This exam contains 8 pages (including this cover page) and 10 questions with multiple parts.

Total of number points is 100.

You are required to show your work on each free-response problem on this exam. The following rules apply:
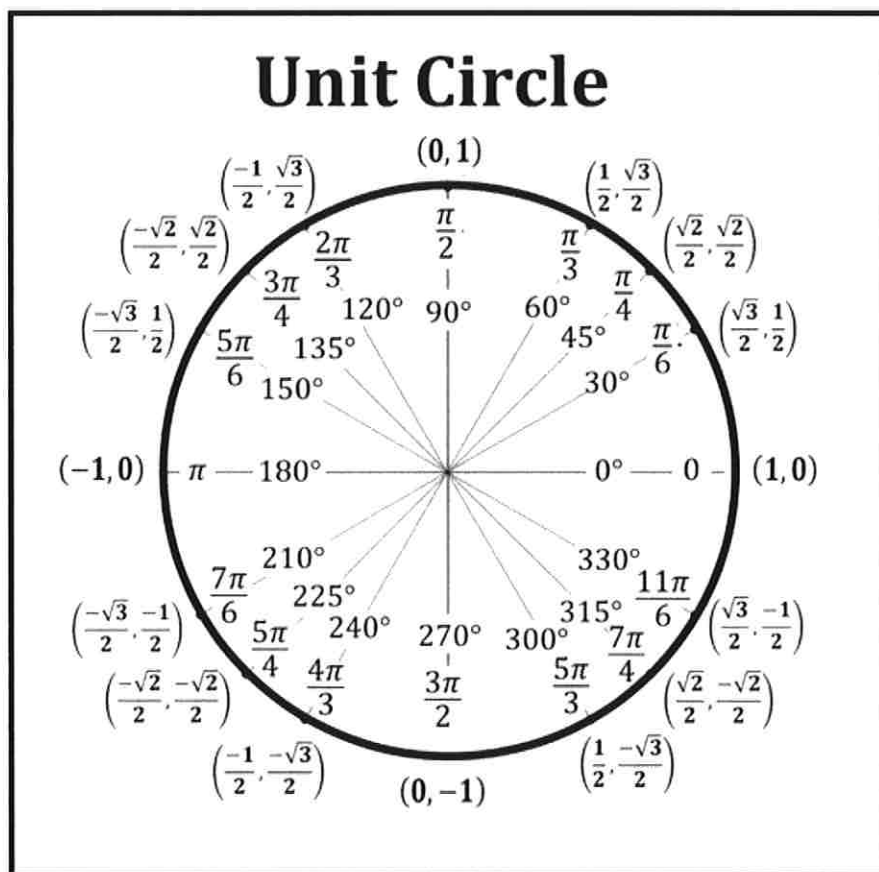
- No outside material is allowed on the exam.

- Use of cellphones, laptops, or similar technology is prohibited.

- Illegible answers will not receive credit.

- Answers without work and justification will not receive credit.

- Only work written on the exam sheet will be graded. If you use a scratch sheet, make sure your complete answer is copied onto the exam sheet.

- On problems with multiple parts, clearly separate your work and mark each part.

- If you need more space, use the back of the pages; clearly indicate when you have done this.

UNC Honor Pledge: I certify that no unauthorized assistance has been received or given in the completion of this work

Signature and Date: _Solution Guide_

Question	Points	Score
1	6	
2	6	
3	4	
4	12	
5	14	
6	10	
7	16	
8	6	
9	10	
10	16	
Total:	100	

