

Mathematics Colloquium

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Counting problems: open questions in number theory

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Abstract. Many questions in number theory can be phrased as counting problems. How many primes are there? How many elliptic curves are there? How many integral solutions to this system of equations are there? How many number fields are there? Sometimes the answer is "infinitely many," and then we want to understand the order of growth for the "family" of objects we are counting. In other settings, we might want to show the answer really is "very few indeed." In this talk, we will explore how both types of counting problem can be related to each other. In particular, we will trace one "simple" problem from the 1600's as it has evolved into a deeply connected web of mysterious "counting problems" that may take decades (centuries?) to solve. This talk intends to be radically accessible to students and researchers in all areas of math.