

**Workshop on Geometric Representation Theory and Moduli spaces**  
 UNC Chapel Hill, November 17-19

Schedule

Nov 17	Nov 18	Nov 19
	8:00-8:30 Registration	
	8:30-9:30 Rozenblyum	8:30-9:30 Achar
	9:00-9:30 Coffee break	9:00-9:30 Coffee break
	10:00-11:00 Boixeda Alvarez	10:00-11:00 Damiolini
	11:15-12:15 Halacheva	11:15-12:15 Shen
	Lunch break	
	2:00-3:00 Weeks	
2:30-3:30 Dhillon	3:15-4:15 Chan	
4:30-5:30 Oblomkov	4:30-5:30 Zhao	
	5:45-6:45 Reception	

The talks on Nov 17 will be held in Phillips Hall 385. The talks on Nov 18-19 will be in PH 332. The coffee breaks and the reception will take place in PH 330.

## **Titles/Abstracts:**

November 17, Friday afternoon

**Gurbir Dhillon** (Yale), 2:30-3:30pm

Title: The Fundamental Local Equivalence and the  $W$ -algebra

Abstract: A basic (and pleasant) assertion in the geometric Langlands program is the Satake isomorphism, which identifies a factorization category of certain  $D$ -modules on the affine Grassmannian of a reductive group  $G$  with the representations of its Langlands dual group  $G^{\vee}$ . Gaitsgory and Lurie proposed a one parameter deformation of this story, which replaces the representations of  $G^{\vee}$  with certain representations of its Kac—Moody algebra, and this 'Fundamental Local Equivalence' plays a similarly basic role in the quantum geometric Langlands correspondence. After giving a gentle introduction to the above circle of ideas for non-specialists, we will describe some joint work with Gaitsgory verifying this conjecture, and its connection to the representation theory of  $W$ -algebras.

**Alexei Oblomkov** (UMass) 4:30-5:30pm

Title: HOMFLYPT homology and categorical Heisenberg action

Abstract: The talk is based on joint work with L. Rozansky. In our previous work we explained how one can associate to a braid on  $n$  strands complex of coherent  $C^{\infty} \times C^{\infty}$ -equivariant sheaves on  $\text{Hilb}_n(C^2)$ . In my talk, I explain how the categorical Nakajima operators fit into the picture. Roughly, the Nakajima operators correspond to adding extra strands to the braid. I will also show how this observation allows us to compute the homology of torus link (and more).

November 18, Saturday morning

**Nick Rozenblyum** (Toronto) 8:30-9:30am

Title: Localization and higher WZW conformal blocks

Abstract: I will describe a general local-to-global principle for  $D$ -modules on the moduli stack of  $G$ -bundles, which provides a natural target for localization functors. As an application, I will explain a derived version of the Verlinde formula for higher WZW conformal blocks. A key ingredient is a uniformization result for perfect complexes on  $\text{Bun}_G$ .

**Pablo Boixeda Alvarez** (Yale) 10:00-11:00am

Title: The center of the small quantum group and affine Springer fibers

Abstract: The quantum group  $U_q$  is Hopf-algebra deforming the enveloping algebra introduced by Lusztig. The representation theory of this algebra is particularly interesting at  $l$ -th roots of one, where it includes a finite dimensional subalgebra known as the small quantum group. In joint work with Bezrukavnikov, Shan and Vasserot we construct an injective map to the center of this algebra from the cohomology of a certain affine Springer fiber  $Fl_s$  for  $s$  a regular semisimple element. In recent progress we check that this map is surjective in type A and get a bound on dimension in general types related to the diagonal coinvariant algebra. We also give an algebro geometric description of the spectrum of the cohomology of the Springer fiber. The work relies on understanding the representation category through a filtration coming from intersection with  $G[[t]]$ -orbits in  $Fl_s$ . In this talk I will present the result and related properties of this filtration of the category.

**Iva Halacheva** (Northeastern), 11:15-12:15pm

Title: Bethe algebras, crystals, and cacti in type A

Abstract: The family of Bethe algebras is a collection of maximal commutative subalgebras of the Yangian parametrized by regular elements of the maximal torus. We will focus on the case of  $gl(n)$ , where this collection is known to extend to the Deligne-Mumford compactification of  $M(0, n+2)$ . For the real locus of this parameter space, the corresponding Bethe subalgebras act with simple spectrum on a given tame representation, and the resulting eigenlines assemble into an unramified covering. Moreover, the fiber over any point can be identified as a  $gl(n)$ -crystal with a collection of Gelfand-Tsetlin keystone patterns. I will discuss the monodromy action under this isomorphism, given by a type of cactus group. This is joint work with Anfisa Gurenkova and Lenya Rybnikov.

