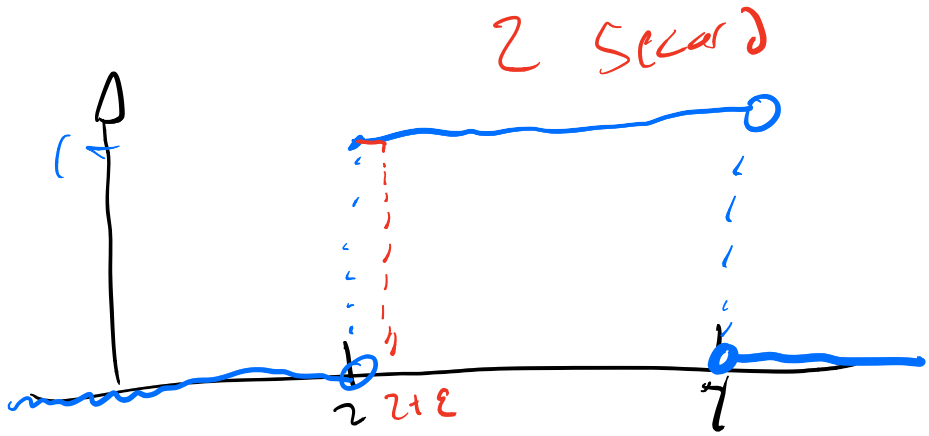
-6.4, 6.5, 6.6, 6.7

6.3 $f(y, y', y'')$

$= f(H(t))$



$H(t-2) - H(t-4)$

$-H(t-2+\varepsilon)$

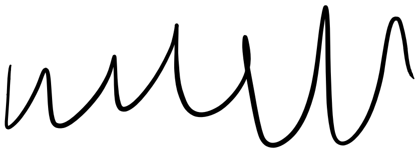
$\varepsilon \to 0$

$$\delta(t-k) = \begin{cases} 1 & \text{if } t = k \\ 0 & \text{if } t \neq k \end{cases}$$

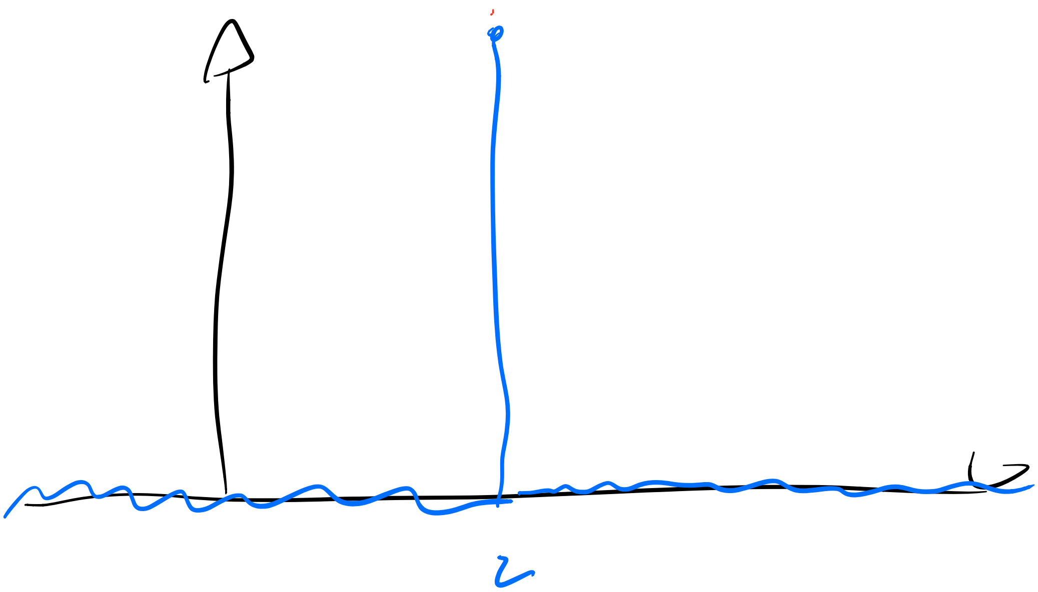
$$\boxed{\int_0^\infty \underbrace{\delta(t-k)}_{} f(t)\, dt}$$

