Two hmwks due tonight!

$\vec{y}' = A\vec{y}$

λ_1, λ_2 eigenvalue
$\vec{e}_1, \vec{e}_2$ eigenvector

→ solution

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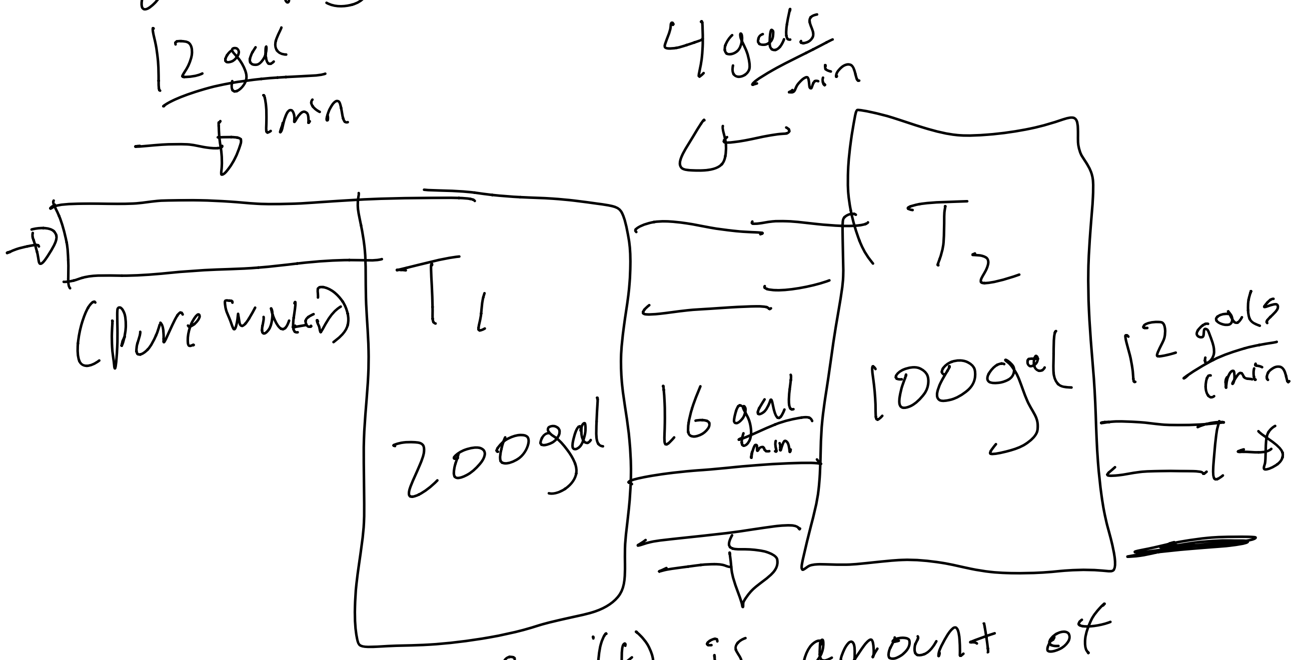


$y_n(t)$ is amount of solute in pounds in tank n.

$$y_1'(t) = \text{ratein}_1 - \text{rateout}_1$$

$$y_2'(t) = \text{ratein}_2 - \text{rateout}_2$$

$$\text{ratein}_1 = \frac{0\ \text{lb}}{1\ \text{gal}} \frac{12\ \text{gal}}{1\ \text{min}} + \frac{y_2(t)\ \text{lb}}{100\ \text{gal}} \frac{4\ \text{gal}}{1\ \text{min}}$$

$$0 \quad 0$$

$$\text{rateout}_1 = \frac{y_1(t)\ \text{lb}}{200\ \text{gal}} \frac{16\ \text{gal}}{\text{min}}$$

$$y_1'(t) = \frac{1}{25} y_2(t) - \frac{2}{25} y_1(t)$$

$$\text{ratein}_2 = \frac{y_1(t)\ \text{lb}}{200\ \text{gal}} \frac{16\ \text{gal}}{\text{min}}$$

$$\text{rate out}_2 = \frac{y_2(t)\,\text{lbs}}{100\,\text{gal}}\,\frac{12\,\text{gal}}{\text{min}} + \frac{y_2(t)\,\text{lbs}}{100\,\text{gal}}\,\frac{4\,\text{gal}}{1\,\text{min}}$$

