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A theory for undercompressive shocks in tears of wine

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Abstract. We revisit the tears of wine problem for thin films in water-ethanol mixtures and present a new model for the climbing dynamics. The new formulation includes a Marangoni stress balanced by both the normal and tangential components of gravity as well as surface tension which lead to distinctly different behavior. The combined physics can be modeled mathematically by a scalar conservation law with a nonconvex flux and a fourth order regularization due to the bulk surface tension. Without the fourth order term, shock solutions must satisfy an entropy condition–in which characteristics impinge on the shock from both sides. However, in the case of a nonconvex flux, the fourth order term is a singular perturbation that allows for the possibility of undercompressive shocks in which characteristics travel through the shock. We present computational and experimental evidence that such shocks can happen in the tears of wine problem, with a protocol for how to observe this in a real life setting.