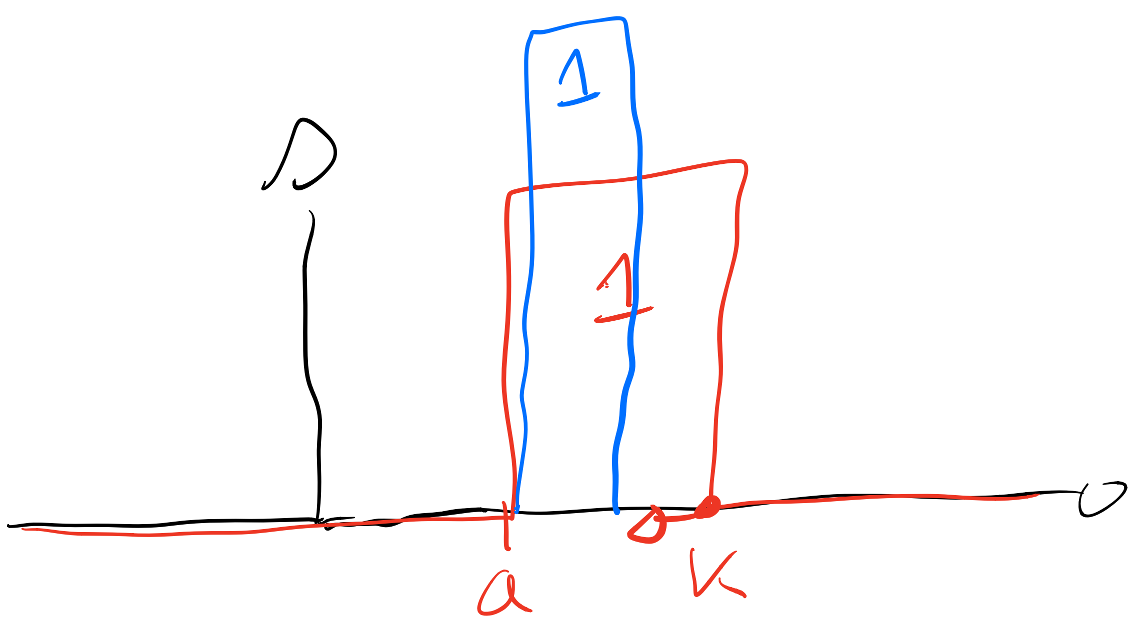
$$= F(k)$$

Note: $\int f(t)\, dt = F(t)$



$= F_0 \, \delta(t-2)$





$$\lim_{k\to 0} \frac{1}{k}\left[H(t-a) - H(t-(a+k))\right] \approx \delta(t-a)$$

$$\frac{1}{k}\left(\mathcal{L}\left(H(t-a)\right) - \mathcal{L}\left(H(t-(a+k))\right)\right)$$