Cheat Sheet

Unit Circle

Slope of secant line is: $\frac{f(b)-f(a)}{b-a}$

Arclength is: $s = r\theta$

Area is: $A = \frac{1}{2}r^2\theta$

Angular speed is: $\omega = \frac{\theta}{t}$

Linear speed is: $v = r\omega$

To Graph:	Draw the Graph of f and:	Functional Change to $f(x)$
Vertical shifts		
$y = f(x) + k, \quad k > 0$	Shift the graph of f up k units.	Add k to $f(x)$.
$y = f(x) - k, \quad k > 0$	Shift the graph of f down k units.	Subtract k from $f(x)$.
Horizontal shifts		
$y = f(x + h), \quad h > 0$	Shift the graph of f to the left h units.	Replace x by $x + h$.
$y = f(x - h), \quad h > 0$	Shift the graph of f to the right h units.	Replace x by $x - h$.
Compressing or stretching		
$y = af(x), \quad a > 0$	Multiply each y-coordinate of $y = f(x)$ by a. Stretch the graph of f vertically if $a > 1$. Compress the graph of f vertically if $0 < a < 1$.	Multiply $f(x)$ by a.
$y = f(ax), \quad a > 0$	Multiply each x-coordinate of $y = f(x)$ by $\frac{1}{a}$. Stretch the graph of f horizontally if $0 < a < 1$. Compress the graph of f horizontally if $a > 1$.	Replace x by ax.
Reflection about the x-axis		
$y = -f(x)$	Reflect the graph of f about the x-axis.	Multiply $f(x)$ by -1.
Reflection about the y-axis		
$y = f(-x)$	Reflect the graph of f about the y-axis.	Replace x by $-x$.

For an angle θ in standard position, let $P = (x, y)$ be the point on the terminal side of θ that is also on the circle $x^2 + y^2 = r^2$. Then

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x} \quad x \neq 0$$

$$\csc\theta = \frac{r}{y} \quad y \neq 0 \qquad \sec\theta = \frac{r}{x} \quad x \neq 0 \qquad \cot\theta = \frac{x}{y} \quad y \neq 0$$

1. (6 points) Match the following descriptions to EXACTLY ONE of the maps below. Write in the corresponding letter to the right of the correct statement.

This mapping is not a function: $\boxed{X}$ one element in domain (B) maps to two elements (2,4) in range
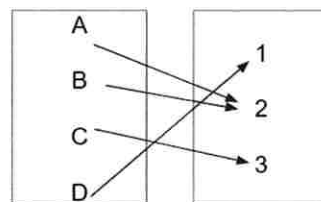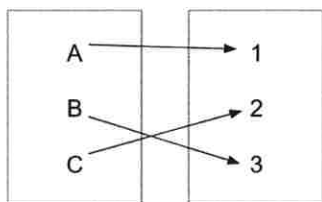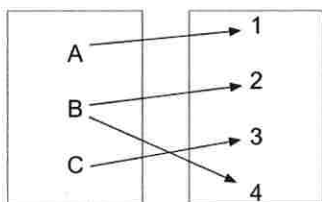
This mapping is a function but is not invertible: $\boxed{Z}$ A → Two elements in domain (A,B) maps to one element (2) in range

This mapping is both a function and invertible: $\boxed{Y}$ One - to - one

2. (6 points) Match the following descriptions to EXACTLY ONE of the plots below. Write in the corresponding letter to the right of the correct statement.
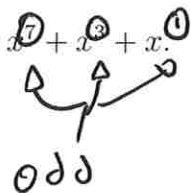
This plot is not a function: $\boxed{Y}$ (fails vertical line test)

This plot is a function but is not invertible: $\boxed{Z}$ (fails horizontal line test)

This plot is both a function and invertible: $\boxed{X}$ (passes both V+H line test)
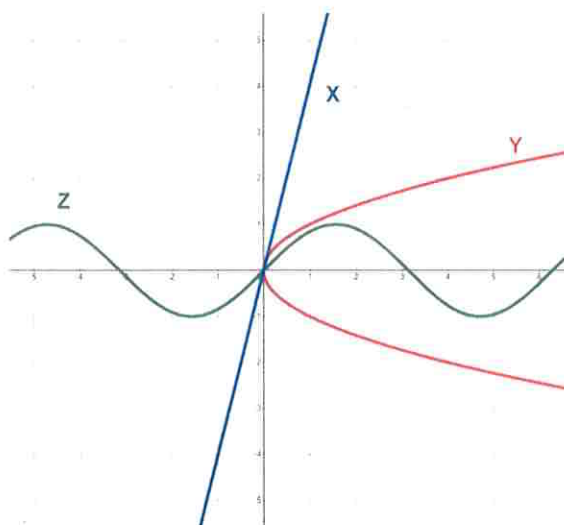
3. (4 points) Decide if the function is even or odd or neither: $f(x) = x^7 + x^3 + x^1$

     A. even

     B. odd

     C. neither

(handwritten) odd

4. (12 points) For the following graph, answer the two questions (assume the domain is infinite).

