Root Finding: Bisection Method

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- Problem: Given a continuous function f(x), we want to find some value x^* such that $f(x^*) = 0$
- Suppose we know two points a and b such that f(a) and f(b) are of different signs.
- For simplicity, suppose f(a) < 0 and f(b) > 0.
- Is there a root in between *a* and *b*?

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- Yes. This follows from the Intermediate Value Theorem:
 If f(a) < u < f(b), then there exists a c ∈ [a, b] such that f(c) = u.

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- Idea: Find the value in between a and b, that is $c = \frac{a+b}{2}$.
- Three cases:
 - f(c) = 0: Then we're done!
 - f(c) < 0: Replace *a* with *c*.
 - f(c) > 0: Replace b with c.
- We can now proceed iteratively.

- When do we stop the iterations? Let c^* be the root, i.e. $f(c^*) = 0$.
- Three ways to determine when we've done enough:
 1) |c^{*} − c| < tol. If we are close enough to the root, we can stop.
 - 2) $|c_n c_{n-1}| < \text{tol.}$ Iterations are getting really close together.
 - 3) N is too big. Finding the root is taking to long.

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 - 2) $|c_n c_{n-1}| < \text{tol.}$ Iterations are getting really close together.
 - 3) N is too big. Finding the root is taking to long.
- We, in general, do not know c^* , so we need to use cases 2 and 3.