$$= \frac{1}{k}\left[\frac{e^{-as}}{s} - \frac{e^{-(a+k)s}}{s}\right]$$

$$!$$

$$\lim_{k\to 0} \quad . \quad . \quad . \quad .$$

$$= \lim_{k\to 0} \frac{e^{-as}}{s}\left(\frac{1-e^{-ks}}{k}\right)$$

$$. \quad . \qquad\qquad 0$$

$$= \frac{1-1}{0} = \frac{\;}{0}$$

$$L'H = \frac{e^{-as}}{s}\left(\frac{0 + se^{-ks}}{1}\right)$$

$$\lim_{k \to 0} = \frac{e^{-as}}{\cancel{s}}\left(\frac{\cancel{s}}{1}\right)$$

$$= e^{-as}$$

$$= \mathcal{L}(\delta(t-a))$$

$$\mathcal{L}(f(t)g(t)) \neq \mathcal{L}(f(t)) \cdot \mathcal{L}(g(t))$$

$$\mathcal{L}^{-1}(F(t)G(t))$$

$$= \mathcal{L}^{-1}(F(t)) * \mathcal{L}^{-1}(G(t))$$

convolution

$$= f(t) * g(t)$$

$$= \int^{t} f(\tau) g(t-\tau)\, d\tau$$

$$\boxed{= \int_0^t f(t-\tau)\, g(\tau)\, d\tau}$$

Suppose we want

$$\mathcal{L}^{-1}\left(\frac{1}{(s-a)s}\right) = \mathcal{L}^{-1}\left(\frac{1}{(s-a)} \cdot \frac{1}{s}\right)$$

$$= \mathcal{L}^{-1}\left(\frac{1}{s-a}\right) * \mathcal{L}^{-1}\left(\frac{1}{s}\right)$$

$$= e^{at} * 1$$

$$= \int^{t} e^{a\tau} \ldots$$

$$= \int_0 e \cdot 1 \, d\tau$$

$$= \frac{1}{a} e^{a\tau}\Big|_0^t$$

$$= \boxed{\frac{e^{at}}{a} - \frac{1}{a}}$$

$$\mathcal{L}^{-1}\left(\frac{w}{s^2(s^2+w^2)}\right)$$

$$= \mathcal{L}^{-1}\left(\frac{1}{s^2}\right) * \mathcal{L}^{-1}\left(\frac{w}{(s^2+w^2)}\right)$$

$$= t * \sin(\omega t)$$

$$= \int_0^t \tau \sin(\omega(t-\tau))\, d\tau$$

$$= \int_0^t \sin(\omega\tau)(t-\tau)\, d\tau$$

$$= t\int_0^t \sin(\omega\tau)\, d\tau$$

$$- \int_0^t \tau \sin(\omega\tau)\, d\tau$$

t

$$= t\left(-\frac{\cos(\omega t)}{\omega}\right)\Big|_0^t$$

$$-\left(\frac{\sin(\omega t) - \omega \tau \cos(\omega t)}{\omega^2}\right)\Big|_0^t$$

$$= \boxed{-\frac{\sin(\omega t)}{\omega^2} + \frac{t}{\omega}}$$

$$y'' + \int^t y(\tau)\sin(t-\tau)\,d\tau$$

$= 0$

0

$$= y(t) * \sin(t)$$

$$= \mathcal{L}^{-1}(Y(s)) * \mathcal{L}^{-1}\left(\frac{w}{s^2+w^2}\right)$$

$$\boxed{= \mathcal{L}^{-1}\left(\frac{Y(s)\,w}{s^2+w^2}\right)}$$

$$s^2 Y(s) + \frac{Y(s)\,w}{s^2+w^2} = 0$$

6.6

$$\mathcal{L}(tf(t)) = -\frac{d}{ds}\mathcal{L}(f(t))$$

$$= -F'(s)$$

$$\mathcal{L}(t\cos(t))$$

$$= -\frac{d}{ds}\mathcal{L}(\cos(t))$$

$$= -\frac{d}{ds}\left(\frac{1}{s^2+1}\right)$$

$$= -\left(\frac{-1}{(s^2+1)^2} \cdot 2s \right)$$

$$= \frac{2s}{(s^2+1)^2}$$

$$\mathcal{L}\left(\underline{t}\,\underline{f(t)}\right) = -\frac{d}{ds}\mathcal{L}\left(f(t)\right)$$

$$\mathcal{L}\left(t^n e^{kt}\right)$$

$$-\frac{d}{ds}\left(\overbrace{\left(t^{n-1} e^{kt}\right)}^{f_1(t)} \right)$$

$$-\mathcal{L}(t(t\; e))$$

$$= -\frac{d}{ds}\underbrace{\mathcal{L}\left(t^{n-1}e^{kt}\right)}$$

$$= -\frac{d}{ds}\mathcal{L}\left(t\left(\underbrace{t^{n-2}e^{kt}}_{f_2(t)}\right)\right)$$

