Root Finding: Bisection Method

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- Suppose we know two points a and b such that $f(a)$ and $f(b)$ are of different signs.
- For simplicity, suppose $f(a) < 0$ and $f(b) > 0$.
- Is there a root in between a and b?

Bisection Method

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Bisection Method

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- Yes. This follows from the Intermediate Value Theorem:
  - If $f(a) < u < f(b)$, then there exists a $c \in [a, b]$ such that $f(c) = u$.

Bisection Method

- How do we find the root between a and b?

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- Idea: Find the value in between a and b, that is $c = \frac{a+b}{2}$.
- Three cases:
  - $f(c) = 0$: Then we're done!
  - $f(c) < 0$: Replace a with c.
  - $f(c) > 0$: Replace b with c.
- We can now proceed iteratively.

Bisection Method

- When do we stop the iterations? Let $c^\star$ be the root, i.e. $f(c^\star) = 0$.

- Three ways to determine when we've done enough:
  1) $|c^\star - c| < \text{tol}$. If we are close enough to the root, we can stop.
  2) $|c_n - c_{n-1}| < \text{tol}$. Iterations are getting really close together.
  3) N is too big. Finding the root is taking to long.

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  1) $|c^\star - c| < \text{tol}$. If we are close enough to the root, we can stop.
  2) $|c_n - c_{n-1}| < \text{tol}$. Iterations are getting really close together.
  3) N is too big. Finding the root is taking to long.
- We, in general, do not know $c^\star$, so we need to use cases 2 and 3.