MATH 130
Test 4
7/24/2023
Time Limit: 90 Minutes

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This exam contains 8 pages (including this cover page) and 8 questions with multiple parts. Total number of points is 100.

You are required to show your work on each free-response problem on this exam. The following rules apply:

- No outside material is allowed on the exam.
- Use of calculators, cellphones, laptops, or similar technology is prohibited.
- Illegible answers will not receive credit.
- Answers without work and justification will not receive credit.
- Only work written on the exam sheet will be graded. If you use a scratch sheet, make sure your complete answer is copied onto the exam sheet.
- On problems with multiple parts, clearly separate your work and mark each part.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

UNC Honor Pledge: I certify that no unauthorized assistance has been received or given in the completion of this work

Signature and Date: SOLUTION GUIDE

Question	Points	Score
1	15	
2	15	
3	20	
4	15	
5	10	
6	15	
7	10	
Total:	100	

Cheat Sheet

Equations of a Parabola: Vertex at (h, k); Axis of Symmetry Parallel to a Coordinate Axis; $a > 0$

Vertex	Focus	Directrix	Equation	Description
(h, k)	$(h+a, k)$	$x = h - a$	$(y-k)^2 = 4a(x-h)$	Axis of symmetry is parallel to the x-axis, opens right
(h, k)	$(h-a, k)$	$x = h + a$	$(y-k)^2 = -4a(x-h)$	Axis of symmetry is parallel to the x-axis, opens left
(h, k)	$(h, k+a)$	$y = k - a$	$(x-h)^2 = 4a(y-k)$	Axis of symmetry is parallel to the y-axis, opens up
(h, k)	$(h, k-a)$	$y = k + a$	$(x-h)^2 = -4a(y-k)$	Axis of symmetry is parallel to the y-axis, opens down

Equations of an Ellipse: Center at (h, k); Major Axis Parallel to a Coordinate Axis

Center	Major Axis	Foci	Vertices	Equation
(h, k)	Parallel to the x-axis	$(h+c, k)$	$(h+a, k)$	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1,$
		$(h-c, k)$	$(h-a, k)$	$a > b > 0$ and $b^2 = a^2 - c^2$
(h, k)	Parallel to the y-axis	$(h, k+c)$	$(h, k+a)$	$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1,$
		$(h, k-c)$	$(h, k-a)$	$a > b > 0$ and $b^2 = a^2 - c^2$

Hyperbolas with Center at (h, k) and Transverse Axis Parallel to a Coordinate Axis

Center	Transverse Axis	Foci	Vertices	Equation	Asymptotes
(h, k)	Parallel to the x-axis	$(h \pm c, k)$	$(h \pm a, k)$	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1, \quad b^2 = c^2 - a^2$	$y - k = \pm\frac{b}{a}(x-h)$
(h, k)	Parallel to the y-axis	$(h, k \pm c)$	$(h, k \pm a)$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1, \quad b^2 = c^2 - a^2$	$y - k = \pm\frac{a}{b}(x-h)$

1. (15 points) Find the center, vertices, and foci of the ellipse $\frac{(x+2)^2}{25} + \frac{(y-3)^2}{9} = 1.$

$(x - \underbrace{(-2)}_{h})$ $(y - \underbrace{(+3)}_{k})$

$\Rightarrow (h,k) = (-2,3)$

Center: $\boxed{(-2,3)}$

$25 > 9 \Rightarrow a^2 = 25,\ b^2 = 9$

$\Rightarrow a = 5$

Vertices $=$ center $\pm (a,0)$

$\Rightarrow (-2,3) \pm (5,0)$

Vertices: $\boxed{(3,3),\ (-7,3)}$

$a^2 - b^2 = c^2 \Rightarrow 25 - 9 = 16 = c^2$

$\Rightarrow 4 = c$

foci $=$ center $\pm (c,0)$

$\Rightarrow (-2,3) \pm (4,0)$

Foci: $\boxed{(2,3),\ (-6,3)}$

2. (15 points) Write the equation for the hyperbola in standard form and find the equations of the asymptotes. $x^2 - 2x - 4y^2 - 40y = 103$

$$(x^2 - 2x + \underline{c_1}) + (-4)(y^2 + 10y + \underline{c_2}) = 103$$

$$\left(\frac{-2}{2}\right)^2 = c_1 = 1 \quad , \quad \left(\frac{10}{2}\right)^2 = c_2 = 25$$

