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On the instability of Hall MHD by degenerate dispersion

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Abstract. In this talk, I will describe a recent series of work with I.-J. Jeong on the incompressible Hall MHD equation. This PDE, first investigated by the applied mathematician M. J. Lighthill, is a one-fluid description of magnetized plasmas with a quadratic second-order correction term (Hall current term), which takes into account the motion of electrons relative to positive ions. Curiously, in the non-resistive case, we demonstrate an extreme form of instability of the trivial solution to this PDE, namely the ill(!)posedness of the Cauchy problem near the trivial solution, despite the apparent linear stability and conservation of energy. Our illposedness mechanism is sharp, in that it remains true under fractional dissipation of any subcritical order. Moreover, this result is to be contrasted with several regimes in which the Cauchy problem is well-posed, which not only includes the original setting that M. J. Lighthill investigated (namely, for initial data close to a uniform magnetic field) but also possibly large perturbations thereof.Central to our proofs is the viewpoint that the Hall current term imparts the magnetic field equation with a quasilinear degenerate(!) dispersive character. With such a viewpoint, the key illposedness mechanism can be understood in terms of a geometric optics (or classical-quantum correspondence) argument. This mechanism is of much wider scope than just Hall MHD; time permitting, I will discuss some other dispersive models where similar illposedness can be proved by extending our ideas.