- Release Test Tonight
- Office hours 11:15 - 12:15 today

$R = 6\,\Omega$ $\quad \frac{1}{0.05} = 20$

$L = 1H$

$C = 0.05F$

$E = 255\,\underline{\sin(t)}$ for $0 < t < 2\pi$

$\boxed{I(0) = 0}$ $\quad Q(0) = 0$

$\Rightarrow \boxed{I'(0) = 0}$

$$\mathcal{L}(y'' + 6y' + 20) = \mathcal{L}(\underline{RHS})$$

$$Y(s)\left(s^2+6s+20\right)+\left(\cancel{I_c}\right)$$

0

RHS:

$E(t)=255\sin(t)$ for $0<t<2\pi$

$E'(t)=255\cos(t)$ for $0<t<2\pi$

$= 0$ elsewhere

$$E'(t)=255\cos(t)\left[H(t)-H(t-2\pi)\right]$$

$$\mathcal{L}\left(\qquad \right)$$

$$25s\left[\mathcal{L}\left(\cos(t)H(t)\right) - \mathcal{L}\left(\overset{\cos(t-2\pi)}{\cos(t)}\, H(t-\underbrace{2\pi}_{a})\right)\right]$$

$$\mathcal{L}\left(f(t-a)H(t-a)\right) = e^{-as}F(s)$$

Note: $\cos(t) = \cos(t-2\pi)$

$$25s\left[\frac{s}{s^2+1} - \frac{s}{s^2+1}e^{-2\pi s}\right]$$

$$\tilde{Y}(s) = 25s\left(\frac{s}{(s^2+1)(s^2+6s+20)} - \frac{s\boxed{e^{-2\pi s}}}{(s^2+1)(s^2+6s+20)}\right)$$

$$\mathcal{L}^{-1}\left(\frac{5}{(s^2+1)(s^2+6s+20)}\right)$$

$$=\frac{1}{397}\left[\mathcal{L}^{-1}\left(\frac{19s+6}{s^2+1}\right)+\mathcal{L}^{-1}\left(\frac{-19s-120}{s^2+6s+20}\right)\right]$$

$$19\mathcal{L}^{-1}\left(\frac{s}{s^2+1}\right)=19\cos(t)$$

$$6\mathcal{L}^{-1}\left(\frac{1}{s^2+1}\right) = 6\sin(t)$$

$$-19\,\mathcal{L}^{-1}\left(\frac{s}{s^2+6s+20}\right)$$

$$s^2+6s+20$$
$$= \underbrace{s^2+6s+9}+11$$
$$= (s+3)^2+11$$

$$= -19\,\mathcal{L}^{-1}\left(\frac{s+3}{(s+3)^2+11}\right)$$

$$\boxed{= -19\,e^{-3t}\cos(\sqrt{11}\,t)}$$

$$\frac{-120-57}{(s+3)^2+11} = \frac{-177}{(s+3)^2+11}$$

$$-177\,\mathcal{L}^{-1}\left(\frac{1}{(s+3)^2+11}\right)$$

$$= -\underline{177} \cdot \frac{1}{\sqrt{11}}\, e^{-3t} \sinh(\sqrt{11}\,t)$$

$$y(t) = \frac{1}{397}\Bigg[19\cos(t) + 6\sin(t) - 19e^{-3t}\cos(\sqrt{11}\,t)$$

$$\underbrace{-\frac{177}{\sqrt{11}}\, e^{-3t}\sinh(\sqrt{11}\,t)\Bigg]}_{f(t)}$$

