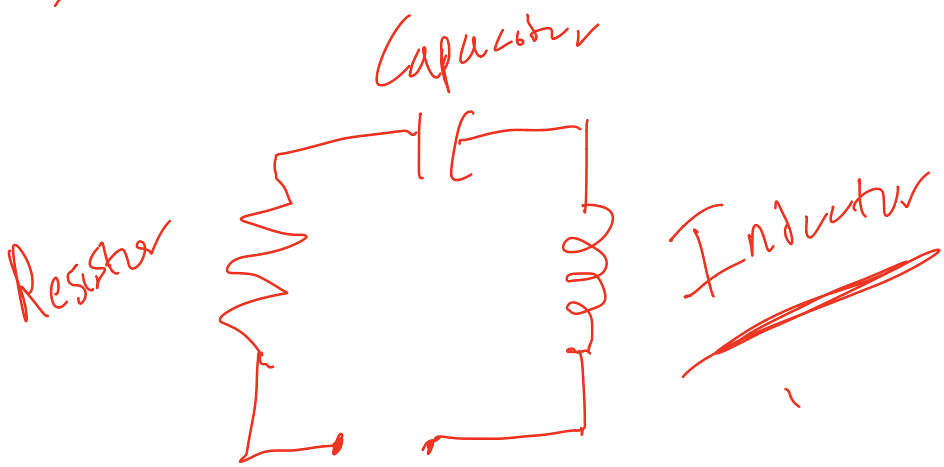
RLC circuits



$$\boxed{L}\, I' + \boxed{R}\, I + \frac{1}{\boxed{C}} \int I \, dt = \underbrace{E_0 \sin(\omega t)}_{E(t)}$$

$$\boxed{L I'' + R I' + \frac{I}{C} = E_0 \omega \cos(\omega t)}$$

(L, R, C are constants)

$I(t)$

9. $R = 4\,\Omega$

$L = 0.1\,H$

$C = 0.1\,F$

$E = 110\,V$

$$\boxed{\underbrace{0.1}_{\text{Mass}} I'' + \underbrace{4}_{\text{drag}} I' + \underbrace{\frac{I}{0.1}}_{\text{Spring constant}} = \overbrace{110}^{\text{Driver}}}$$

$$y(t) = C_1 e^{-\lambda_1 t} + C_2 e^{-\lambda_2 t}$$

+ particular solution

$t=0$

$$LI' + RI + \frac{1}{C}\int I\,dt = E$$

$$Q(0) = 0 \quad , \quad I(0) = 0$$

$$Q(t) = \int I\,dt$$

$$\boxed{0 = \int I\,dt}$$

$t = 0$ $\quad LI' = E$

$$\boxed{I'(0) = \frac{E}{L}}$$

$$y'' + ay' + by = F\sin(\omega t)$$

vs

$$= F$$

what is a matrix, vector
matrices

$$A^{(2\times 2)} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 4 \\ 6 & 2 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

vector

$$\bar{x} = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}$$

$(\bar{x})$

$(\hat{x})$

$$\vec{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$(m \times n)(n, 1) = (m \times 1)$

$$A \cdot \vec{x} = \vec{b}$$

Ex (3×1) vector

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

2×3 1   2   3

$$\begin{array}{c} 1 \\ 2 \end{array}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$\underset{(2\times\cancel{2})}{\begin{bmatrix} a & b \\ c & d \end{bmatrix}}\underset{(\cancel{2}\times 1)}{\begin{bmatrix} e \\ f \end{bmatrix}} = \underset{(2\times 1)}{\begin{bmatrix} A \\ B \end{bmatrix}}$$

$$= \begin{bmatrix} a \cdot e + b \cdot f \\ \\ e \cdot c + d \cdot f \end{bmatrix}$$

$(2\times\cancel{3})\ (\cancel{3}\times 2)\qquad (2\times 2)$

$$A \times B = C$$

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} g & h \\ i & j \\ k & l \end{bmatrix}$$

$$C = \begin{bmatrix} r_1 c_1 & r_1 c_2 \\ r_2 c_1 & r_2 c_2 \end{bmatrix}$$

$$5 \cdot 2 = 2 \cdot 5$$

$$A \cdot B \neq B \cdot A$$

$$A \cdot A^{-1} = I = A^{-1} A$$

$$5 \cdot \frac{1}{5} = 1$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$\boxed{A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} a_{22} & a_{12} \\ -a_{21} & a_{11} \end{bmatrix}}$$

$$A = \begin{bmatrix} 3 & 4 \\ 2 & 5 \end{bmatrix}$$

1

$$\frac{1}{3\times5-2\times4}=\frac{1}{7}$$

$$\begin{bmatrix} 5 & -4 \\ -2 & 3 \end{bmatrix}$$

$$A^{-1}=\frac{1}{7}\begin{bmatrix} 5 & -4 \\ -2 & 3 \end{bmatrix}$$

$$=\begin{bmatrix} \frac{5}{7} & -\frac{4}{7} \\ -\frac{2}{7} & \frac{3}{7} \end{bmatrix}$$

$$A \cdot A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\det\left(A^{(n \times n)}\right) = C$$

$$C = 0$$

$$\begin{array}{cc} \begin{bmatrix} [1 & 2] \\ [2 & 4] \end{bmatrix} & \begin{array}{l} - v_1 \\ - v_2 \end{array} \end{array} \qquad 1 \cdot 4 - 2 \cdot 2 = 0$$

$$\boxed{v_1 \cdot 2 = v_2}$$

$$c_1 v_1 + c_2 v_2 = 0$$

if possible then v_1 and v_2 are linearly dependents

If $\det(A) = 0$

$\Rightarrow$ not exist A^{-1}

$$A\vec{x} = \underline{\lambda}\,\vec{x}$$

$$\underbrace{\begin{bmatrix}1 & 1\\ 4 & 1\end{bmatrix}}_{A}\underbrace{\begin{bmatrix}1\\ 2\end{bmatrix}}_{\vec{x}} = \begin{bmatrix}3\\ 6\end{bmatrix} = \underbrace{3}_{\lambda}\underbrace{\begin{bmatrix}1\\ 2\end{bmatrix}}_{\vec{x}}$$

