$$y_2 - y_2^2 = 0$$

$$y_1 - y_1^2 = 0$$

$$y_2(1 - y_2) = 0$$

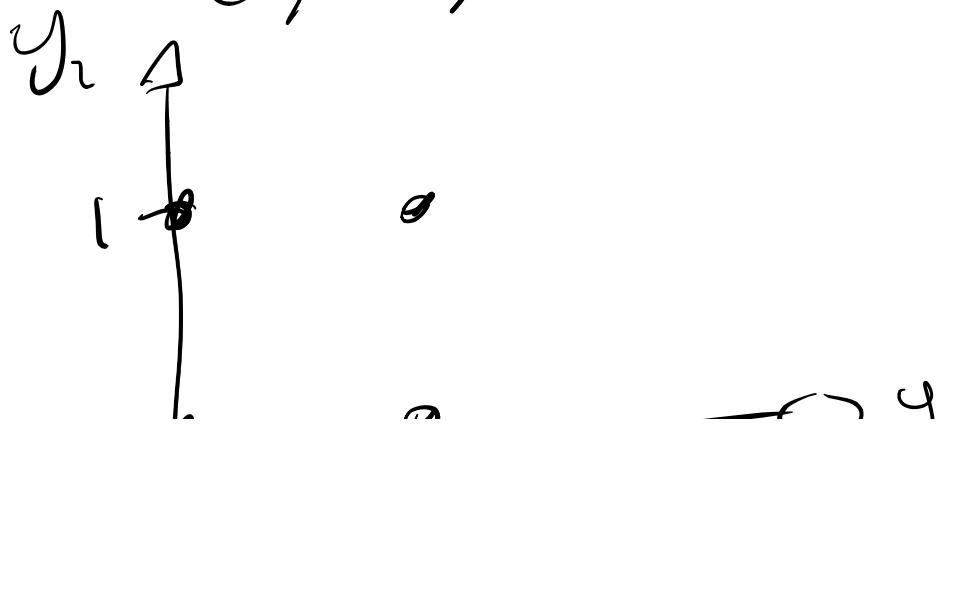
$$y_1(1 - y_1) = 0$$

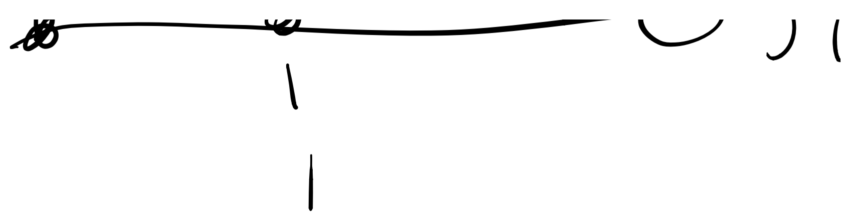
$$y_2 = 0, 1$$

$$y_1 = 0, 1$$

$$(0,0), \quad (0,1)$$

$$(1,0), \quad (1,1)$$





$$\begin{pmatrix} \frac{\partial f(y_1,y_2)}{\partial y_1} & \frac{\partial f(y_1,y_2)}{\partial y_2} \\ \frac{\partial g(y_1,y_2)}{\partial y_1} & \frac{\partial g(y_1,y_2)}{\partial y_2} \end{pmatrix}$$

$$f(t) = y_2 - y_2^2$$

$$g(t) = y_1 - y_1^2$$

$$J_{(y_1,y_2)}(f,g) = \begin{pmatrix} 0 & 1-2y_2 \\ 1-2y_1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1-2y_1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1-2y_1 & 0 \\ 0 & 1-2y_2 \end{pmatrix}$$

