MATH 130
Test 2
7/10/2023
Time Limit: 90 Minutes

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This exam contains 8 pages (including this cover page) and 10 questions with multiple parts.
Total of number points is 100.

You are required to show your work on each free-response problem on this exam. The following rules apply:

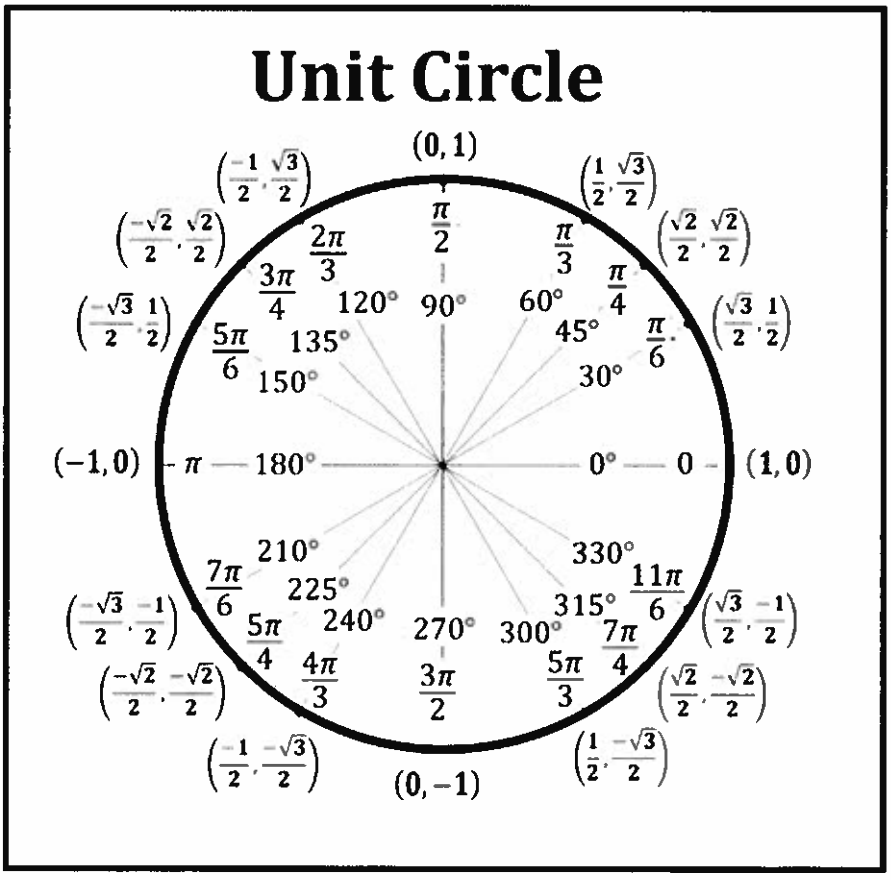
- No outside material is allowed on the exam.
- Use of cellphones, laptops, or similar technology is prohibited.
- Illegible answers will not receive credit.
- Answers without work and justification will not receive credit.
- Only work written on the exam sheet will be graded. If you use a scratch sheet, make sure your complete answer is copied onto the exam sheet.
- On problems with multiple parts, clearly separate your work and mark each part.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

UNC Honor Pledge: I certify that no unauthorized assistance has been received or given in the completion of this work

Signature and Date: SOLUTION GUIDE

Question	Points	Score
1	18	
2	4	
3	18	
4	8	
5	12	
6	8	
7	8	
8	24	
Total:	100	

Cheat Sheet



FORMULAS

$\cos^2(\theta) + \sin^2(\theta) = 1$

$\tan^2(\theta) + 1 = \sec^2(\theta)$

$\cot^2(\theta) + 1 = \csc^2(\theta)$

For an angle θ in standard position, let $P = (x, y)$ be the point on the terminal side of θ that is also on the circle $x^2 + y^2 = r^2$. Then

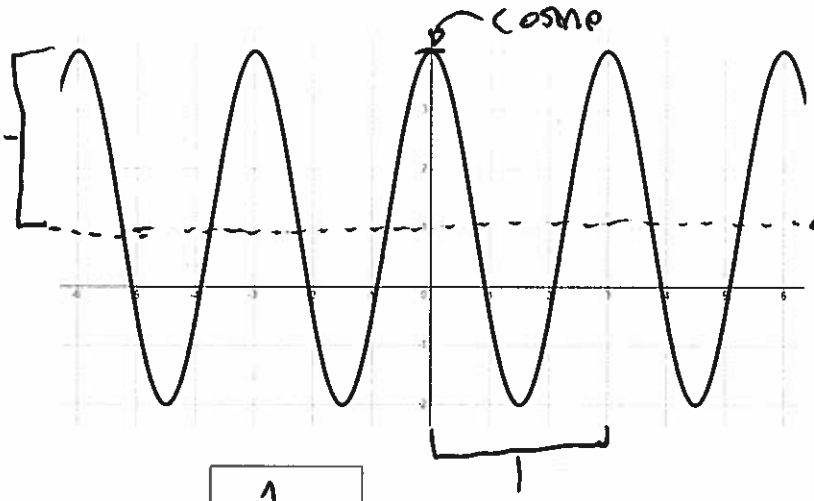
$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x} \quad x \neq 0$$

$$\csc\theta = \frac{r}{y} \quad y \neq 0 \qquad \sec\theta = \frac{r}{x} \quad x \neq 0 \qquad \cot\theta = \frac{x}{y} \quad y \neq 0$$

