Identity for hmwk 3 1c

$$\boxed{\cosh(x) - \cosh(y) = 2\sinh\left(\frac{x+y}{2}\right) \cdot \sinh\left(\frac{x-y}{2}\right)}$$

2.7

$$\boxed{y'' + ay' + by = \underline{r(x)}}$$

0

describes motion of external force

first you solve the homogeneous

problem

proof...

$$\boxed{y_h'' + a y_h' + b y = 0} \quad 1$$

$$\Rightarrow y_h$$

$$\boxed{y_p'' + a y_p' + b y = r(x)} \quad 2$$

$$\Rightarrow y_p$$

$$y = y_p + y_n$$

$$(y_p + y_n)'' + a(y_p + y_n)'$$

$$+ b(y_p + y_n) = r(x)$$

$$y_p'' + a y_p' + b y_p$$

$$+ \boxed{y_h'' + a y_h' + b y_h} = r(x)$$

1

$= 0$

$$\Rightarrow \boxed{y_p'' + a y_p' + b y_p = r(x)}$$

2 $\Rightarrow$ ✓

$$y = y_p + y_h$$

Solve for y_p using method of undetermined coeff.

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if $r(x)$ is:

$r(x)$	guess
$\cos(\omega x), \sin(\omega x)$	$\alpha \sin(\omega x) + \beta \cos(\omega x)$
$e^{\omega x}$	$\alpha e^{\omega x}$
x^N	$\sum_{n=0}^{N} \alpha_n x^n$

Ex

$$r(x) = x^3 \Rightarrow \boxed{\alpha x^3 + \beta x^2 + \gamma x + \delta}$$

Ex

$$r(x) = e^{2x} \Rightarrow \alpha e^{2x}$$

$$r(x) = x^2 \sin(x) \, ?$$

$$(ax^2+bx+c)(d\sin(x)+e\cos(x))$$

$$\boxed{\begin{aligned} y_p(x) = {} & Ax^2\sin(x) + Bx^2\cos(x) \\ & + Cx\sin(x) + Dx\cos(x) \\ & + E\sin(x) + F\cos(x) \end{aligned}}$$

$$r(x) = x^2 + \sin(x)$$

$$\boxed{y_p = (ax^2+bx+c) + (d\sin(x)+e\cos(x))}$$

$$y'' + ay' + by = r(x)$$

1) solve homogeneous (y_h)

2) make guess for particular solution (y_p)

2*) (if y_h has sim. terms to y_p multiply that piece of the guess by x)

3) Plug in y_p into your DE

4) Solve for your undetermined coeffs.

5) $y = y_h + y_p$ (add them)

6) Solve for c_1, c_2 in y_h via your initial conds.

Ex $y'' - 4y' - 12y = te^{4t}$

$\lambda^2 - 4\lambda - 12 = 0$

$(\lambda - 6)(\lambda + 2) = 0$

$\boxed{\lambda_{1,2} = 6, -2}$

$\boxed{y_h = c_1 e^{6t} + c_2 e^{-2t}}$

$y_p = Ce^{4t}$

$r(x) = te$

b

$$y_p = (at+b)\left(ce^{4t}\right)$$

$$= \boxed{Ate^{4t} + Be^{4t}}$$

$$y_p = Ate^{4t} + Be^{4t}$$

$$y_p' = Ae^{4t} + 4Ate^{4t} + 4Be^{4t}$$

$$y_p'' = 4Ae^{4t} + 4Ae^{4t} + 16Ate^{4t} + 16Be^{4t}$$

$$y_p'' - 4y_p' - 12y_p = te^{4t}$$

$$e^{4t}\cdot\left(\cancel{16A}\right) - 4\left(\cancel{4A}\right)$$

$$-12(A) = 1$$

$$\boxed{A = -\frac{1}{12}}$$

e^{4t}:

$$(4A + 4A + 16B) - 4(A + 4B)$$

$$-12(B) = 0$$

$$\boxed{B = -\frac{1}{36}}$$

$$y_p = -\frac{1}{12} te - \frac{1}{36} e$$

$$y = y_h + y_p$$

$$\boxed{y = c_1 e^{-2t} + c_2 e^{6t} - \frac{1}{12} te^{-4t} - \frac{1}{36} e^{-4t}}$$

Special case that!

