$\cos(2x)$, $\sin(2x)$

$y(0) = 7$

$y'(0) = -5$

$y'' + ay' + by = 0$

D

$\lambda^2 + a\lambda + b = 0$

$\Rightarrow \lambda_{1,2} = \frac{-a \pm \sqrt{a^2 - 4b}}{2}$

$$\frac{\sqrt{a^2-46}}{2} = 2i$$

$-a/2$

$$-\frac{a}{2} = 0$$

$\cos(ax)$, $\sin(ax)$

e^{xx}

Office hours

8:40-9:40 am Tuesday

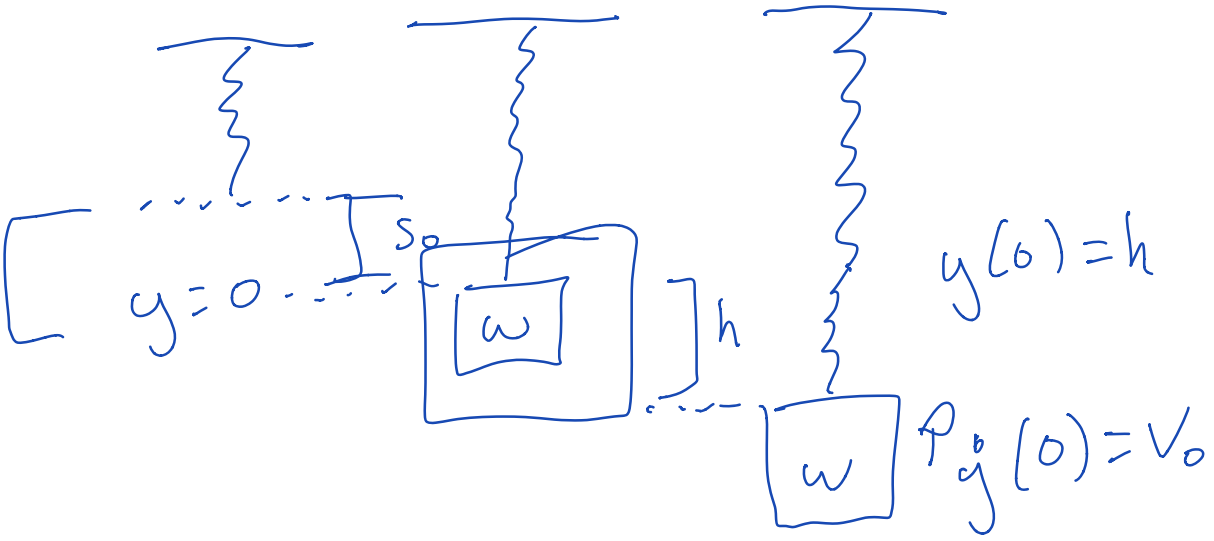
11:30-12:30 pm Thursday

Monday 2.4, 2.7

$$y'' + ay' + by = 0$$

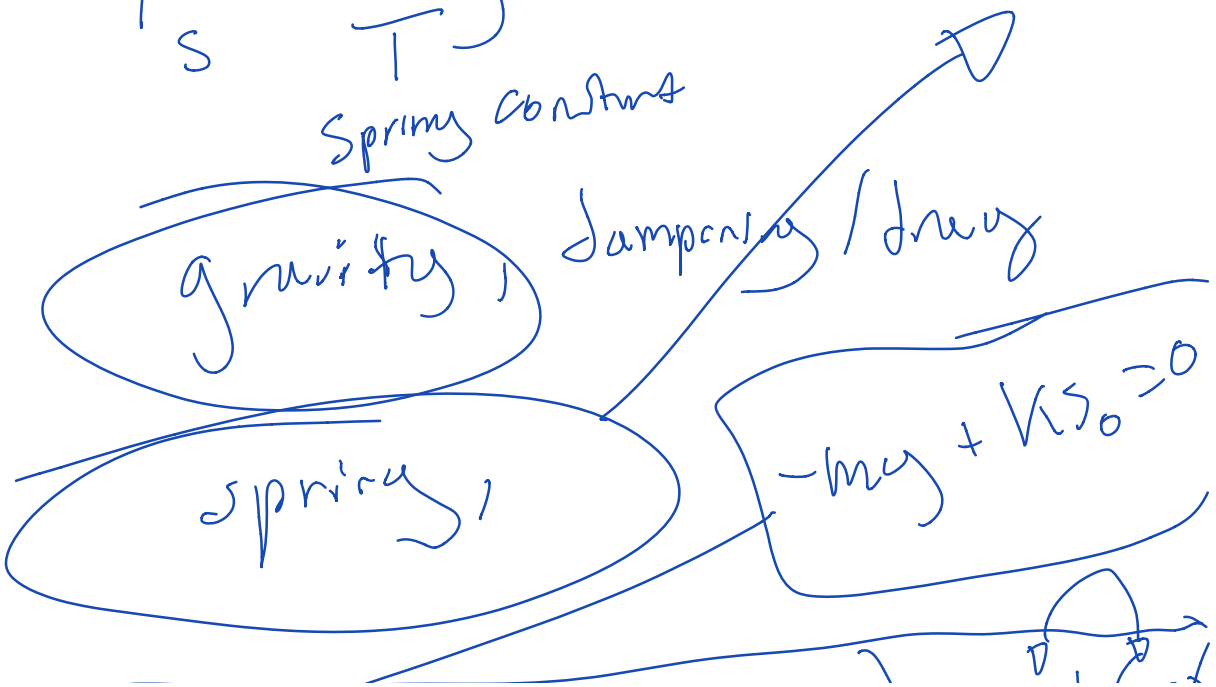
Newton's second law

$$\sum f = ma = m\ddot{x}$$



$$F_s = -\underset{\uparrow}{k}y$$

spring constant



$\sum f = ma = m\ddot{y} = -mg + -k(y-s_0)$

$= -mg - ky + ks_0$

$= -mg + ks_0 - ky$

$= 0 - ky$

$m\ddot{y} = -ky$

notation: $y = \frac{dy}{dt}$

$y'' = \frac{d^2y}{dx^2}$

$-mg = -ks_0$

+

+y

−

$m\ddot{y} = -ky$

$y = 0$

(+) (−)

$-ky$

$-mg$

$$\ddot{y} + \frac{k}{m} y = 0$$

Undamped spring eqn.