November 18, Saturday afternoon

**Alex Weeks** (Saskatchewan), 2:00-3:00pm

Title: Affine Grassmannians, Coulomb branches, and closed embeddings

Abstract: I discuss joint work with Dinakar Muthiah (arXiv:2211.04788). We construct closed embeddings between Coulomb branches for quiver gauge theories. For finite type ADE quivers, these Coulomb branches are affine Grassmannian slices, and we recover their usual embeddings into one another. Following the paradigm suggested by Braverman, Finkelberg and Nakajima, one should interpret Coulomb branches for arbitrary quivers (without edge loops) as affine Grassmannian slices of Kac-Moody type. In this way, our result provides closed embeddings between these slices.

**Charlotte Chan** (Michigan), 3:15-4:15pm

Title: Generic character sheaves for parahoric subgroups

Abstract: Lusztig's theory of character sheaves for connected reductive groups is one of the most important developments in representation theory in the last few decades. In this talk, I will describe a construction which extends this "depth zero" picture to give positive-depth character sheaves associated to generic data. In the simplest nontrivial case, this resolves a conjecture of Lusztig and produces perverse sheaves associated to sufficiently generic multiplicative local systems whose trace-of-Frobenius function coincides with parahoric Deligne Lusztig induction. This is joint work with R. Bezrukavnikov.

**Lutian Zhao** (Maryland), 4:30-5:30pm

Title: Nonabelian Hodge Theory of parahoric torsors

Abstract: Modern mathematical research has been enriched by exploring the bijection between Higgs bundles and local systems over noncompact Riemann surfaces, an incredible feat accomplished profoundly by C. Simpson by introducing weighted filtrations. In this discussion, our focus will be on further unraveling this relationship by delving into the correlation between the Dolbeault and de Rham moduli spaces for generally complex reductive structure groups, utilizing the influential language of parahoric group schemes as initially established by Bruhat-Tits. This presentation is based on collaborative work with Georgios Kydonakis, Pengfei Huang, and Hao Sun.

November 19, Saturday morning

**Pramod Achar** (Louisiana State), 8:30-9:30am

Title: Cleanness in Springer theory

Abstract: In Lusztig's papers from 1985-1986 that invented the theory of character sheaves, he proved (in nearly all cases) a remarkable property of cuspidal perverse  $\mathbb{Q}$ -sheaves on the nilpotent variety: they are "clean," meaning that their stalks vanish outside a single orbit. This property is crucial to making character sheaves computable by an algorithm, and it is a precursor of various "block decompositions" of the derived category studied by various authors (Gunningham, Rider, Russell, and others) later. About 10 years ago, Mautner conjectured that these perverse sheaves remain clean after reduction modulo  $p$  (with some exceptions for small  $p$ ). In this talk, I will discuss the history and context of the cleanness phenomenon, along with the current status of Mautner's conjecture. This is joint work with T. Chatterjee.

**Chiara Damiolini** (UT Austin), 10:00-11am

Title: Conformal blocks through Mode Transition Algebras

Abstract: Vertex operator algebras (VOAs) and their modules define sheaves of conformal blocks over the moduli space of stable curves. In today's talk I will describe how we can study the

properties of a VOA, and their conformal blocks, through properties of a family of associative algebras associated with it, which we call Mode Transition Algebras (MTAs). In particular we will see how the degeneration of conformal blocks to nodal curves---behavior which is not fully understood for non-rational VOAs---can be translated into intrinsic properties of the MTAs. If time permits I will also discuss how MTAs relate to higher-level Zhu algebras. This is based on a joint work with A. Gibney and D. Krashen.

**Linhui Shen** (Michigan state), 11:15-12:15pm

Title: Cluster Nature of Quantum Groups

Abstract: We present a rigid cluster model to realize the quantum group  $U_q(\mathfrak{g})$  for  $\mathfrak{g}$  of type ADE. That is, we prove that there is a natural Hopf algebra isomorphism from the quantum group to a quotient algebra of the Weyl group invariants of a Fock-Goncharov quantum cluster algebra. By applying the quantum duality of cluster algebras, we show that the quantum group admits a cluster canonical basis  $\Theta$  whose structural coefficients are in  $\mathbb{N}[q^{\frac{1}{2}}, q^{-\frac{1}{2}}]$ . The basis  $\Theta$  satisfies an invariance property under the braid group action, the Dynkin automorphisms, and the star anti-involution. Based on the preprint arXiv: 2209.06258