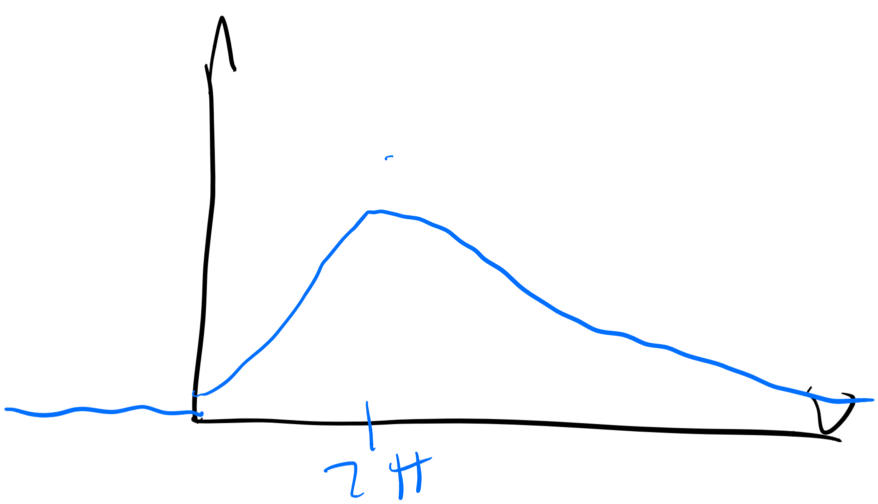
$$\mathcal{L}\left(e^{-2\pi s}F(s)\right)$$

$$\mathcal{L}^{-1}\left(e^{-as}F(s)\right) = f(\underline{t-a})H(t-a)$$

$$\Bigg(\frac{255}{397}\left[19\cos(t) + 6\sin(t) - 19e^{-3t}\cos(\sqrt{11}\,t) - \frac{177}{\sqrt{11}} e^{-3t}\sinh(\sqrt{11}\,t) \right]$$

$$\sqrt{11}$$

$$-H(t-2\pi)\left[19\cos(t-2\pi) + 6\sin(t-2\pi) - 19e^{-3(t-2\pi)}\cos(\sqrt{11}\,(t-2\pi)) - \frac{177}{\sqrt{11}} e^{-3(t-2\pi)}\sinh(\sqrt{11}\,(t-2\pi)) \right] \Bigg)$$



$$\begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} -\frac{8}{100} y_1 \ \frac{2}{100} y_2 \\ \frac{8}{100} y_1 \ -\frac{8}{100} y_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 6 \\ 0 \end{bmatrix}}_{g(t)}$$

$$\vec{y}' = A\vec{y} + g(t)$$

$$y_1(0) = 0, \quad y_2(0) = 150$$

$$sY_1(s) - 0 = -\frac{8}{100} Y_1(s) + \frac{2}{100} Y_2(s) + \frac{6}{s}$$

$$sY_2(s) - 150 = \frac{8}{100} Y_1(s) - \frac{8}{100} Y_2(s)$$

$$\begin{bmatrix} (-0.08 - s) & 0.02 \end{bmatrix} \begin{bmatrix} Y_1 \end{bmatrix} \quad \begin{bmatrix} -\frac{6}{s} \end{bmatrix}$$

$$\begin{bmatrix} & \\ 0.08 & (-0.08-s) \end{bmatrix} \begin{bmatrix} \vdots \\ Y_2 \end{bmatrix} = \begin{bmatrix} \vdots \\ -150 \end{bmatrix}$$

$$A\vec{y} = \vec{b}$$

$$A^{-1}\left(A\vec{y} = \vec{b}\right)$$

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad \vec{y} = A^{-1}\vec{b}$$

$$A^{-1} = \boxed{\frac{1}{\det(A)} \begin{bmatrix} d & b \\ -c & a \end{bmatrix}}$$

$$\vec{Y}(s) = \frac{1}{s^2 + .16s + .0048} \begin{bmatrix} -0.08-s & -.08 \\ -0.02 & -0.08-s \end{bmatrix}$$

$$\begin{bmatrix} -\frac{6}{s} \\ \end{bmatrix}$$

$$-150$$

$$Y_1(s) = \frac{9s + 0.48}{s(s+0.12)(s+0.04)}$$

$$Y_2(s) = \frac{150s^2 + 12s + 0.48}{s(s+0.12)(s+0.04)}$$

$$y_1(t) = 100 - 62.5e^{-12t} - 37.5e^{-.04t}$$
$$y_2(t) = 100 + 125e^{-0.12t} - 75e^{-0.04t}$$