(handwritten, point labels) (-2, 3), (1, 0)
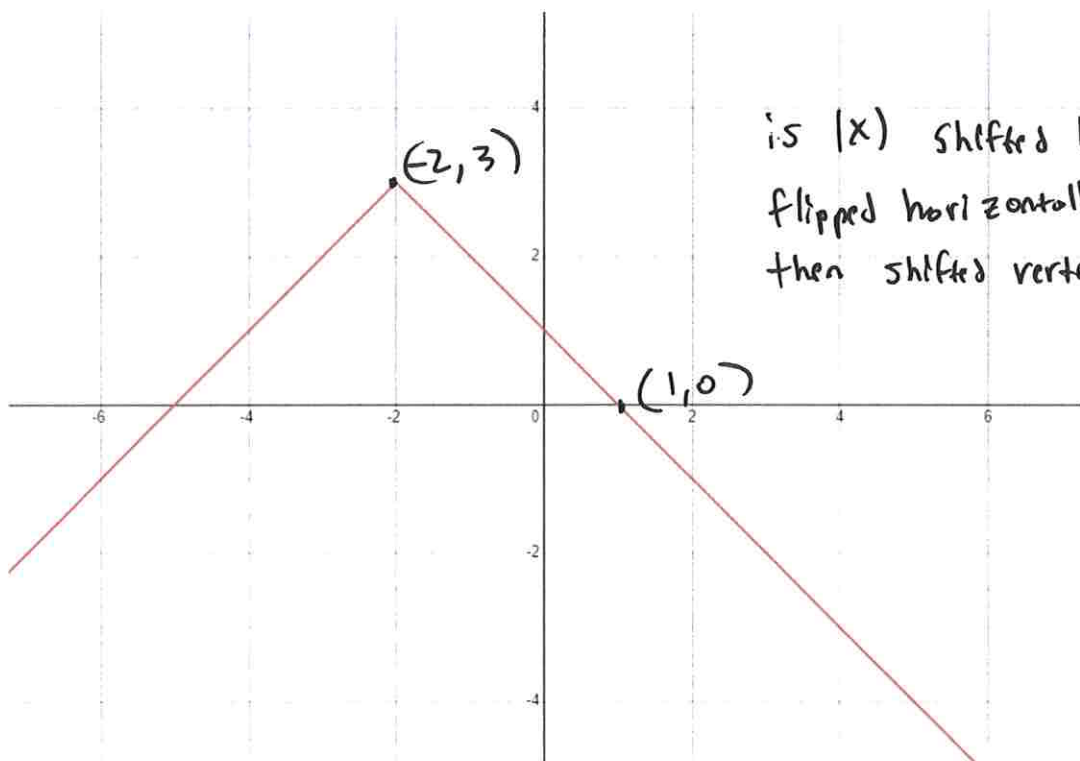
(handwritten note) is |x| shifted left 2, flipped horizontally, and then shifted vertically.

(a) Find the equation of the function graphed above:

     A. $y = -|x + 2| + 3$

     B. $y = -|x - 2| - 3$ *(handwritten) (-2,3) doesn't work*

     C. $y = |-x - 2| + 3$ *(handwritten) (1,0) doesn't work*

     D. $y = -(x + 2)^2 - 3$

     E. $y = (-x + 2)^2 - 3$ *(handwritten) not absolute value*

(b) The absolute maximum is (put a number or DNE): **3**

The absolute minimum is (put a number or DNE): **PNE** *(handwritten) (goes to -infinity)*

5. (14 points) For the function $g(x) = \dfrac{x+1}{x}$:

(a) Find $g^{-1}(x)$

$$y = \frac{x+1}{x} \implies x = \frac{y+1}{y}$$

$$\implies yx - y = 1$$

$$\implies y(x-1) = 1$$

$$\implies y = \frac{1}{x-1}$$

$$g^{-1}(x) = \boxed{\frac{1}{x-1}}$$

(b) Find the domain and range of of $g(x)$ and $g^{-1}(x)$. Write your answers in interval notation.

