Double Pendulums:

Kinematics of Pendulums:

Position of the masses:

```
In[46]:= x1[t_] := L1 Sin[θ1[t]];
y1[t_] := -L1 Cos[θ1[t]];
x2[t_] := x1[t] + L2 Sin[θ2[t]];
y2[t_] := y1[t] - L2 Cos[θ2[t]];
```

Calculate velocity and acceleration of the two pendulums:

```
In[21]:= x1'[t];
y1'[t];
x2'[t];
y2'[t];
x1''[t];
y1''[t];
x2''[t];
y2''[t];
```

Forces on Pendulums:

T is the Tension in the rod
m is the mass of the pendulum
g is gravitational constant

For the upper pendulum:

```
m1 x1''[t] = -T1[t] Sin[θ1[t]] + T2[t] Sin[θ2[t]];
m1 y1''[t] = -T1[t] Cos[θ1[t]] - T2[t] Cos[θ2[t]] - m2 g;
```

For the lower pendulum:

```
m2 x2''[t] = -T2[t] Sin[θ2[t]];
m2 y2''[t] = T2[t] Cos[θ2[t]] - m2 g;
```

Finding Equations of motion:

```
m1 x1''[t] = -T1[t] Sin[θ1[t]] - m2 x2''[t];
m1 y1''[t] = T1[t] Cos[θ1[t]] - m2 y2''[t] - m2 g - m1 g;
```

```
Sin[θ1[t]] (m1 y1''[t] + m2 y2''[t] + m2 g + m1 g) = -Cos[θ1[t]] (m1 x1''[t] + m2 x2''[t])
```

```
T2[t] Sin[θ2[t]] Cos[θ2[t]] = -Cos[θ2[t]] (m2 x2''[t])
T2[t] Sin[θ2[t]] Cos[θ2[t]] = Sin[θ2[t]] (m2 y2''[t] + m2 g)
```

```
Sin[θ2[t]] (m2 y2''[t] + m2 g) = -Cos[θ2[t]] (m2 x2''[t])
```

Solve for $\theta 1''$ and $\theta 2''$:

```
In[53]:= Solve[{Sin[θ2[t]] (m2 y2''[t] + m2 g) == -Cos[θ2[t]] (m2 x2''[t]),
    Sin[θ1[t]] (m1 y1''[t] + m2 y2''[t] + m2 g + m1 g) ==
     -Cos[θ1[t]] (m1 x1''[t] + m2 x2''[t])}, {θ1''[t], θ2''[t]}] // FullSimplify
```

```
{{θ1''[t] → -((g (2 m1 + m2) Sin[θ1[t]] + g m2 Sin[θ1[t] - 2 θ2[t]] +
        2 m2 Sin[θ1[t] - θ2[t]] (L1 θ1'[t]^2 Cos[θ1[t] - θ2[t]] + L2 θ2'[t]^2)) /
      (L1 (2 m1 - m2 Cos[2 (θ1[t] - θ2[t])] + m2))), θ2''[t] → (2 Sin[θ1[t] - θ2[t]]
      ((m1 + m2) (g Cos[θ1[t]] + L1 θ1'[t]^2) + L2 m2 θ2'[t]^2 Cos[θ1[t] - θ2[t]])) /
    (L2 (2 m1 - m2 Cos[2 (θ1[t] - θ2[t])] + m2))}}
```

Numerical Solution

We' ll use RK4 to solve these ODEs

```
θ1'[t] = ω1[t]
θ2'[t] = ω2[t]
w1'[t] = -((g (2 m1 + m2) Sin[θ1[t]] + g m2 Sin[θ1[t] - 2 θ2[t]] +
       2 m2 Sin[θ1[t] - θ2[t]] (L1 ω1[t]^2 Cos[θ1[t] - θ2[t]] + L2 ω2[t]^2)) /
     (L1 (2 m1 - m2 Cos[2 (θ1[t] - θ2[t])] + m2)))
```

```
w2'[t] = (2 Sin[θ1[t] - θ2[t]]
     ((m1 + m2) (g Cos[θ1[t]] + L1 ω1[t]^2) + L2 m2 ω2[t]^2 Cos[θ1[t] - θ2[t]])) /
   (L2 (2 m1 - m2 Cos[2 (θ1[t] - θ2[t])] + m2))
```