$$(n) \quad (n-1) \quad (n-2)$$

$y + a_0(x)y + a_1(x)y \dots a_n(x)y =$
$f(x)$

$\boxed{5.1}$ Guessing ~~solutions~~ Power Series

$$y(x) = \sum_{m=0}^{\infty} a_m (x - x_0)^m$$

$$y' - y = 0$$

$$\frac{dy}{dt} - y = 0$$

$$\int \frac{1}{y}\,dy = \int dt$$

$$c \cdot e^t$$

$$\Rightarrow \boxed{y(t) = 4}$$

$$y(x) = \sum_{n=0}^{\infty} a_n (x - x_0)^n$$

$$x_0 = 0$$

$$y'(x) = \sum_{n=1}^{\infty} a_n \cdot n \, x^{n-1}$$

$$y' - y = 0$$

$$\sum_{n=1}^{\infty} a_n n \, x^{n-1} - \sum_{n=0}^{\infty} a_n x^n = 0$$

$$O(x^0): a_1 - a_0 = 0$$

$$O(x^1): 2a_2 - a_1 = 0$$

$$\vdots$$

$$O(x^m): a_{m+1} \cdot (m+1) - a_m = 0$$

$$\Rightarrow \boxed{a_{m+1} = \frac{a_m}{m+1}}$$

$$a_0 = a_0$$

$$a_1 = \frac{a_0}{1}$$

$$a_2 = \frac{a_1}{2} = \frac{a_0}{2 \cdot 1}$$

$$a_3 = \frac{a_2}{3} = \frac{a_1}{3 \cdot 2} = \frac{a_0}{3 \cdot 2 \cdot 1}$$

$$a_{m+1} = \frac{a_0}{(m+1)!}$$

$$\Rightarrow \boxed{a_m = \frac{a_0}{m!}}$$

$$y(x) = \sum_{m=0}^{\infty} a_m x^m = a_0 \sum_{m=0}^{\infty} \frac{x^m}{m!} = a_0 e^x$$

$$y'' + a(x)y' + b(x)y = r(x)$$

what being analytic means:

$$\sum^{\infty} a \, (x - x_0)^m$$ on some subset

$f(x) = \sum_{m=0} a_m$ of $\mathbb{R}$

A solution to this DE in a domain Ω will be solvable via anzats by power series if $a(x)$ and $b(x)$ and $r(x)$ are all analytic in Ω.

What is the radius of conv.?

$$f(x) = \sum_{m=0}^{\infty} a_m (x-x_0)^m$$

$$\lim_{m\to\infty} \left| \frac{a_{m+1}}{a_m} \right| \text{ if } \begin{cases} = 0 \to R = \infty \\ = \alpha \in \mathbb{R}/0 \; R = \frac{1}{\alpha} \end{cases}$$

$= \infty \quad R = 0$

$$R = \lim_{m \to \infty} \left(\frac{1}{\left| \frac{a_{m+1}}{a_m} \right|} \right)$$

Power series rules

add them termwise

subtract them termwise

$$\sum_{m=0}^{\infty} a_m x^m \ \& \ \sum_{m=0}^{\infty} b_m x^m$$

$$\sum (a_m \pm b_m) x^m$$

$$(a_0 \pm b_0)x^0$$

$$(a_0 + a_1 x + a_2 x^2 + \ldots\ldots)$$

$$\cdot\, (b_0 + b_1 x + b_2 x^2 + \ldots)$$

$$\theta(x^0):\ a_0 b_0$$

$$\theta(x^1):\ a_0 b_1 x + b_0 a_1 x$$

$$\theta(x^2):\ a_1 b_1 x + b_0 a_2 x^2$$

$$+\ a_0 b_2 x^2$$