MATH 130
Test 3
7/17/2023
Time Limit: 90 Minutes

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This exam contains 7 pages (including this cover page) and 8 questions with multiple parts. Total number of points is 100.

You are required to show your work on each free-response problem on this exam. The following rules apply:

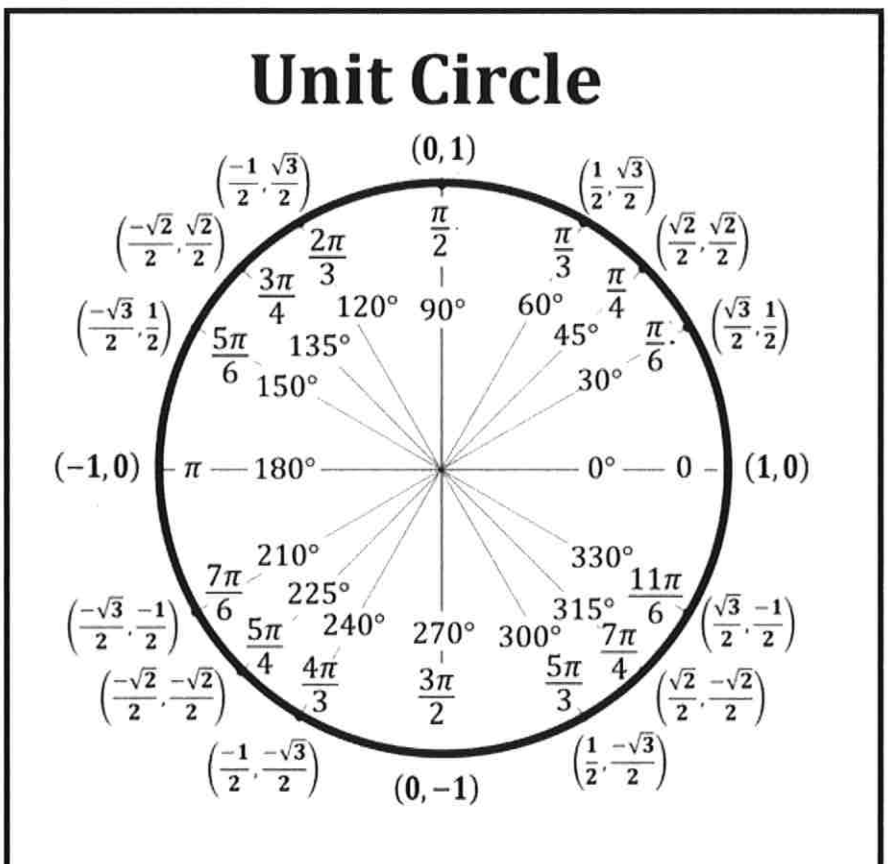
- No outside material is allowed on the exam.
- Use of cellphones, laptops, or similar technology is prohibited.
- Illegible answers will not receive credit.
- Answers without work and justification will not receive credit.
- Only work written on the exam sheet will be graded. If you use a scratch sheet, make sure your complete answer is copied onto the exam sheet.
- On problems with multiple parts, clearly separate your work and mark each part.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

UNC Honor Pledge: I certify that no unauthorized assistance has been received or given in the completion of this work

Signature and Date: SOLUTION GUIDE

Question	Points	Score
1	8	
2	8	
3	8	
4	8	
5	16	
6	16	
7	18	
8	18	
Total:	100	

Cheat Sheet



FORMULAS

$\cos^2(\theta) + \sin^2(\theta) = 1$

$\tan^2(\theta) + 1 = \sec^2(\theta)$

$\cot^2(\theta) + 1 = \csc^2(\theta)$

$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$

$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$

$\sin(\alpha - \beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$

$\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$

Double Angle

$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$

$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$

Half Angle

$\sin\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{2}}$

$\cos\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$

Law of Cosines

$c^2 = a^2 + b^2 - 2ab\cos(C)$ Is useful for SSS, SAS Triangles

$a^2 = c^2 + b^2 - 2cb\cos(A)$

$b^2 = a^2 + c^2 - 2ac\cos(B)$

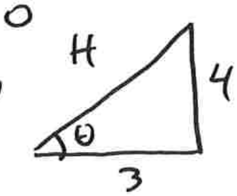
Law of Sines

$\dfrac{\sin(A)}{a} = \dfrac{\sin(B)}{b} = \dfrac{\sin(C)}{c}$ Is useful for SAA, ASA, SSA Triangles

1. (8 points) If $\tan\theta = \frac{4}{3}$ and θ terminates in the first quadrant, find $\sin(2\theta)$.

double angle

θ is in quadrant I $\Rightarrow \sin(\theta) > 0, \cos(\theta) > 0$



$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$

$= 2 \cdot \frac{4}{5} \cdot \frac{3}{5} = \frac{24}{25}$

$\tan(\theta) = \frac{O}{A} = \frac{4}{3} \Rightarrow$

$\Rightarrow H = \sqrt{4^2 + 3^2} = 5$

$\Rightarrow \sin(\theta) = \frac{O}{H} = \frac{4}{5}, \cos(\theta) = \frac{A}{H} = \frac{3}{5}$

Answer: $\boxed{\frac{24}{25}}$

2. (8 points) Provide 3 solutions of $\sec(\theta) = \sqrt{2}$ in degrees.