$$\Rightarrow (x^2 - 2x + 1) + (-4)(y^2 + 10y + 25) = 103 + 1 - \overbrace{100}^{(4\cdot 25)}$$

$$\Rightarrow (x-1)^2 + (-4)(y+5)^2 = 4$$

$$\Rightarrow \frac{(x-1)^2}{+4} + \frac{(y+5)^2}{-1} = 1$$

$$\Rightarrow \frac{(x-1)^2}{4} - \frac{(y+5)^2}{1} = 1 \qquad (x - \underbrace{(+1)}_{h})$$

$$(y - \underbrace{(-5)}_{k})$$

$$\Rightarrow a^2 = 4 \,,\, b^2 = 1$$

$$\Rightarrow a = 2 \,,\, b = 1 \qquad (h,k) = (1,-5)$$

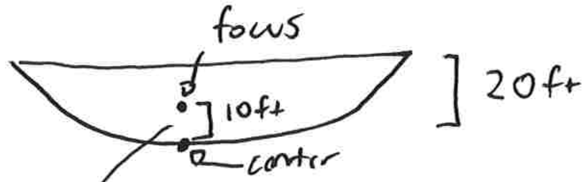
$$y - k = \pm \frac{b}{a}(x-h) \Rightarrow y + 5 = \pm \frac{1}{2}(x-1)$$

$$\Rightarrow y = \frac{1}{2}x - \frac{11}{2} \,,\, y = -\frac{1}{2}x - \frac{9}{2}$$

Equation of hyperbola in standard form: $\boxed{\frac{(x-1)^2}{4} - \frac{(y+5)^2}{1} = 1}$

Equations of asymptotes: $\boxed{y = \frac{1}{2}x - \frac{11}{2} \,,\, y = -\frac{1}{2}x - \frac{9}{2}}$

3. (20 points) A satellite dish is in the shape of a paraboloid (i.e. it has a cross section that is a parabola) has a height of 20 feet. Its focus is halfway between the top and the bottom of the dish. Find the equation and then width of the dish.



$10 = a$ also set center at $(0,0)$ for convenience $\Rightarrow 0=h, 0=k$

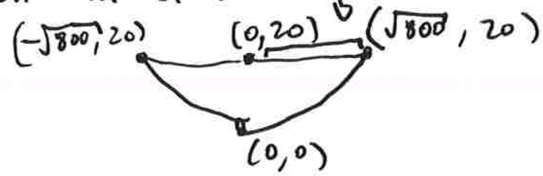
$\Rightarrow x^2 = 4 \cdot 10\, y$

$\Rightarrow x^2 = 40y$

To find width set $y = \text{height} = 20$

$\Rightarrow x^2 = 40 \cdot 20 = 800 \Rightarrow x = \pm\sqrt{800}$

This is half the distance

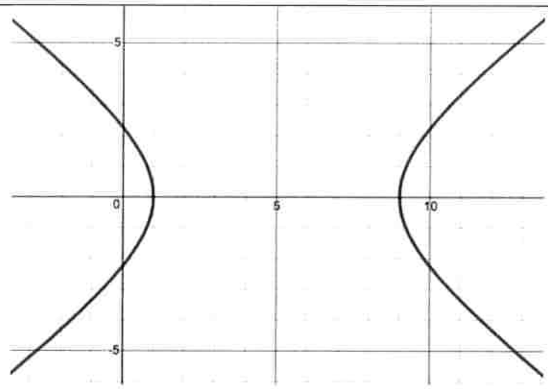


$\Rightarrow W = 2\sqrt{800}$

Equation of model parabola: $\boxed{x^2 = 40y}$

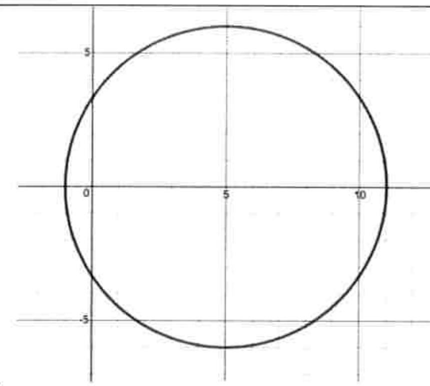
Width: $\boxed{2\sqrt{800} \text{ feet}}$

4. (15 points) Match the graph with the equation:



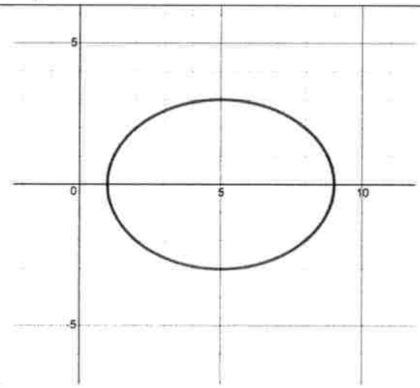
1.