λ is an eigenvalue of A

$\vec{x}$ is an eigenvector of A

$$\boxed{A\vec{x} = \lambda\vec{x}}$$

$$(A - I\lambda)\vec{x} = 0$$

$$\left(\begin{bmatrix}a_{11} & a_{12} \\ \end{bmatrix} \begin{bmatrix}\lambda & 0 \\ \end{bmatrix}\right)\begin{bmatrix}x_1 \\ \end{bmatrix}$$

$$\left(\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$= 0$$

$$A - I\lambda = B$$

$$B\vec{x} = 0$$

$$\Rightarrow \det(B) = 0$$

$$\begin{bmatrix} \begin{bmatrix} | \\ b_1 \\ \end{bmatrix} & \begin{bmatrix} \\ b_2 \\ \end{bmatrix} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\boxed{x_1 b_1 + x_2 b_2 = 0}$$

Remnder $c_1 y_1 + c_2 y_2 = 0$

$\Rightarrow$ y_1 and y_2 are lin dep.

$\Rightarrow$ b_1 and b_2 are linearly dep.

$\Rightarrow$ $\det(B) = 0$

$$\underbrace{(A - \lambda I)}\vec{x} = 0$$

B

$$\det(A - \lambda I) = 0$$

$$\det\left(\begin{bmatrix} a_{11}-\lambda & a_{12} \\ a_{21} & a_{22}-\lambda \end{bmatrix}\right) = 0$$

$$= (a_{11}-\lambda)(a_{22}-\lambda) - a_{21}a_{12}$$

$$= a_{11}a_{22} - (a_{11}+a_{22})\lambda$$

$$+ \lambda^2 - a_{21}a_{12}$$

$$= \lambda^2 - (a_{11}+a_{22})\lambda$$

$$+ (a_{11} a_{22} - a_{21} a_{12})$$

$$= \boxed{\lambda^2 - \underline{\mathrm{Tr}(A)}\,\lambda + \det(A) = 0}$$

$$\lambda = \frac{\mathrm{Tr}(A) \pm \sqrt{\mathrm{Tr}(A)^2 - 4\det(A)}}{2}$$

$$A = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} \Rightarrow A - \lambda I = \begin{bmatrix} 1-\lambda & 1 \\ 4 & 1-\lambda \end{bmatrix}$$

$4 \quad 1 \quad \lambda$

$$(1-\lambda)(1-\lambda) - 4 = 0$$

$$\lambda^2 - 2\lambda + \overset{-3}{\cancel{1 - 4}} = 0$$

$$(\lambda+1)(\lambda-3) = 0$$

$$\boxed{\lambda = -1, 3}$$

$$(A - \lambda I)\vec{x} = \vec{0}$$

$$\lambda_1 = -1 \qquad \begin{bmatrix} -1/2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$[\,2x_1 + x_2 = 0$$

$$\cancel{[\,4x_1 + 2x_2 = 0}$$

$$\left[\begin{array}{cc:c} \boxed{\begin{matrix} 2 \\ 4 \end{matrix}} & \begin{matrix} 1 \\ 2 \end{matrix} & \begin{matrix} 0 \\ 0 \end{matrix} \end{array}\right]$$

Rules:

$cR_n \Rightarrow R_n$

$$\boxed{R_n + CR_m \Rightarrow R_n}$$

What do we want?

Column one, we want the first entry to be 1

Column two, we want the second entry to be 1

We want all other entries to be 0.

$$\begin{bmatrix} 2 & 1 & \vdots & 0 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2 & \vdots & 0 \end{bmatrix}$$

$$R_1 \cdot \tfrac{1}{2} \Rightarrow R_1$$

$$\begin{bmatrix} 1 & \tfrac{1}{2} & \vdots & 0 \\ 4 & 2 & \vdots & 0 \end{bmatrix}$$

$$R_2 - 4R_1 \Rightarrow R_2$$

$$\begin{bmatrix} 1 & \tfrac{1}{2} & \vdots & 0 \\ 0 & 0 & \vdots & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & \tfrac{1}{2} \\ & \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} 1/2 \end{bmatrix} \begin{bmatrix} \ \end{bmatrix}$

$$x_1 + \frac{x_2}{2} = 0$$

$$\boxed{\begin{aligned} x_1 &= -\frac{x_2}{2} \\ x_2 &= x_2 \end{aligned}}$$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -\frac{x_2}{2} \\ x_2 \end{bmatrix}$$

$$= x_2 \begin{bmatrix} -1/2 \\ 1 \end{bmatrix}.$$

Gaussian elimination $\tilde{e}_1$

RREF : Reduced Row Echelon Form