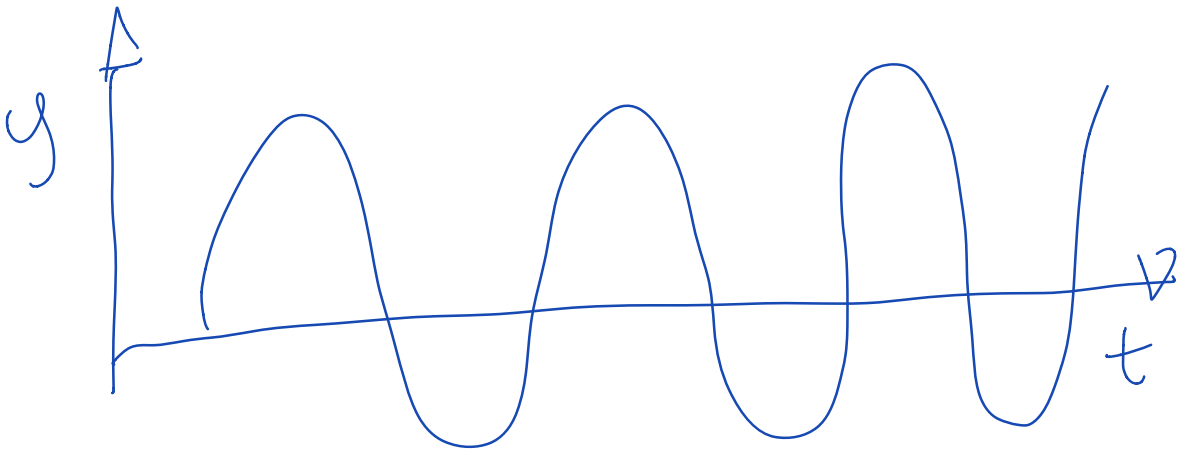
$$x^2 + \frac{k}{m} = 0$$

$$y(t) = A\cos(\omega_0 t) + B\sin(\omega_0 t)$$

$$\omega_0 = \sqrt{\frac{k}{m}}$$

What is the period

Period: $\left(\frac{\omega_0}{2\pi}\right)$ (unit of time) (usually seconds)



$$y(t) = C\cos(\omega_0 t - \delta)$$

$$C = \sqrt{A^2 + B^2}$$

$$\delta = a\tan\left(\frac{B}{A}\right)$$

$$\max(|y'(t)|) = \max(C\omega_0 \sin(\omega_0 t - \delta$$

$$= \boxed{C\omega_0}$$

Often they will just give you weight (w), s_0 (initial stretch) and wont give you k or m.

$$F_s = ky$$

if $y = s_0 \Rightarrow ks_0 = mg$

$$k = \frac{mg}{s_0}$$

$$mg = w$$

$$k = \frac{w}{s_0}$$

$$mg = w$$

$$\Rightarrow \quad m = \frac{w}{g}$$

Ex) 192 lb man on a bungee cable (assume undamped) initially stretches the cable 6 ft by his own weight.

Suppose he is lowered 10 ft and thrusted upwards with a velocity of $\sqrt{3}\ \frac{ft}{s}$.