Inverse Trigonometric Functions

Function	Domain	Range	Quadrant of the Unit Circle Range Values come from
$y = \arcsin x$	$[-1,1]$	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	I and IV
$y = \arccos x$	$[-1,1]$	$[0,\pi]$	I and II
$y = \arctan x$	$(-\infty,\infty)$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$	I and IV
$y = \operatorname{arccot} x$	$(-\infty,\infty)$	$(0,\pi)$	I and II
$y = \operatorname{arcsec} x$	$(-\infty,-1]\cup[1,\infty)$	$[0,\pi], y \neq \frac{\pi}{2}$	I and II
$y = \operatorname{arccsc} x$	$(-\infty,-1]\cup[1,\infty)$	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right], y \neq 0$	I and IV

1. (18 points) For the following graph, answer the ~~two~~ six questions below:



(a) The Midline is: $y =$ 1

(b) The Amplitude is: 3

(c) The Period is: 3

(d) The Frequency (using the Period) is: $\frac{1}{3}$ $= \frac{1}{\text{Period}}$

(e) The Phase (using the Period) is: $\frac{2}{3}\pi$ $= \frac{2\pi}{\text{Period}}$

(f) Write the equation for the plot above using the prior information:

$f(x) =$ $3\cos\left(\frac{2\pi}{3}x+1\right)$

I know it's cosine since at $x=0$, $f(x) \neq 0$.

2. (4 points) Which of the following is not in the range of $\csc(x)$:

A. -1

(B.) 0

C. 1

D. 2

E. All real numbers are in the range

range of csc = domain of $\csc^{-1} = (-\infty,1]\cup[1,\infty$

0 is not in

3. (18 points) For the function $g(x) = 2\sin(x) - 1$ where $\frac{-\pi}{2} \leq x \leq \frac{\pi}{2}$

(a) Find $g^{-1}(x)$

$y = 2\sin(x) - 1$

$\Rightarrow x = 2\sin(y) - 1$

$\Rightarrow \frac{x+1}{2} = \sin(y)$

$\Rightarrow y = \sin^{-1}\left(\frac{x+1}{2}\right)$

Also for range

$-1 \leq \frac{x+1}{2} \leq 1$

$-2 \leq x+1 \leq 2$

$-3 \leq x \leq 1$

$[-3, 1]$

$g^{-1}(x) = \boxed{\sin^{-1}\left(\frac{x+1}{2}\right)}$

(b) Find the domain and range of of $g(x)$ and $g^{-1}(x)$. Write your answers in interval notation.

known in advance of from range of $\sin^{-1}$

Domain of $g(x)$: $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

Copies

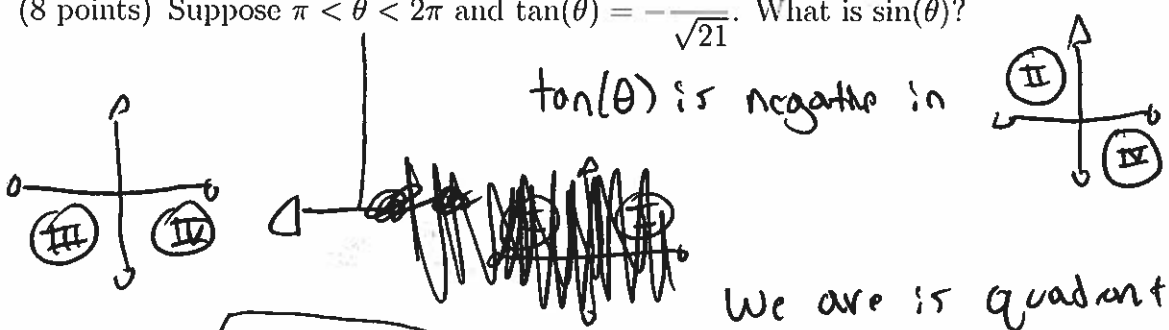
Domain of $g^{-1}(x)$: $[-3, 1]$

Range of $g(x)$: $[-3, 1]$

Copies

Range of $g^{-1}(x)$: $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

4. (8 points) Suppose $\pi < \theta < 2\pi$ and $\tan(\theta) = -\frac{2}{\sqrt{21}}$. What is $\sin(\theta)$?



