

Mathematics Colloquium

March 30, 2023

On positivity for flag manifolds and Hessenberg spaces

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Abstract. "Positivity" is a phenomenon involving the intersection of subvarieties in the presence of enough structure to ensure that intersection points are positively oriented, among other generalizations. It has a direct combinatorial interpretation, which has engaged mathematicians for at least 150 years, in one form or another. Such phenomena occur with frequency in the context of complex homogenous spaces such as flag varieties. In this talk, I will introduce and define positivity in a limited context, including some generalizations to the (torus) equivariant cohomology ring of G/B, where G is a complex reductive group and B is a Borel subgroup. I will then turn to Hessenberg varieties, a special class of subvarieties of G/B. In some cases, these subvarieties also exhibit positivity properties, and have combinatorial formulas describing it. In other cases, positivity is unknown but lots of hints exist, leading to conjectural behavior. I will close with several open problems. The work I present consists of outcomes from various joint projects with L. Mihalcea, R. Singh, B. Gorbutt, M. Precup, and J. Tymoczko.* (Almost) everything you need to know about equivariant cohomology will be presented during the talk.