$$y_2'(t) = \frac{2}{25}y_1(t) - \frac{4}{25}y_2(t)$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}' = \begin{bmatrix} -2/25 & 1/25 \\ 2/25 & -4/25 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$\vec{y}\,' = \begin{bmatrix} -2/25 & 1/25 \\ 2/25 & -4/25 \end{bmatrix} \vec{y}$$

$$A = \begin{bmatrix} -2/25 & 1/25 \\ 2/25 & -4/25 \end{bmatrix}$$

$$A^* = \begin{bmatrix} -2 & 1 \\ 2 & -4 \end{bmatrix}$$

$$\frac{\lambda_n^*}{25} = \lambda_n$$

$$\vec{e}_n^{\,*} = \vec{e}_n$$

$$\det(A - I\lambda) = 0$$

$$(-2-\lambda^*)(-4-\lambda^*) - 2 = 0$$

$$\lambda^{*2} + 6\lambda^* + 6 = 0$$

$$\lambda_{1,2}^* = -3 \pm \sqrt{3}$$

$$\lambda = \frac{-3 \pm \sqrt{3}}{25}$$

$\lambda_{1,2}$

$\lambda_1 \approx -4.73$

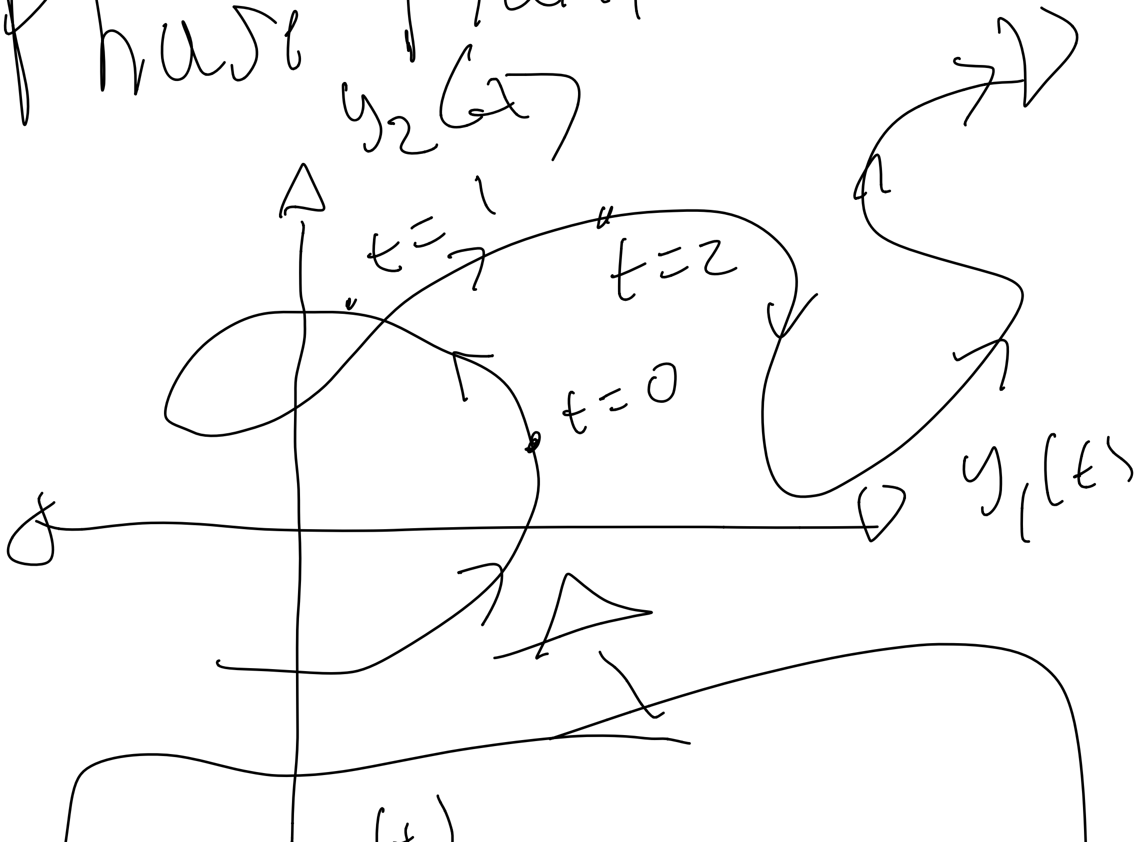
$\lambda_2 \approx -1.267$

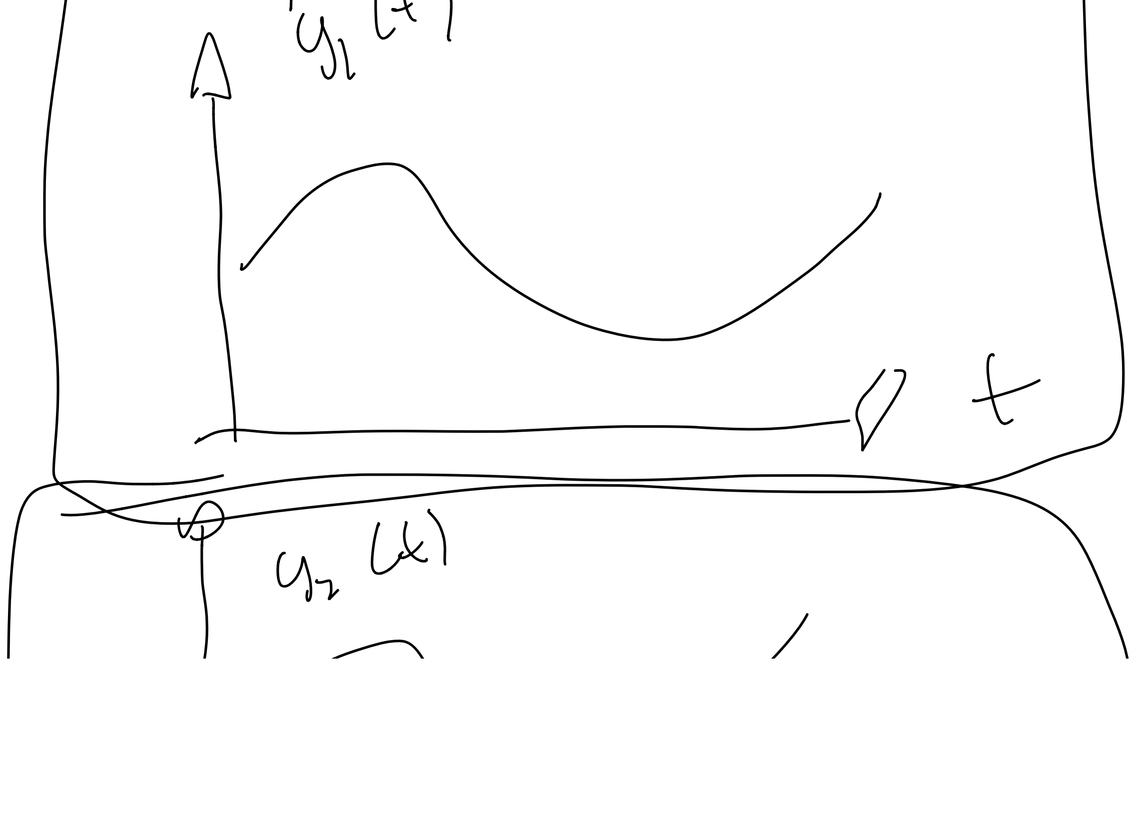
$\vec{e}_1 \approx \begin{bmatrix} -0.366 \\ 1 \end{bmatrix}$

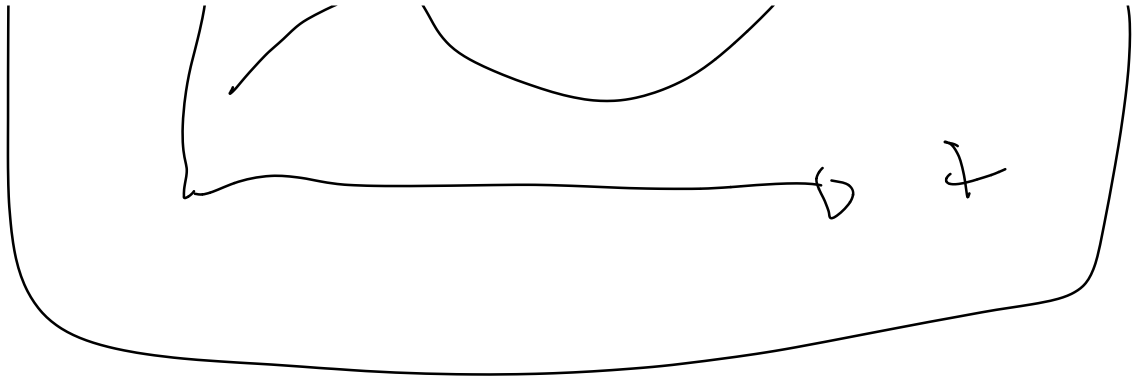
$\vec{e}_2 \approx \begin{bmatrix} 1.37 \\ 1 \end{bmatrix}$