$(0, 0)$

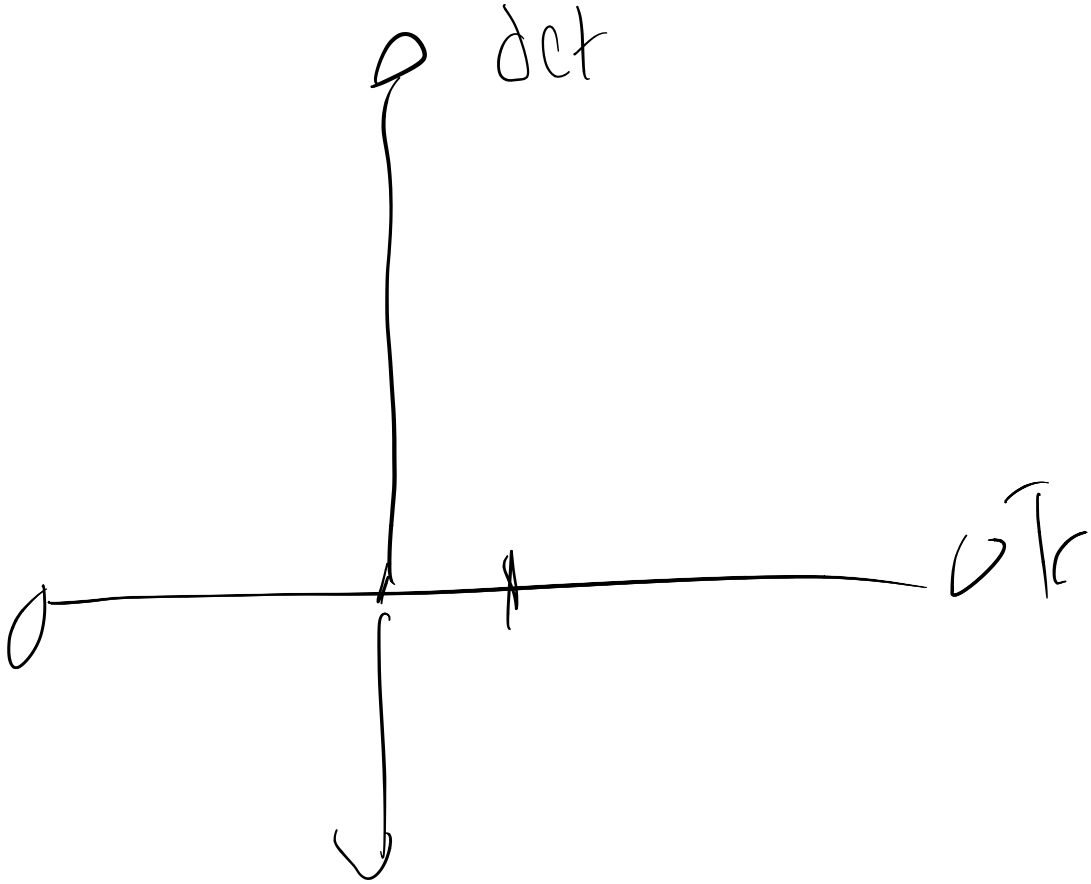
$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$\Rightarrow \mathrm{Tr}(A) = 2$

$\mathrm{Tr}(B) = 0$

- $\mathrm{Tr}(B) =$
- $\det(A) = 1$
- $\det(B) = -1$



$$y'' + A(x)\,y' + B(x)\,y = r(x)$$

are analytic

$\Rightarrow$ they have a power series rep

Anzats: $$\sum_{m=0}^{\infty} a_m (x-x_0)^m$$

$x_0 = 0$

$$= \sum_{m=0}^{\infty} a_m x^m$$

$$y'' + (1+x^2)y = 0$$

#1 on pg 124

analytic

calculate to order $O(x^6)$

standard form

$$\Rightarrow \boxed{y = \sum_{m=0}^{\infty} a_m x^m}$$

$$\boxed{y'' = \sum_{m=2}^{\infty} a_m (m)(m-1) x^{m-2}}$$

$$\boxed{x^2 y = \sum_{m=0}^{\infty} a_m x^{m+2}}$$

$$\Rightarrow \sum_{m=2}^{\infty} a_m (m)(m-1) x^{m-2} + \sum_{m=0}^{\infty} a_m x^m + \sum_{m=0}^{\infty} a_m x^{m+2} = 0$$

$$\sum^{\infty} a_{m+2}(m+2)(m+1) x^m + \sum^{\infty} a_m x^m + \sum^{\infty} a_{m-2} x^m$$

$m=0$ $m+2$

$m=0$ $m=2$

$$\theta(x^0): (a_2 \cdot 2 \cdot 1 + a_0) = 0$$

$$\theta(x^1): (a_3 \cdot 3 \cdot 2 + a_1) = 0$$

$$\theta(x^2): (a_4 \cdot 4 \cdot 3 + a_2 + a_0) = 0$$

$$\theta(x^3): (a_5 \cdot 5 \cdot 4 + a_3 + a_1) = 0$$

$$\theta(x^4): (a_6 \cdot 6 \cdot 5 + a_4 + a_2) = 0$$

$$\boxed{\theta(x^m): \quad a_{m+2}(m+2)(m+1) + a_m + a_{m-2}}$$

$$a_0 = a_0$$

$$a_1 = a_1$$

$$(a_2 \cdot 2 \cdot 1 + a_0) \overset{=0}{\Rightarrow} a_2 = \frac{-a_0}{2 \cdot 1}$$