... the function of

what is [illegible]
his height in time? What
is the max velocity?

$$m = \frac{192}{g} = \frac{192}{32}$$

$$\boxed{m = 6}$$

$$K = \frac{w}{50} = \frac{192}{6}$$

$$\boxed{K = 32}$$

$$\ddot{y} + \frac{K}{m} y = 0 \qquad \frac{32}{6} = \frac{16}{3}$$

$$y(0) = 10$$

$$\dot{y}(0) = \sqrt{3}$$

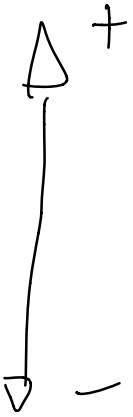
y

$$y(t) = A\cos\left(\frac{4}{\sqrt{3}}t\right) + B\sin\left(\frac{4}{\sqrt{3}}t\right)$$

$$y(0) = 10 = A$$

$$\dot{y}(0) = \sqrt{3} = B\frac{4}{\sqrt{3}}$$

$$B = \frac{3}{4}$$



$$y(t) = C\cos(\omega_0 t - \delta)$$

$$\sqrt{A^2+B^2} = C$$

$$\approx 10.028$$

$$\delta = \operatorname{atan}\left(\frac{(3/4)}{-10}\right)$$

$$\approx -0.075$$

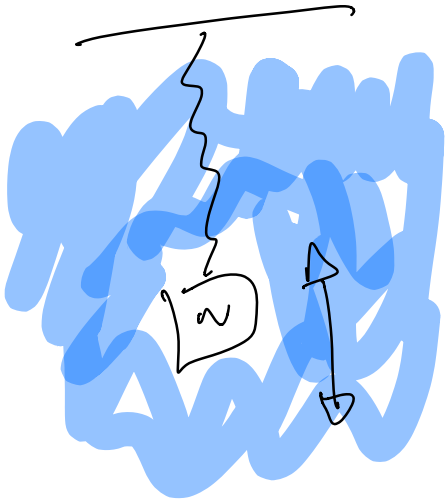
$$y(t) = 10.028 \cos\left(\frac{4}{\sqrt{3}}t + 0.075\right)$$

$$\max\left|y'(t)\right| = \max\left|10.028 \cdot \frac{4}{\sqrt{3}} \sin(\cdots)\right|$$