$\sec(\theta) = \sqrt{2} \Rightarrow \cos(\theta) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \Rightarrow \theta = 45°$ or $315°$

need one more solution so just add $360°$

$45° + 360° = 405°$

Answer: $\boxed{45°, 315°, 405°}$

3. (8 points) Use the half angle formula to find $\sin(165°)$.

$165° = \frac{330°}{2} \Rightarrow \sin\left(\frac{330}{2}\right) = \pm\sqrt{\frac{1-\cos(330)}{2}} = \pm\sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}}$

$\sin(165°)$ is in II $\Rightarrow \sin(165°) > 0$

Answer: $\boxed{\sqrt{\frac{1}{2} - \frac{\sqrt{3}}{4}}}$

4. (8 points) Find the EXACT value of $\sin(\frac{5\pi}{12})$. Hint: Sum formula will work here

$\frac{5\pi}{12} = \frac{\pi}{4} + \frac{\pi}{6}$

or $= \frac{3\pi}{12} + \frac{2\pi}{12}$

$\Rightarrow \sin\left(\frac{5\pi}{12}\right) = \sin\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$

$= \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) + \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= \frac{\sqrt{6} + \sqrt{2}}{4}$

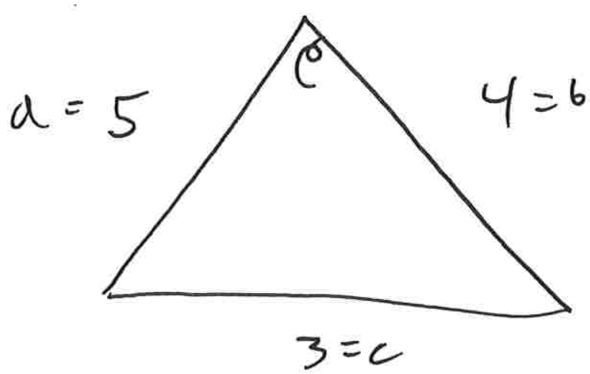
Answer: $\boxed{\frac{\sqrt{2} + \sqrt{6}}{4}}$

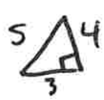
5. (16 points) Show the identity $\frac{1+\sin(\theta)}{1+\csc(\theta)} = \sin(\theta)$ is true.

$$\frac{1+\sin\theta}{1+\csc\theta} = \frac{1+\sin\theta}{1+\frac{1}{\sin\theta}} = \frac{(1+\sin\theta)}{\left(1+\frac{1}{\sin\theta}\right)} \cdot \frac{\sin\theta}{\sin\theta} = \frac{\sin\theta\,(1+\sin\theta)}{(\sin\theta+1)} = \sin(\theta)$$

Answer: $\boxed{}$

6. (16 points) A triangle has sides of length $a = 5$ inches, $b = 4$ inches, and $c = 3$ inches. What is the measure of angle C? Put your answer in a form that could be typed into a calculator to get a numerical answer.



(you can also recognize this as the 3-4-5 triangle )

Law of cosines $\Rightarrow c^2 = a^2 + b^2 - 2ab\cos(C)$

$$\Rightarrow C = \cos^{-1}\left(\frac{c^2-a^2-b^2}{-2ab}\right)$$

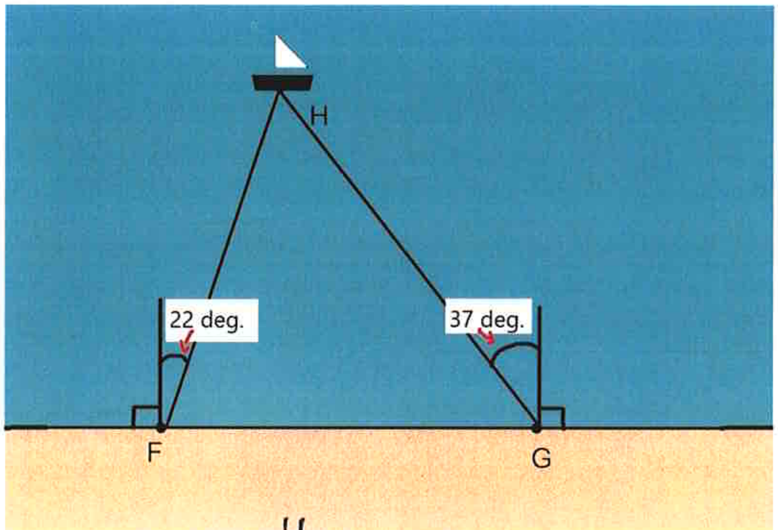
$$= \cos^{-1}\left(\frac{9-25-16}{-2\cdot5\cdot4}\right)$$