$$\left(a_3 \cdot 3 \cdot 2 + a_1\right) \overset{=0}{\Rightarrow} a_3 = \frac{-a_1}{3 \cdot 2}$$

$$\left(a_4 \cdot 4 \cdot 3 + a_2 + a_0\right) = 0$$

$$\Rightarrow a_4 = \frac{-(a_2 + a_0)}{4 \cdot 3} = \frac{-a_0}{4 \cdot 3 \cdot 2}$$

$$\left(a_5 \cdot 5 \cdot 4 + a_3 + a_1\right) = 0$$

$$\Rightarrow a_5 = \frac{-(a_3 + a_1)}{5 \cdot 4} \Rightarrow \frac{-\cancel{5}a_1}{6 \cdot \cancel{5} \cdot 4}$$

$$\left(a_6 \cdot 6 \cdot 5 + a_4 + a_2\right)$$

$$\Rightarrow a_6 = \frac{-(a_4 + a_2)}{6 \cdot 5}$$

$$= \frac{-\left(\frac{-a_0}{4\cdot3\cdot2} - \frac{a_0}{2\cdot1}\right)}{6\cdot5} = \frac{13a_0}{24\cdot6\cdot5}$$

$$y = a_0 + a_1 x$$

$$- \frac{a_0}{2} x^2 - \frac{a_1}{6} x^3$$

$$- \frac{a_0}{24} x^4 - \frac{a_1}{24} x^5$$

$$+ \frac{13a_0}{720} x^6 + \ldots$$

$$y = a_0 \overbrace{\left(1 - \frac{1}{2}x^2 - \frac{1}{24}x^4 + \frac{13}{720}x^6 + \ldots\right)}^{y_1}$$

$$+ a_1 \left(x - \frac{1}{\ } x^3 - \frac{1}{\ } x^5 + \ldots\right)$$

$$+ a_1 \underbrace{\left(x - \frac{x}{6} + \frac{x}{24} + \dots \right)}_{y_2}$$

$$y'' - y = 0 \Rightarrow y = C_1 \underbrace{\cos(x)} + C_2 \underbrace{\sin(x)}$$

$$y'' + A(x)\, y' + B(x)\, y = r(x)$$

$\Omega \subset \mathbb{R}$

$A(x)$, $B(x)$, $r(x)$ are analytic in Ω

then $\Rightarrow$ y is also analytic in Ω

$$y'' - \boxed{\frac{2x}{(1-x^2)}}\, y' + \boxed{\frac{n(n+1)\, y}{(1-x^2)}} = 0$$

5. $\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^{n} x^{2n} = \sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^{\frac{n}{2}} x^{n}$

$$R = \lim_{n \to \infty} \frac{1}{\left| \frac{a_{n+1}}{a_n} \right|}$$

$$R = \lim_{n \to \infty} \frac{1}{\sqrt[n]{|a_n|}}$$

$$R = \lim_{n \to \infty} \frac{1}{\sqrt[n]{\left(\frac{2}{3}\right)^{\frac{n}{2}}}} = \frac{1}{\sqrt{\left(\frac{2}{3}\right)}}$$

$$= \lim_{n \to \infty} \frac{1}{\sqrt[2n]{\left(\frac{2}{3}\right)^{n}}}$$

$$\boxed{\sqrt{\left(\frac{3}{2}\right)}}$$

$$R = \sqrt{(2)}$$

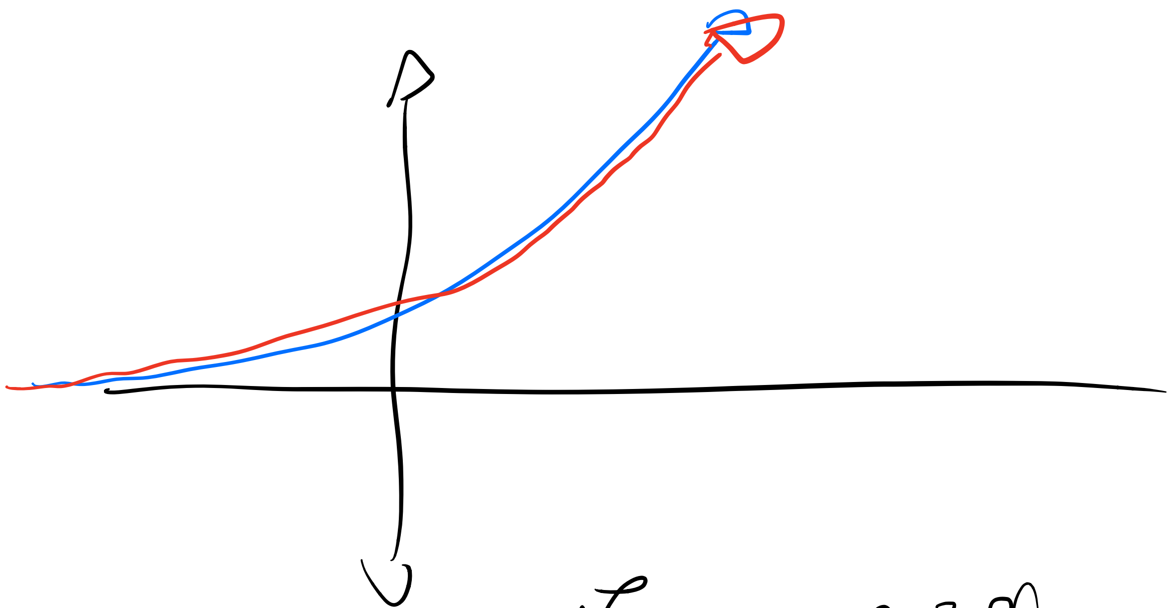
$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \cdots$$