$$\vec{y} = c_1 \vec{e}_1 e^{\lambda_1 t} + c_2 \vec{e}_2 e^{\lambda_2 t}$$

Phase Plane







$$\vec{y}\,' = A\vec{y}$$

Critical points are point in the phase plane that undf.

$$\frac{y_1'(t)}{y_2'(t)} = \frac{0}{0}$$

$$y_1'(t) = 0$$
$$y_2'(t) = 0$$

$$\vec{y}\,' = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \vec{y}$$

$$a y_1 + b y_2 = 0$$

$$c y_1 + d y_2 = 0$$

$$y_1 = \frac{-b}{a} y_2$$

$$-\frac{cb}{a} y_2 + d y_2 = 0$$

$$\left(-cb + d\right) y_2 = 0$$

$$-\frac{b}{a} y_2 = y_1$$

$$y_2 = 0 \Rightarrow y_1 = 0$$

or

$$-\frac{cb}{a} + d = 0$$

$$da - cb = 0$$

$$\det(A) \neq 0$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$ad - bc \neq 0$

$\frac{y_1'}{y_2'} = \frac{0}{0}$ is $(y_1, y_2) = (0, 0)$

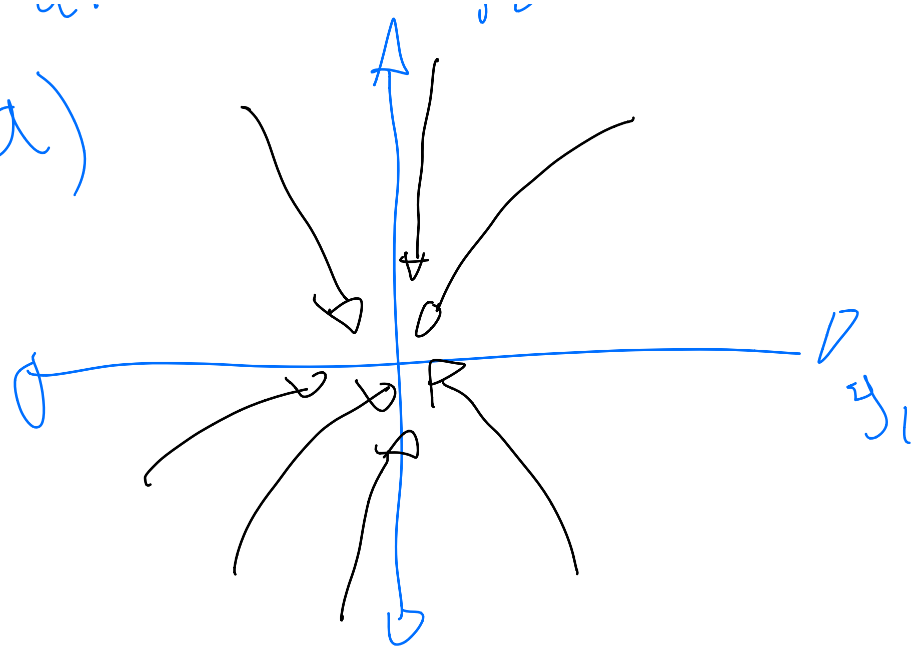
For all constant coeff. matrices s.t. we have $\vec{y}' = A\vec{y}$, the only critical point is at $y_1 = y_2 = 0$.

