that $W^s \geq V^s$. Furthermore, if $W^s > V^s$, (A6) may be satisfied for all $\lambda \geq 0$ only if $\lambda^s = 0$. Hence, we have the complementary slackness condition $\lambda^s(W^s - V^s) = 0$. A similar argument shows that $\ell^s \leq 1$, and that $\mu^s_\ell(1 - \ell^s) = 0$. Consider now (A4). Following an argument similar to the previous one, we have now that for all (μ_x, θ_x) we have

$$(\mu_x - \mu^s_x)(1 - x^s) \geq 0 , \quad (\theta_x - \theta^s_x)x^s \geq 0 . \tag{A7}$$

Again, (A7) implies that $0 \leq x^s \leq 1$, and that $\theta^s_x x^s = \mu^s_x(1 - x^s) = 0$. From (A3) we have that

$$-\lambda^s V(x, B^s, D^s) - \mu^s_x(1 - x) - \theta^s_x x \geq -\lambda^s V^s - \mu^s_x(1 - x^s) - \theta^s_x x^s . \tag{A8}$$

This, and the complementary slackness conditions implied by (A7), gives that

$$\lambda^s(V^s - V(x, B^s, D^s)) \geq \mu^s_x(1 - x) + \theta^s_x x . \tag{A9}$$

Hence $V^s \geq V(x, B^s, D^s)$ for all $0 \leq x \leq 1$, and x^s is the optimal deviation for a deviant bank, given (B^s, D^s). Finally, from (A1), and the slackness conditions implied by (A2), we have that

$$W(\ell^s, B^s, D^s) \geq W(\ell, B, D) + \lambda^s[W(\ell, B, D) - V(x^s, B, D)] + \mu^s_\ell(1 - \ell) , \tag{A10}$$

and (ℓ^s, B^s, D^s) solves (14) subject to (13), (15) and $0 \leq \ell \leq 1$. Q.E.D.

Proof of Proposition 2. (i) Suppose otherwise that at an optimum $W > V$, so that $\lambda = 0$. Then, from (13), we obtain that $U'(C_{2b}) = U'(C_{2b}) = U'(C_2)$. This, using (14), implies that $U'(C_{1a}) = U'(C_{1a}) = U'(C_1)$. Using (12), this implies that $(1 - p)U'(C_1) = (R - p)U'(C_2)$. Hence, $\{C_1, C_2\}$ are at their first-best levels, which we know, from Lemma 1, not to be incentive-compatible, violating (8). (ii) Equation (13) and $\lambda > 0$ imply that $C_{2b} < C_{2a}$. This, with (14), implies also that $C_{1b} < C_{1a}$. (iii) $C_{1a} > C_{1b}$ and (6) imply that $B < (1 - p)\ell$ and $L < p\ell$. Similarly, $C_{2a} > C_{2b}$ and (6) imply that $DB > (1 - p)\ell$ and $DL > p\ell$. (iv) Consider the incentive-compatibility constraint (8), and compare the consumption levels under the proposed plan and the one available to a deviant bank. We have that $\hat{C}_{1b} = C_{1b}$, and

$$\hat{C}_{2b} = (1 - x)R + x - DB \geq (1 - \ell)R + \ell - DB = C_{2b} \quad \text{for all } x \leq \ell . \tag{A11}$$

Choose then $x_0 = \ell - B/(1 - p)$. From (iii), $x_0 > 0$. At $x = x_0$, we have that $\hat{C}_{1a} = C_{1a}$, and

$$\hat{C}_{2a} \equiv (1 - x_0)R - DB = (1 - \ell)R + B\left(\frac{R}{1 - p} - D\right) \geq (1 - \ell)R + DB\frac{p}{1 - p} \equiv C_{2a} , \tag{A12}$$

for $D \leq R$. Hence, if $D \leq R$, setting $x = x_0$ strictly dominates the prescribed ℓ^s, preventing incentive-compatibility. Hence, $D > R$. Q.E.D.

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