$$= -1\cdot -1\cdot \frac{d^2}{ds^2}\mathcal{L}\left(t^{n-2}e^{kt}\right)$$

$$\vdots$$

$$= (-1)^n \frac{d^n}{ds^n}\mathcal{L}\left(t^{\cancel{(n-n)}}e^{kt}\right)$$

$$-(-1)\,\frac{d}{ds^n}\,\mathcal{L}(t \cdots)$$

$$= (-1)^n \frac{d^n}{ds^n}\left(\frac{1}{s-k}\right)$$

$$= (-1)^n \frac{d^{n-1}}{ds^n}\left(\frac{-1}{(s-k)^2}\right)$$

$$= \cancel{(-1)^n(-1)^n}\,\frac{n!}{(s-k)^{n+1}}$$

$$\boxed{= \frac{n!}{(s-k)^{n+1}}}$$

$$= \mathcal{L}\left(t^n e^{kt}\right)$$

$$\mathcal{L}\left(t^2 \cos(3t)\right)$$

$$= \mathcal{L}\left(t\left(t\cos(3t)\right)\right)$$

$$= -d\ \mathcal{L}\left(t\cos(3t)\right)$$

$$-\frac{d}{ds}\mathcal{L}$$

$$= \frac{d^2}{ds}\mathcal{L}\left(\cos(3t)\right)$$

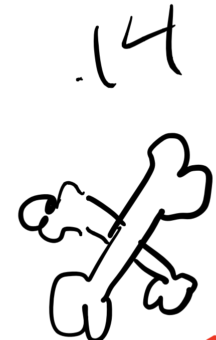
$$= \frac{d^2}{ds^2}\left(\frac{1}{s^2+9}\right)$$

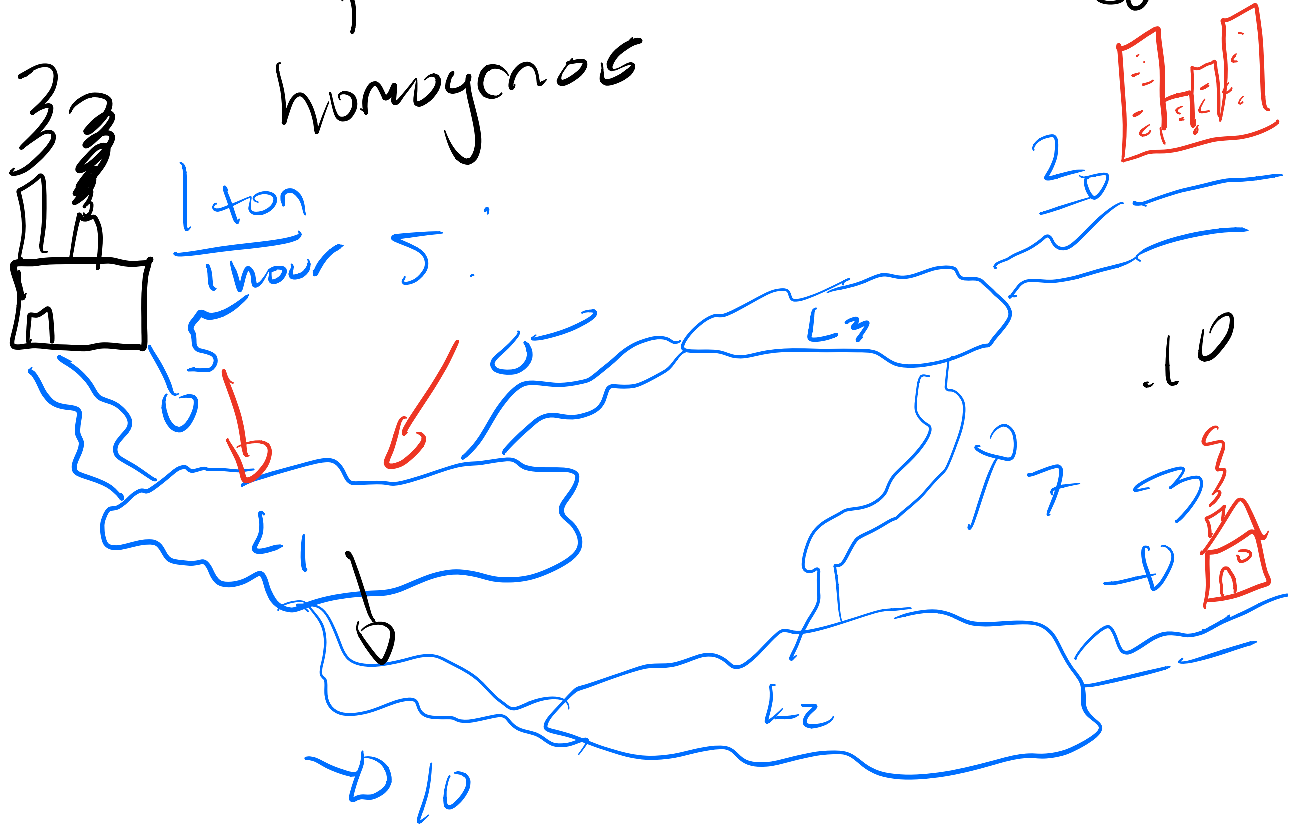
$$\mathcal{L}\left(\frac{f(t)}{t}\right) = \int_s^\infty F(s)\,ds$$