Domain of $g(x)$

$\boxed{(-\infty, 0) \cup (0, \infty)}$ division by 0 if $x=0$

Domain of $g^{-1}(x)$

$\boxed{(-\infty, 1) \cup (1, \infty)}$ division by 0 if $x=1$

Range of $g(x)$

Copied $\boxed{(-\infty, 1) \cup (1, \infty)}$

Range of $g^{-1}(x)$

$\boxed{(-\infty, 0) \cup (0, \infty)}$ Copied

6. (10 points) Suppose $0 < \theta < \pi$ and $\cos(\theta) = \dfrac{3}{5}$. What is $\sin(\theta)$?

is positive

Answer: $\boxed{4/5}$

$\cos(\theta) = \frac{3}{5} = \frac{A}{H}$

1st Quadrant

$3^2 + \theta^2 = 5^2 \implies \theta^2 = 16 \implies \theta = 4$

7. (16 points) Find the exact values of the following expressions. "Exact value" means your answers should have square roots in them, if necessary, instead of decimal approximations. If undefined, write "undefined".

(a) $\sin(390°)$.

$$390° - 360° = 30°$$

$$\sin(390°) = \sin(30°) = \tfrac{1}{2} \quad \text{(unit circle)}$$

$$\boxed{\tfrac{1}{2}}$$

(b) $\cos\left(-\dfrac{15\pi}{4}\right)$

$$\boxed{\dfrac{\sqrt{2}}{2}}$$

① $\quad \dfrac{-15\pi}{4} + 2\pi$

$$= \dfrac{-15\pi}{4} + \dfrac{8\pi}{4} = \dfrac{-7\pi}{4}$$

$$\cos\left(\dfrac{-15\pi}{4}\right) = \cos\left(\dfrac{\pi}{4}\right) = \dfrac{\sqrt{2}}{2}$$

(unit circle)

② $\quad \dfrac{-7\pi}{4} + 2\pi = \dfrac{-7\pi}{4} + \dfrac{8\pi}{4} = \dfrac{\pi}{4}$

(c) $\csc(-90°)$

$$\csc(-90°) = \dfrac{1}{\sin(-90°)} = \dfrac{1}{-1} = -1$$

$$\boxed{-1}$$

(d) $\cot(\pi) = \dfrac{\cos(\pi)}{\sin(\pi)} = \dfrac{-1}{0}$

$$\boxed{DNE}$$

8. (6 points) What is the slope of the secant line of the plot $f(x) = x^2 + x - 1$ through the points at $x = 1, 3$?

$$\overset{\overset{Y}{a}}{f(6)} = 3^2+3-1 = 11$$
$$\overset{\overset{Y}{b}}{f(a)} = 1^2+1-1 = 1$$

$$\frac{f(b)-f(a)}{b-a} = \frac{11-1}{3-1} = \frac{10}{2}$$
$$= 5$$

Answer: $\boxed{5}$

9. (10 points) A satellite moves in a circular orbit around the Earth at a distance of 36,000 miles from the center of the Earth. Each hour, it travels through an angle of 30° as measured from the center of the Earth. What is the (linear) speed of the satellite in miles per hour?

linear speed $= V = r \cdot \underset{\text{in radians}}{\theta}$

$$36,000 \text{ miles} = r$$

$$30° \cdot \frac{\pi}{180°} = \frac{\pi}{6} \text{ radians}$$

$$\Rightarrow V = \frac{36000}{6} \pi = 6000\pi \text{ mph}$$

Answer: $\boxed{6000\pi \text{ mph}}$

10. (16 points) A salvage ship's sonar locates wreckage at a 12° angle of depression. A diver is lowered 40 meters to the ocean floor. How far does the diver need to walk along the ocean floor to the wreckage? Write your answer in a form that could be typed into a calculator to get a numerical answer.

12°

$$\overset{\circ}{90}-12 = 78°$$

40 m

diver x wreckage

Answer: $\boxed{\dfrac{40}{\tan(12°)} \text{ meters}}$

or $\tan(78°) \cdot 40$ meters

12°

40m

12° x

$$\Rightarrow \tan(12°) = \frac{40m}{x}$$

$$x = \frac{40m}{\tan(12°)}$$