Stability (3 types)

stable attractive y_2

Stabl…

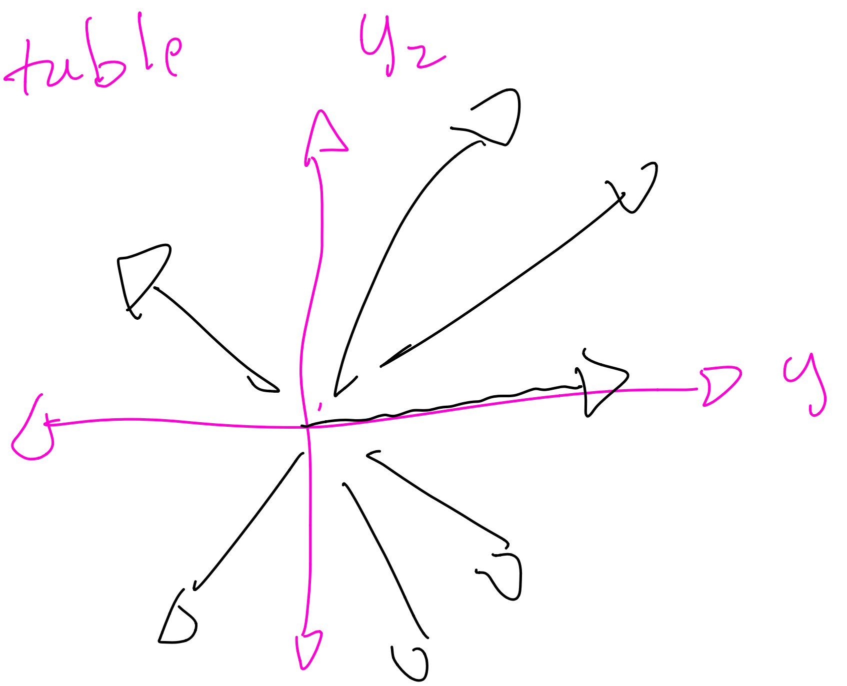
a)



