

Mathematics Colloquium

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Unipotent representations and quantization

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Abstract. A fundamental question in the representation theory of semisimple Lie groups is to classify their irreducible unitary representations. A guiding principle here is the Orbit method, first discovered by Kirillov in the 60's for nilpotent Lie groups. It states that the irreducible unitary representations should be related to coadjoint orbits, i.e., the orbits of the Lie group action in the dual of its Lie algebra. Passing from orbits to representations could be thought of as a quantization problem and it is known that in this setting this is very difficult. For semisimple Lie groups it makes sense to speak about nilpotent orbits, and one could try to study representations that should correspond to these orbits via the yet undefined Orbit method. These representations are called unipotent: they are expected to be nicer than general ones, while one hopes to reduce the study of general representations to that of unipotent ones. I will concentrate on the case of complex Lie groups. I will explain how recent advances in the study of deformation quantizations of singular symplectic varieties allow to define unipotent representations and obtain some results about them. The talk is based on the joint work with Lucas Mason-Brown and Dmytro Matvieievskyi.