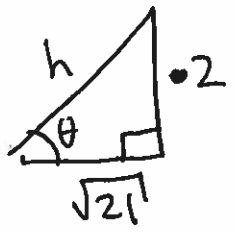
$\tan(\theta)$ is negative in II and IV

We are in quadrant IV

$\Rightarrow \sin\theta < 0$

Answer: $\boxed{-\frac{2}{5}}$

$\tan(\theta) = \frac{\text{Opp.}}{\text{Adj.}} \Rightarrow$



$2^2 + \sqrt{21}^2 = 25$

$\Rightarrow h = 5$

$\sin\theta = \frac{\text{Opp.}}{\text{Hyp.}} = \frac{2}{5}$

5. (12 points) The height of a bouncing ball is approximated by the eqn $g(t) = -3\sin(\pi t) + 3$, where t is time in seconds after 12:00 noon and $g(t)$ is height in meters.

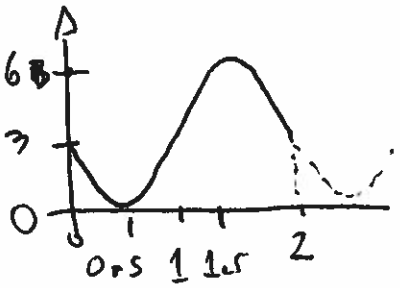
   (a) What is the maximum height of the ball?

3 = mid line

3 = amp

$\frac{2\pi}{\pi}$ = period = 2

$\frac{\pi}{3}$ =



[midline + amp] = 6

Answer: 6 meters

   (b) How long does it take for the ball to go from the maximum height to the ground and back up to its maximum height again?

Period

Answer: 2 seconds

   (c) What is the time in seconds that the ball first strikes the ground.

$\frac{\text{Period}}{4}$ = 0.5 or from image

Answer: $\frac{1}{2}$ seconds

6. (8 points) The expression $\dfrac{\sec\theta}{\cos\theta} - \dfrac{\tan\theta}{\cot\theta}$ simplifies to a number. What is it?

$\sec(\theta) = \frac{1}{\cos\theta} \Rightarrow \frac{\sec\theta}{\cos\theta} = \frac{1}{\cos^2\theta}$

$\cot(\theta) = \frac{1}{\tan\theta} \Rightarrow \frac{\tan\theta}{\cot\theta} = \tan^2\theta = \frac{\sin^2}{\cos^2\theta}$

$\Rightarrow \frac{\sec\theta}{\cos\theta} - \frac{\tan\theta}{\cot\theta} = \frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}$

$= \frac{1-\sin^2\theta}{\cos^2\theta} = \frac{\cos^2\theta}{\cos^2\theta} = 1$

Answer: 1

($\sin^2\theta + \cos^2\theta = 1$
$\Rightarrow 1 - \sin^2(\theta) = \cos^2\theta$)

7. (8 points) The expression $\sin(\theta - \frac{\pi}{2}) + \cos(\theta)$ simplifies to a number. What is it?

$\sin(\theta - \frac{\pi}{2})$

$= -\sin(-\theta + \frac{\pi}{2}) = -\cos(-\theta) = -\cos(\theta)$

[sin is odd] [cos is even]

$\Rightarrow \sin(\theta - \frac{\pi}{2}) + \cos(\theta)$

$= -\cos(\theta) + \cos(\theta) = 0$

Answer: 0

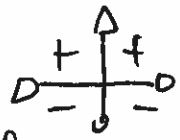
8. (24 points) Find the exact values of the following. If the value does not exist, write "DNE".

(a) $\sin^{-1}(2)$

[redacted] Domain of $\sin^{-1}(x)$ is $[-1, 1]$, 2 is outside of that!

Answer: DNE

(b) $\cos(\sin^{-1}(-\frac{4}{5}))$

negative →  → Quadrant III or IV

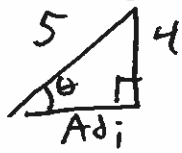
$\sin^{-1}$ ranges from Quadrant IV to I

(in other words $[\frac{-\pi}{2}, \frac{\pi}{2}]$)

Positive

Thus, we are in quadrant IV => cos is. ~~negative~~

$\sin(\theta) = \frac{4}{5} = \frac{Opp}{Hyp}$ =>



=> Adj = 3

$\cos(\theta) = \frac{Adj}{Hyp} = \frac{3}{5}$

Answer: $\frac{3}{5}$

(c) $\tan^{-1}(\tan(\frac{7\pi}{8}))$

Answer: $\frac{-\pi}{8}$

tan has a period of π!

$\tan(\frac{7\pi}{8}) = \tan(\frac{7\pi}{8} - \pi)$

$= \tan(\frac{-\pi}{8})$

$\tan^{-1}(\tan(\frac{-\pi}{8}))$

$= \frac{-\pi}{8}$

$\frac{-\pi}{8}$ is between $[\frac{-\pi}{2}, \frac{\pi}{2}]$!