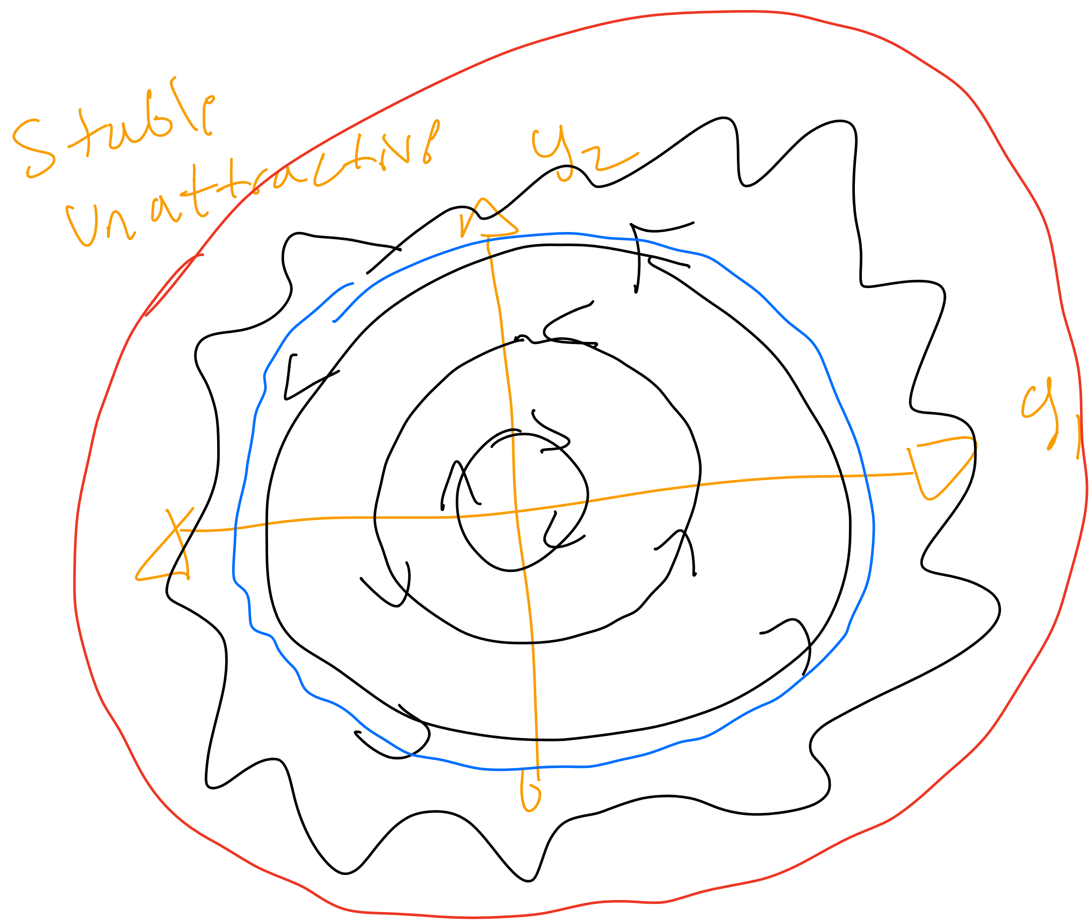
$t \to \infty$, $y_1 \to 0$

$y_2 \to 0$

Unstable



$t_1 \to \infty$, $y_1 \to \infty$ or $y_2 \to \infty$



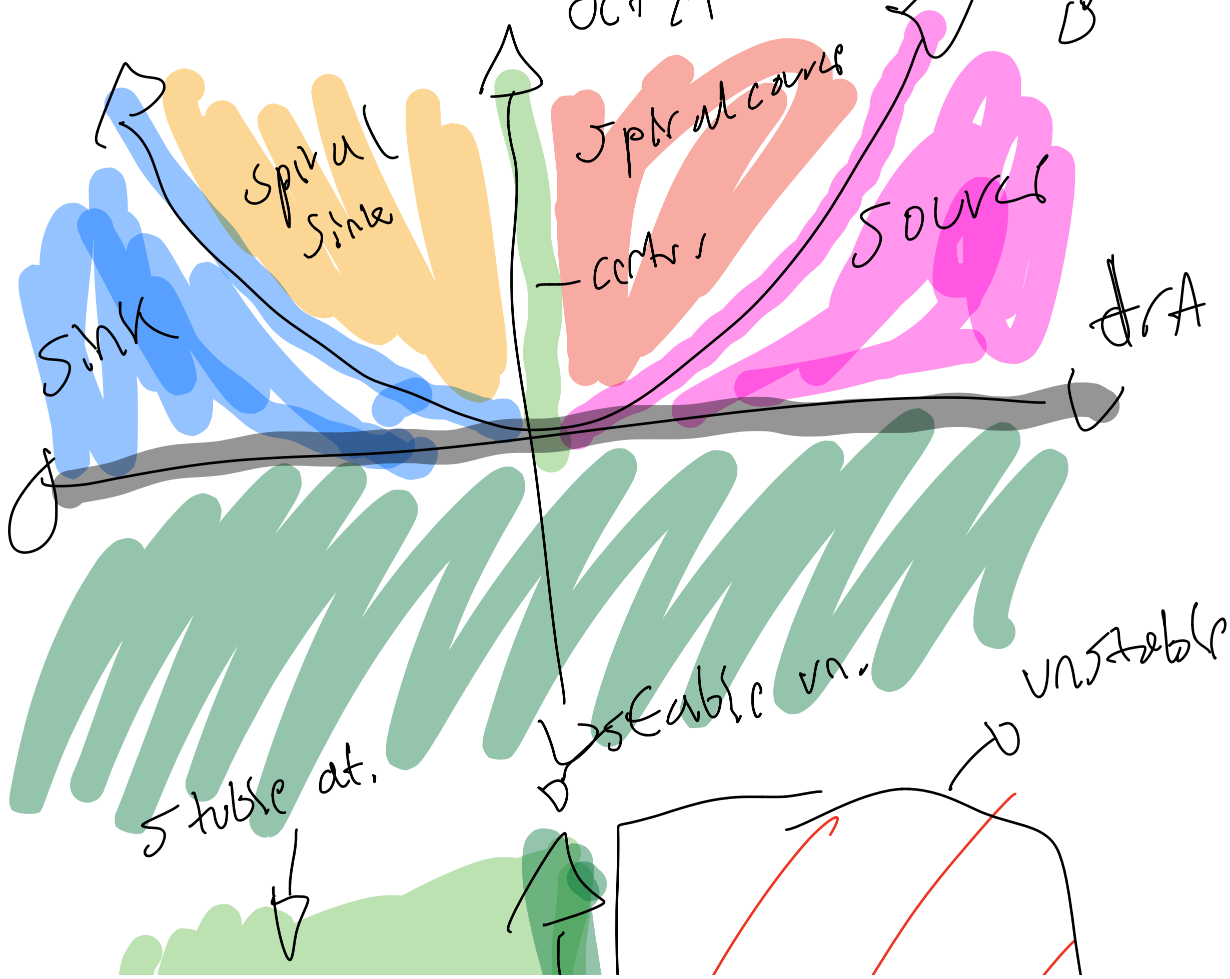
y_1 and y_2

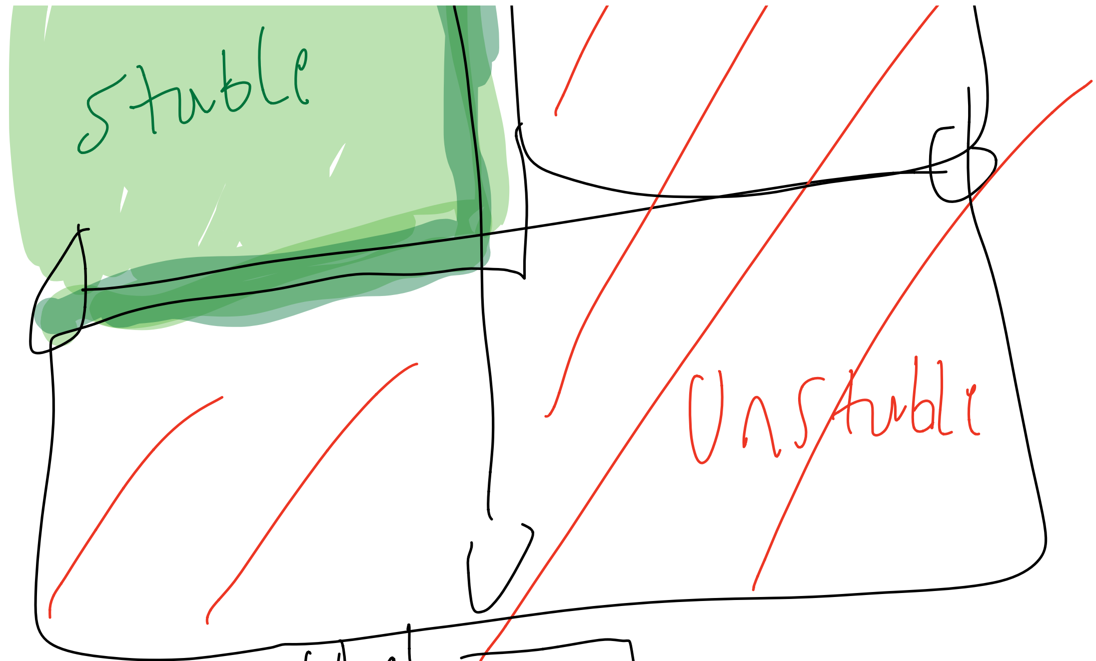
$a < y_1 < b$

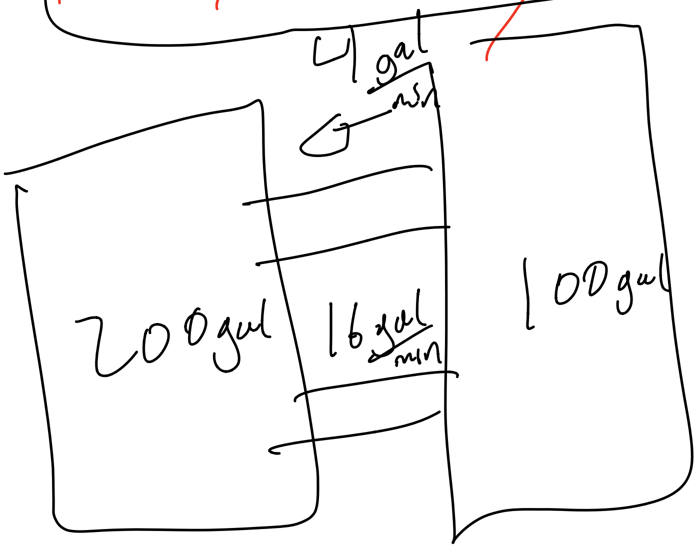
$a < y_2 < b$

$$\lambda = \frac{trA \pm \sqrt{(trA)^2 - 4\,detA}}{2}$$