$$\vec{y}\,' = A\vec{y} + \boxed{g(t)}$$

$$\boxed{\vec{y}\,' - A\vec{y} = 0}$$

homogenous





$L_1(0) = 0$

$L_2(0) = 0$

$L_3(0) = 0$

$$\boxed{L_n' = \text{ratein} - \text{rateout}}$$

$$L_1' = \boxed{1 \frac{\text{ton}}{\text{hr}}} + 5L_3 - 10L_1$$

$$\left(L_3 \frac{\text{tons}}{\text{gallon}} \cdot \frac{5 \text{ gallon}}{\text{hrs}}\right) = \frac{\text{tons}}{\text{hour}}$$

$$L_2' = 10L_1 \underbrace{-3L_2 - 7L_2}_{-10L_2}$$

$$L_3' = 7L_2 \underbrace{-3L_3 - 5L_3}_{-8L_3}$$

$$\begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix}' = \begin{bmatrix} -10 & 0 & 5 \\ 10 & -10 & 0 \\ 0 & 7 & -7 \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix}$$

$$+\underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \nabla}_{f(t)}$$

$$s\tilde{L}_1 = -10\tilde{L}_1 + 5\tilde{L}_3 + \frac{1\nabla}{5}$$

$$s\tilde{L}_2 = 10\tilde{L}_1 - 10\tilde{L}_2$$

$$s\tilde{L}_3 = 7\tilde{L}_2 - 7\tilde{L}_3$$

$$\begin{bmatrix} s+10 & 0 & -s \\ -10 & s+10 & 0 \\ 0 & -7 & (s+7) \end{bmatrix} \begin{bmatrix} \tilde{L}_1(s) \\ \tilde{L}_2(s) \\ \tilde{L}_3(s) \end{bmatrix} = \begin{bmatrix} V_s \\ 0 \\ 0 \end{bmatrix}$$

$$\vec{\tilde{L}}(s) = \vec{f}_1(s)$$

$$A\vec{\tilde{L}}(s) = \vec{f}_2(s)$$

$$A^{-1}\left(A\vec{\tilde{L}}(s) = \vec{f}_2(s)\right)$$

$$\left(I\tilde{L}(s) = A^{-1} f_2(s) \right)$$

$$\Rightarrow \tilde{L}(s) = \tilde{f}_3(s)$$