Step 2^{*}

$$y'' \; y' - 12y = e^{6t}$$

$y'' - 4y$ ... y

$y_h = c_1 e^{-2t} + c_2 \boxed{e^{6t}}$

$y_p = A\boxed{e^{6t}}$

$\Rightarrow \boxed{y_p = Ate^{6t}}$

$y'' - 100y = 9t^2 e^{10t} + \cos(t) - t\sin(t)$

$y_h = c_1 \boxed{e^{10t}} + c_2 e^{-10t}$

$y_p = (at^2 + bt + c)\, d\, e^{10t}$

$$+ t\left(\cos(t) + j\sin(t)\right) + (ft+g)\cdot\left(h\sin(t) + i\cos(t)\right)$$

$$\Rightarrow At\sin(t) + Bt\cos(t) + C\sin(t) + E\cos(t)$$

$$t\,(at^2+bt+c)\,(d\,e^{10t})$$

$$\boxed{At^3 e^{10t} + Bt^2 e^{10t} + Cte^{10t}}$$

2.8

$$\boxed{\boxed{m}y'' + \boxed{\gamma}y' + ky = \boxed{F_0}\cos(\boxed{\omega t})}$$

$$y_h = e^{-\frac{\gamma}{2m}}\left(\sin\left(\frac{\sqrt{\gamma^2-4mk}}{2m}t\right) + \cos\left(\frac{\sqrt{\gamma^2-4mk}}{2m}t\right)\right)$$

$$y_p = A\cos(\omega t) + B\sin(\omega t)$$

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$$A = \boxed{F_0}\ \boxed{m}\left(\omega_0^2 - \omega^2\right)$$

$$\frac{\quad}{m^2(\omega_0^2-\omega^2)^2+\omega^2\gamma^2}$$

$$B=\frac{F_0\,\omega\gamma}{m^2(\omega_0^2-\omega^2)^2+\omega^2\gamma^2}$$

$$\boxed{\omega_0=\frac{\sqrt{\gamma^2-4mk}}{2m}}$$

$\omega\to\omega_0$ $\quad A\to 0$ $\quad B\to$ $\boxed{\dfrac{F_0}{\omega\gamma}}$ Amp

$$\gamma=0$$

$$B=0$$

$$A=\frac{F_0}{m(\omega_0^2-w^2)}$$

$\omega \to \omega_0$

$A \to \infty$

$$my'' + ky = F_0 \cos(t)$$

$$y_h(t) = C_1 \boxed{\cos\left(\sqrt{\frac{k}{m}}\, t\right)} + C_2 \boxed{\sin\left(\sqrt{\frac{k}{m}}\, t\right)}$$