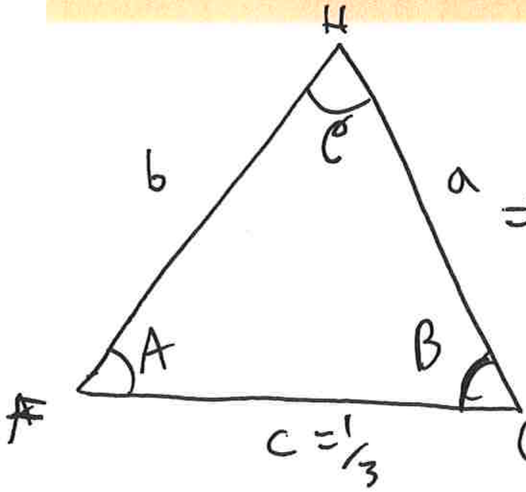
$$= \cos^{-1}\left(\frac{-32}{-40}\right) = \cos^{-1}\left(\frac{4}{5}\right)$$

Answer: $\boxed{\cos^{-1}\left(\frac{4}{5}\right)}$ inches

7. (18 points) Two observers are standing on shore $\frac{1}{3}$ mile apart at points F and G. The observer at point F sees the ship at a bearing of $N22°E$. The observer at point G sees the ship at a bearing of $N37°W$ (see image below for help). Find the distance from each observer to the sailboat. Assume the shoreline runs west to east with point F west of point G.

Put your answer in a form that could be typed into a calculator to get a numerical answer.





$A = 90° - 22° = 68°$

$B = 90° - 37° = 53°$

$\Rightarrow C = 180° - 68° - 53° = 59°$

Law of sines

$\frac{b}{\sin(53°)} = \frac{1/3}{\sin(59°)} \Rightarrow b = \frac{\frac{1}{3}\sin(53)}{\sin(59)}$

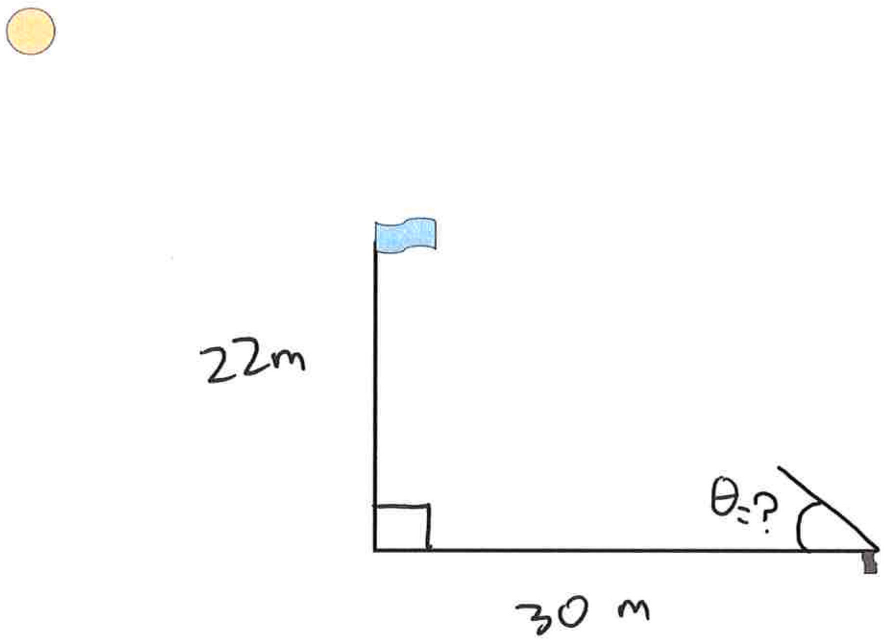
Distance from point F: $\boxed{\frac{1}{3}\frac{\sin(53)}{\sin(59)} \text{ miles}}$

Distance from point G: $\boxed{\frac{1}{3}\frac{\sin(68)}{\sin(59)} \text{ miles}}$

$\frac{a}{\sin(68)} = \frac{1/3}{\sin(59)} \Rightarrow a = \frac{\frac{1}{3}\sin(68)}{\sin(59)}$

8. (18 points) A vertical flagpole that is 22 meters tall makes a shadow on the ground that is 30 meters long. What is the angle of elevation of the sun?

Put your answer in a form that could be typed into a calculator to get a numerical answer.



$$\tan(\theta) = \frac{22m}{33m} \Rightarrow \theta = \tan^{-1}\left(\frac{22}{33}\right)$$

Answer: $\boxed{\tan^{-1}\left(\frac{22}{33}\right) \text{ meters}}$