$\lambda(trA, detA)$



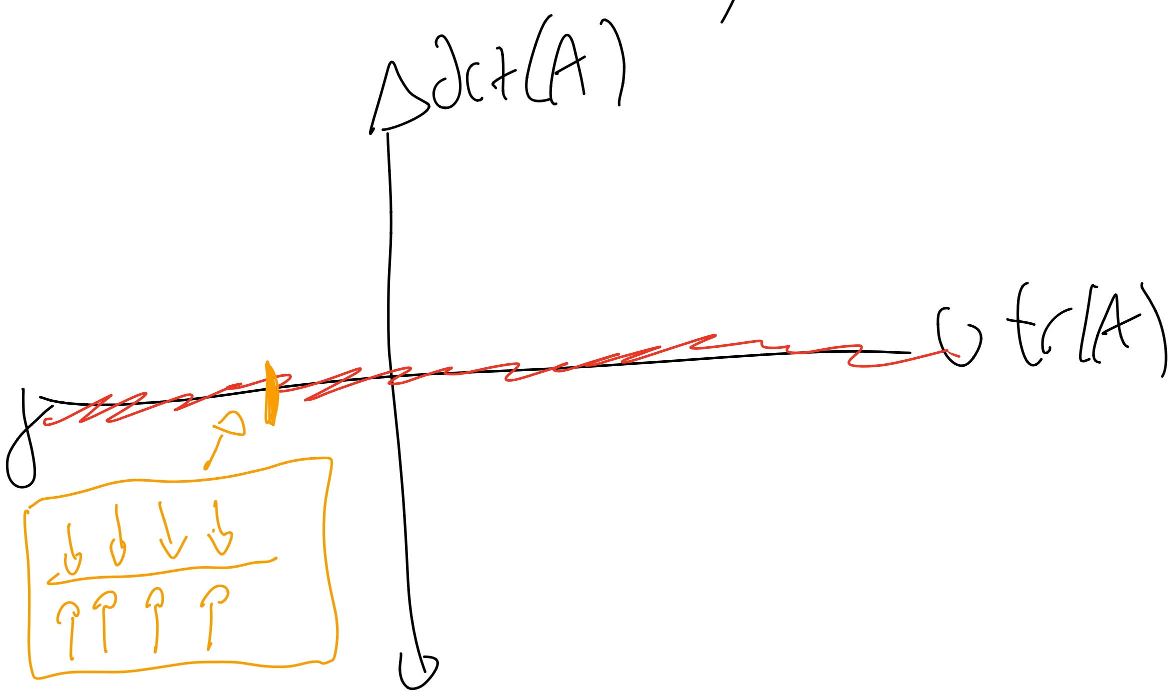




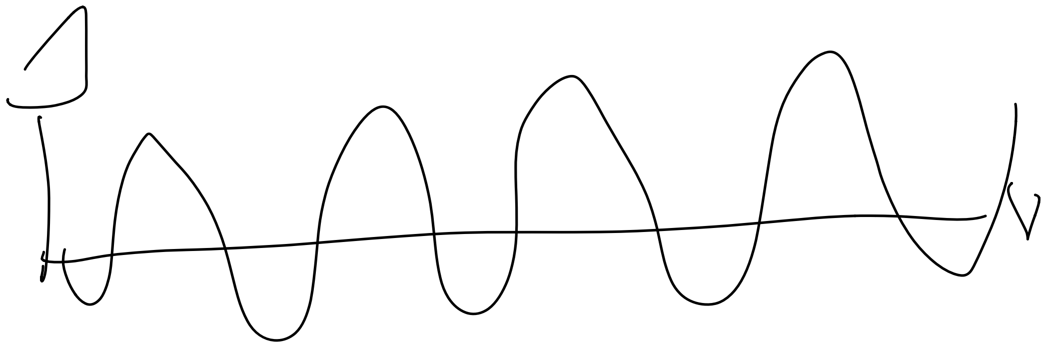
$$A = \begin{bmatrix} \frac{-2}{25} & \frac{1}{25} \\ \frac{2}{25} & \frac{-1}{25} \end{bmatrix}$$

$$tr(A) = \frac{-3}{2s}$$

$$det(A) = \frac{2}{2s^2} - \frac{2}{2s^2} = 0$$



$$my'' + ky = 0$$



$$y_1 = y$$

$$y_2 = y'$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}' = \begin{bmatrix} y' \\ y'' \end{bmatrix} = \begin{bmatrix} y_2 \\ \frac{-ky}{m} \end{bmatrix}$$

$$= \begin{bmatrix} y_2 \\ \frac{-ky_1}{m} \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$\mathrm{Tra}(A)$ $\qquad$ $\det(A)$

$\| \qquad\qquad \downarrow \qquad\qquad k > 0$

$0 \qquad\qquad \frac{k}{m} \qquad\qquad m > 0$