Hmwk 10

$$LI'' + RI' + \frac{I}{C} = E'(t)$$

$$I(0) = 0$$

$$0 = Q(0) \Rightarrow LI'(0) + RI(0) + \cancel{Q(0)}^{0}$$

$$= LI'(0) + R\cancel{I(0)}_{0} = E(0)$$

$$(\sin(0) = 0)$$

$$\Rightarrow I'(0) = 0$$

1a) $255\cos(t) \quad 0 < t < 2\pi$

$$\Rightarrow \boxed{E'(t) = 255\cos(t)\left(H(t) - H(t-2\pi)\right)}$$

1b) $\mathcal{L}(LI'') = L(s^2\tilde{I} - sI(0) - I'(0))$

$= Ls^2\tilde{I}$

$\mathcal{L}(RI') = R(s\tilde{I} - \cancel{I(0)}^{0})$

$= Rs\tilde{I}$

$\mathcal{L}(255\cos(t)(H(t) - H(t-2\pi)))$

$= 255(\mathcal{L}(\cos(t)H(t)) - \mathcal{L}(\cos(t)H(t-2\pi)))$

(s-shifting)

$= 255\left(\left(\frac{s}{s^2+1}\right) - \left(\frac{e^{-2\pi s}s}{s^2+1}\right)\right)$

$$= 225\frac{s}{s^2+1}\left(1-e^{-2\pi s}\right)$$

$$Ls^2\tilde{I} + Rs\tilde{I} + \frac{\tilde{I}}{C} = 225\frac{s}{s^2+1}\left(1-e^{-2\pi s}\right)$$

$$\boxed{\tilde{I} = 225\frac{s}{(s^2+1)(Ls^2+Rs+\frac{1}{C})}\left(1-e^{-2\pi s}\right)}$$

$$\frac{s}{(s^2+1)(s^2+2s+10)}$$

$$= \frac{1}{8}\left(\frac{-9s-20}{(s^2+2s+10)} + \frac{9s+2}{(s^2+1)}\right)$$

$$= \frac{1}{85}\left(-9\left(\frac{s+1}{(s+1)^2+9}\right) - \frac{11}{3}\left(\frac{3}{(s+1)^2+9}\right)\right.$$

$$\left. + 9\left(\frac{s}{s^2+1}\right) + 2\left(\frac{1}{s^2+1}\right)\right)$$

$$= \frac{1}{85}\left(-9\,\mathcal{L}^{-1}\left(\frac{s+1}{(s+1)^2+9}\right) - \frac{11}{3}\mathcal{L}^{-1}\left(\frac{3}{(s+1)^2+9}\right)\right.$$

$$\left. + 9\mathcal{L}^{-1}\left(\frac{s}{s^2+1}\right) + 2\,\mathcal{L}^{-1}\left(\frac{1}{s^2+1}\right)\right)$$

(s-shifting)

$$= \left(\frac{-9}{85}\cos(3t) - \frac{11}{175}\sin(3t)\right)e^{-t}$$

$$+ 9\cos(t) + 2\sin(t)$$

$$\underbrace{\frac{1}{8s} \qquad 8s}_{f(t)}$$

Notice that we have $-F(s)e^{-2\pi s}$

as the second piece to invert

$$\Rightarrow -\mathcal{L}^{-1}\left(F(s)\,e^{-2\pi s}\right)$$

$$= -f(t-2\pi)\,H(t-2\pi)$$

Thus:

$$y(t) = 225\left(f(t) - f(t-2\pi)H(t-2\pi)\right)$$

Notice that

$$f(t) = \left(\frac{-9}{85}\cos(3t) - \frac{11}{175}\sin(3t)\right)e^{-t} + 9\cos(t) + 2\sin(t)$$

$\Rightarrow f(t-2\pi)$ (sin & cos(x) and sin(x) are 2pi periodic)

$$= \frac{-9}{85}\cos(3t) - \frac{20}{255}\sin(3t)\, e^{-t+2\pi} + 9\cos(t) + 2\sin(t)$$

$$\Rightarrow f(t) - f(t-2\pi)$$

$$= \left[\left(\frac{-9}{85}\cos(3t) - \frac{11}{175}\sin(3t)\right)\underbrace{e^{-t}\left(1-e^{2\pi}\right)}_{g(t)}\right]$$

Thus for

	$y(t)$
$t<0$	0
$0<t<2\pi$	$f(t)\cdot 225$
$2\pi<t$	$g(t)\cdot 225$

2d