$$R = \lim_{n \to \infty} \left| \frac{1}{\left| \frac{a_{n+1}}{a_n} \right|} \right| = \lim_{n \to \infty} \left| \frac{1}{\left| \frac{\frac{1}{(n+1)!}}{\frac{1}{n!}} \right|} \right|$$

$$= \lim_{n \to \infty} \left| \frac{(n+1)!}{n!} \right|$$

$$= \lim_{n \to \infty} \left| n+1 \right|$$

$$\approx \infty$$



$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2n!}$$

$$R = \lim_{n \to \infty} \left| \frac{1}{\left| \frac{a_{n+1}}{a_n} \right|} \right| = \lim_{n \to \infty} \left| \frac{1}{\frac{\frac{1}{(2n+2)!}}{\frac{1}{2n!}}} \right|$$

$$= \lim_{n \to \infty} \left| \frac{(2n+2)!}{2n!} \right|$$

$$\lim_{n\to 0} \left| (2n+2)(2n+1) \right|$$

$$= \infty$$

$$(1-x^2)y'' - 2xy' + \boxed{n(n+1)}\,y = 0$$

$$y'' - \frac{2x}{(1-x^2)}y' + \frac{n(n+1)}{1-x^2}\,y = 0$$

$$1-x^2 = 0$$

$$x^2 = 1$$

$$x = \pm 1$$

A(x) and B(x) get divided by zero

$$y = \sum_{m=0}^{\infty} a_m x^m$$

$$\sum_{m=2}^{\infty} m(m-1) a_m x^{m-2} - \sum_{m=2}^{\infty} m(m-1) a_m x^m$$

$$- \sum_{n=1}^{\infty} 2m\, a_m x^m + \sum_{m=0}^{\infty} \boxed{k}\, a_m x^m = 0$$

$\to n(n+1)$

$$= \sum_{s=0}^{\infty} (s+2)(s+1) a_{s+2} x^s$$

$$- \sum_{s=2}^{\infty} s(s-1) a_s x^s$$

$$- \sum_{s=1}^{\infty} 2s\, a_s x^s + \sum_{s=0}^{\infty} k a_s x^s$$

$$= 0$$

$-v$

$$O(x^s): (s+2)(s+1)a_{s+2} - s(s-1)a_s - 2s a_s - k a_s$$

$$\Rightarrow a_{s+2} = \frac{a_s(s^2 - s + k)}{(s+2)(s+1)}$$

$$\boxed{a_{s+2} = -\frac{a_s(n-s)(n+s+1)}{(s+2)(s+1)}}$$

Usually $a_0 = a_0$
$a_1 = a_1$

$$\boxed{(1-x^2)y'' - 2xy' + 6y = 0}$$

$n(n+1) \Rightarrow n=3$

$$a_n = \frac{(2n)!}{2^n (n!)^2}$$

$$a_3 = \frac{6!}{2^n (3!)^2}$$

$$P_n(x) = \sum_{m=0}^{M} (-1)^m \frac{(2n-2m)!}{2^n m! (n-m)! (n-2m)!} x^{n-2m}$$

$$P_3(x) = \sum_{m=0}^{1} (-1)^m \frac{(6-2m)!}{8 m! (3-m)! (3-2m)!} x^{3-2m}$$

$$M = \boxed{n/2} \text{ or } \boxed{\frac{(n-1)}{2}}$$

$$P_3(x) = \frac{1}{2}(5x^3 - 3x)$$

Only valid between

$(-1, 1)$