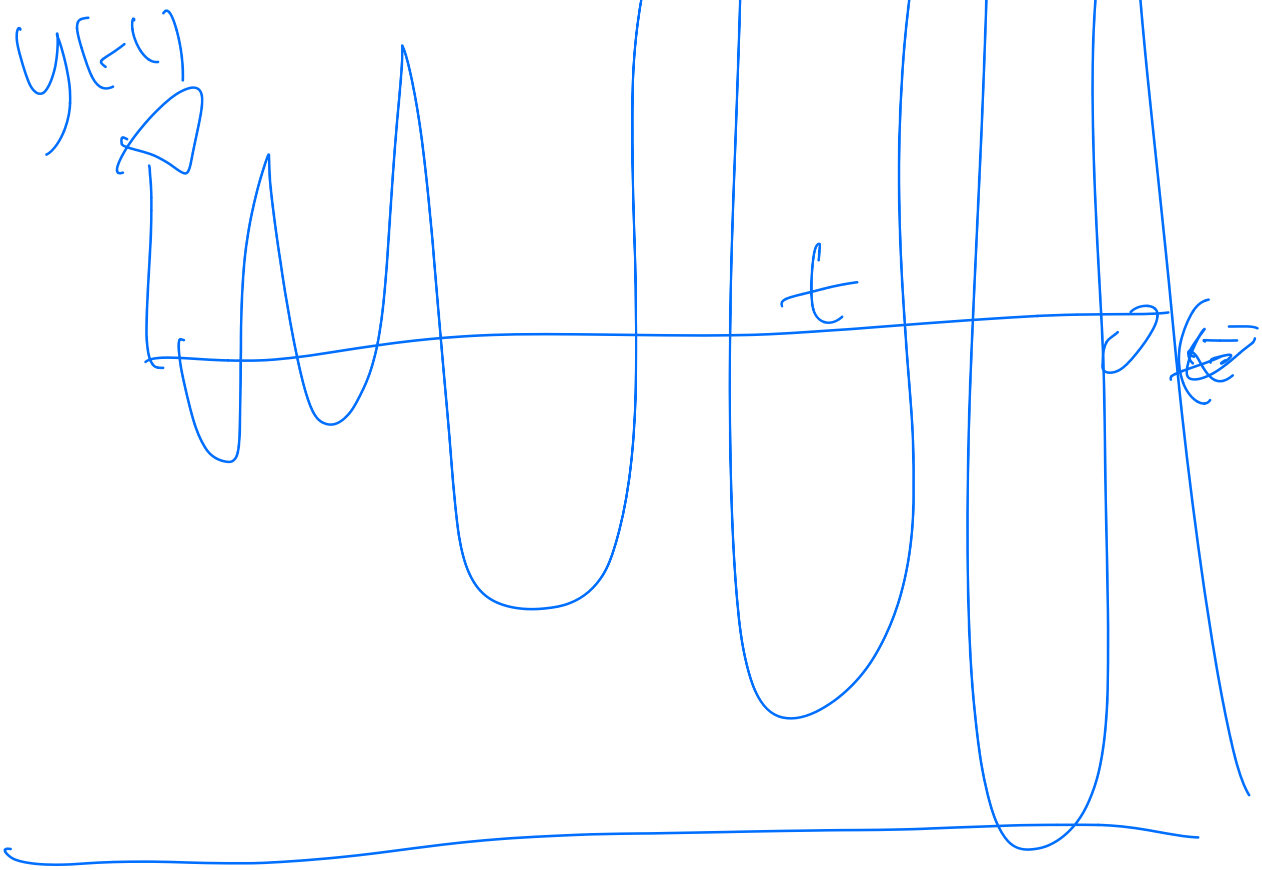
if $m = 1$
$k = 1$

$$y_p = A \boxed{\cos(t)} + B \boxed{\sin(t)}$$

$$\boxed{y_p = A\,t\cos(t) + B\,t\sin(t)}$$

$t \to \infty$

$y_p \to \infty$



2.10

$y'' + p(x)\,y' + q(x)\,y = r(x)$

y_1, y_2

$y_h = c_1 y_1 + c_2 y_2$

$$\boxed{y_p(x) = -y_1 \int \frac{y_2 r(x)}{W(y_1, y_2)}\, dx + y_2 \int \frac{y_1 r(x)}{W(y_1, y_2)}\, dx}$$

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$y'' + y = \frac{1}{\cos(x)}$

$$y_h = c_1 \sin(x) + c_2 \cos(x)$$

$$y_p = -\sin(x) \int \frac{\cos(x) \cdot \frac{1}{\cos(x)}}{-1} dx + \cos(x) \int \frac{\frac{\sin(x)}{\cos(x)}}{-1} dx$$

$$W(\sin(x), \cos(x)) = -\sin(x)\sin(x) - \cos(x)\cos(x)$$

$$= -(\sin^2 x + \cos^2 x)$$

$$= -1$$

$$y_p = \sin(x) \int 1 \, dx - \cos(x) \int \tan(x) \, dx$$

$$y_p = x\sin(x) - \cos(x)\left(-\ln(\cos(x))\right)$$

$$\boxed{y_p = x\sin(x) + \cos(x)\ln(\cos(x))}$$