$$= 10.028 \cdot \frac{4}{\sqrt{3}}$$

$$= 23.159 \frac{ft}{s}$$

$$= 15.5 \text{ mph}$$



$$F_d = -c\dot{y}$$

$$m\ddot{y} = -ky - c\dot{y}$$

$$m\ddot{y} + c\dot{y} + ky = 0$$

$$\ddot{y} + \frac{c}{m}\dot{y} + \frac{k}{m}y = 0$$

$$\lambda_{1,2} = -\frac{c}{2m} \pm \sqrt{\frac{c^2 - 4mk}{4m^2}}$$

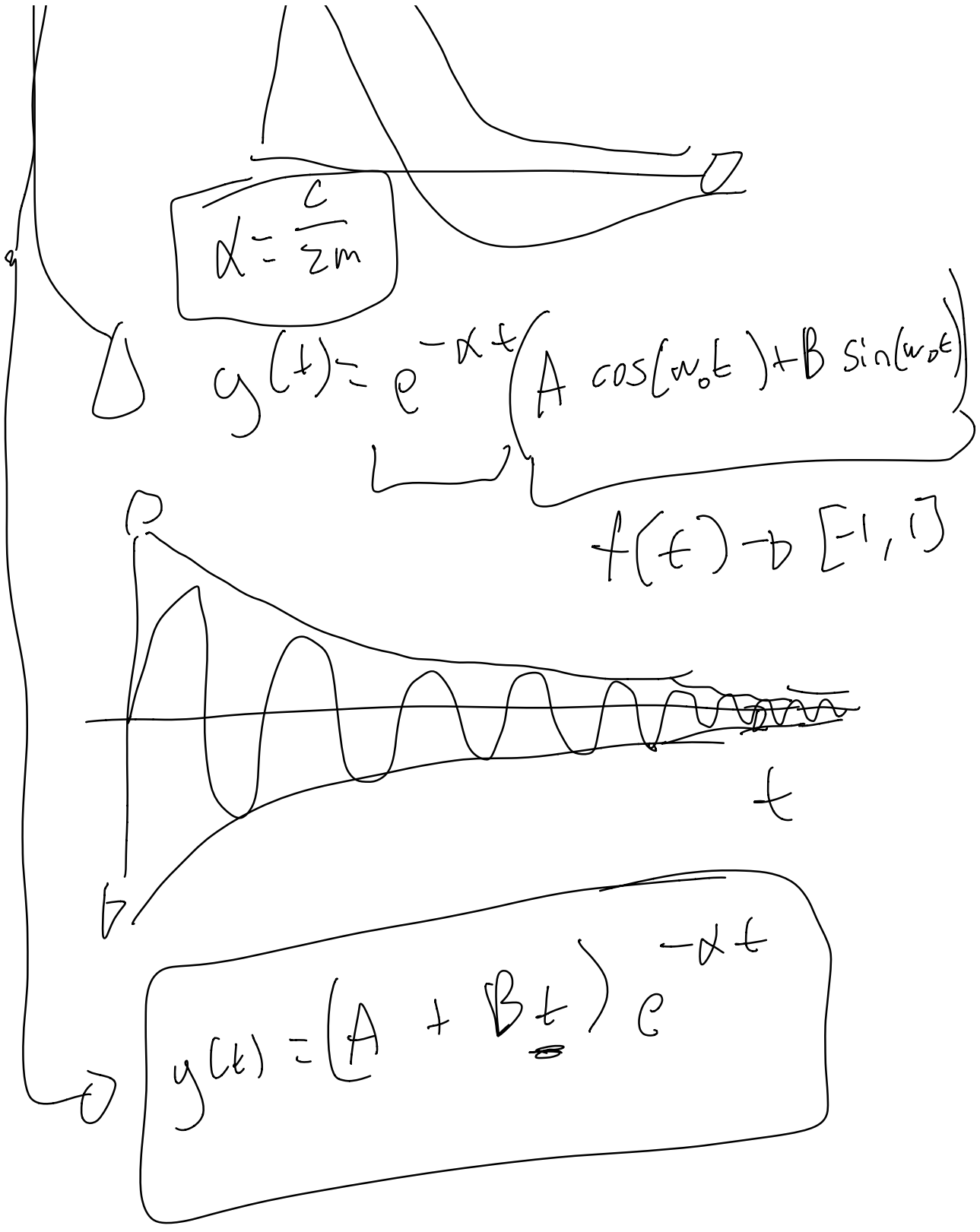
$$\lambda^2 + \frac{c}{m}\lambda + \frac{k}{m} = 0$$

$c^2 > 4mk$ Over dumped

$c^2 = 4mk$ Critially dumped regime

$c^2 < 4mk$ Under dumped regime

$$y(t) = \frac{e^{\lambda_1 t} + e^{\lambda_2 t}}{\lambda^2}$$


$\alpha = \frac{c}{2m}$
$y(t) = e^{-\alpha t}(A\cos(w_0 t) + B\sin(w_0 t))$
$f(t) \to [-1, 1]$
t
$y(t) = (A + Bt)e^{-\alpha t}$

$$\ddot{y} + \frac{c}{m}\dot{y} + \frac{k}{m}y = 0$$

$$-\frac{c}{m} \pm \sqrt{\frac{c^2}{m^2} - \frac{4km}{m^2}}$$

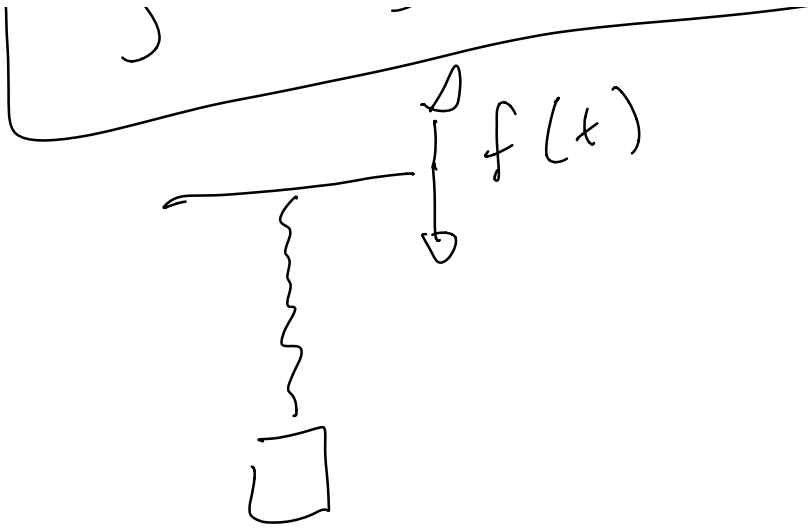
$$2$$

$$-\frac{c}{2m} \pm \frac{\sqrt{c^2 - 4km}}{m2}$$

$$\frac{-c \pm \sqrt{c^2 - 4km}}{2m}$$

2.7 nonhomogenous

$$\ddot{u} + a\dot{y} + by = f(t)$$



$y_h \quad (f(t) = 0)$

$y_p \quad (f(t) \neq 0)$

$$y = y_h + y_p$$

$$f(t) = \sin(2x)$$

$$A\sin(2x) + B\cos(2x)$$