Equation letter: C



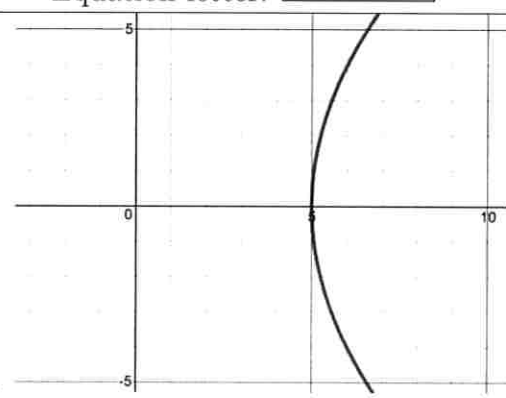
2.

Equation letter: G



3.

Equation letter: A



4.

Equation letter: F

ircles/ Ellipses

hyperbolas

A. $\frac{(x-5)^2}{16} + \frac{y^2}{9} = 1$ ellipses

~~B.~~ $\frac{(x-5)^2}{9} + \frac{y^2}{16} = 1$ (major axis of 3 is horizontal, not vertical)

C. $\frac{(x-5)^2}{16} - \frac{y^2}{9} = 1$

~~D.~~ $\frac{y^2}{9} - \frac{(x-5)^2}{16} = 1$ (Tranverse axis is horizontal for 1 not vertical)

~~E.~~ $(x-5)^2 = 16y$ (Parabola doesn't open correct direction)

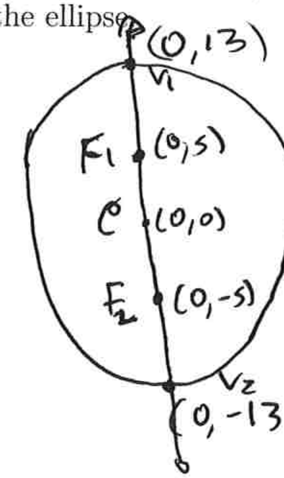
F. $y^2 = 16(x-5)$

G. $x^2 + y^2 - 10x = 11$ circles

~~H.~~ $x^2 + y^2 - 10y = 11$ (shifted in y, but 2 is shifted in x)

Parabola

5. (10 points) An ellipse has foci at the points (0,-5) and (0,5) and vertices at the points (0,-13) and (0,13). Find an equation for the ellipse.



Major axis is vertical and center at (0,0)

$\Rightarrow \frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$

$c = 5$ (distance from center to foci)

$a = 13$ (distance from center to vertex)

Equation: $\boxed{\frac{y^2}{169} + \frac{x^2}{144} = 1}$

$13^2 = 169$ $169 - 25 = 144$ $b^2 = a^2 - c^2 \Rightarrow 13^2 - 5^2 = b^2$

$\Rightarrow b = \sqrt{13^2 - 5^2}$

6. (15 points) True or False. No justification needed. Circle your answer.

(a) In the equation $x^2 = 4py$, the number p represents the distance between the focus and the directrix.

A. True B. False

(b) The equation $4x^2 + 8x - 7y^2 + 48y + 9 = 0$ represents a hyperbola.

A. True B. False

(c) The equation $2(x + 3)^2 + (y - 4)^2 = 10$ represents an ellipse that is elongated horizontally (wider than it is tall).

A. True B. False

7. (10 points) Provide two applications of conics (they can be the same conic or different). No need to write a paragraph, a sentence will suffice.

Conic: $\boxed{\text{Parabola}}$

Application: When Jordan shoots a 3-pointer, the ball takes an approximately parabolic trajectory to the basket.

Conic: $\boxed{\text{Ellipse}}$

Application: The crossection of the dome of the old well is approximately an ellipse.