# Root Finding: Bisection Method 

September 13, 2022

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- Suppose we know two points $a$ and $b$ such that $f(a)$ and $f(b)$ are of different signs.
- For simplicity, suppose $f(a)<0$ and $f(b)>0$.
- Is there a root in between $a$ and $b$ ?


## Bisection Method

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## Bisection Method

- Is there a root in between $a$ and $b$ ?
- Yes. This follows from the Intermediate Value Theorem:
- If $f(a)<u<f(b)$, then there exists a $c \in[a, b]$ such that $f(c)=u$.


## Bisection Method

- How do we find the root between $a$ and $b$ ?


## Bisection Method

$\square$ How do we find the root between $a$ and $b$ ?
$\square$ Idea: Find the value in between $a$ and $b$, that is $c=\frac{a+b}{2}$.

- Three cases:
$-f(c)=0$ : Then we're done!
$-f(c)<0$ : Replace $a$ with $c$.
$-f(c)>0$ : Replace $b$ with $c$.
- We can now proceed iteratively.


## Bisection Method

- When do we stop the iterations? Let $c^{\star}$ be the root, i.e. $f\left(c^{\star}\right)=0$.
- Three ways to determine when we've done enough:

1) $\left|c^{\star}-c\right|<$ tol. If we are close enough to the root, we can stop.
2) $\left|c_{n}-c_{n-1}\right|<$ tol. Iterations are getting really close together.
3) $N$ is too big. Finding the root is taking to long.

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- We, in general, do not know $c^{\star}$, so we need to use cases 2 and 3 .

