$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\boxed{\lambda = \frac{\mathrm{Tr}(A) \pm \sqrt{\mathrm{Tr}(A)^2 - 4\det(A)}}{2}}$$

$$y'' + 3y' - 4y = 6e^{2t-3}$$

$$\begin{cases} y(1.5) = 4 \\ y'(1.5) = 5 \end{cases}$$

$$t = \tilde{t} + 1.5$$

$$\tilde{y}'' + 3\tilde{y}' - 4\tilde{y} = 6e$$

$$\tilde{y}(0) = 4$$

$$\tilde{y}'(0) = 5$$

$$\tilde{Y}(s) = \frac{\frac{6}{(s-2)} + 4(s+3) + 5}{s^2 + 3s - 4}$$

$$\mathcal{L}(\tilde{y}'')$$

$$= s^2 \mathcal{L}(\tilde{y})$$

$$-s\tilde{y}(0) - \tilde{y}'(0)$$

$$= s^2 \tilde{Y}(s) - s4 - 5$$

$$3 \cdot \mathcal{L}(\tilde{y}')$$

$$= 3\left(s\,\mathcal{L}(\tilde{y}) - \tilde{y}(0)\right)$$

$$\left(s\tilde{Y}(s) - 4\right) \cdot 3$$

$$-4\,\mathcal{L}(\tilde{y}) = -4\tilde{Y}(s)$$

$$s^2 \tilde{Y}(s) - s4 - 5 + 3s\tilde{Y}(s) - 12$$
$$- 4\tilde{Y}(s) = \mathcal{L}(RHS)$$

$$\tilde{Y}(s)\underline{(s^2+3s-4)}\ \underline{-17-s4} = \mathcal{L}(RHS)$$

$$\tilde{Y}(s) = \frac{\frac{6}{s-2} + 17 + s4}{s^2+3s-4}$$

$$\underline{y(t)} = \underline{\mathcal{L}^{-1}(RHS)}$$

$$RHS = \underbrace{\frac{17+4s}{(s+4)(s-1)}} + \frac{6}{(s-2)(s+4)(s-1)}$$

$$\frac{4s+17}{(s+4)(s-1)} = \boxed{\frac{A}{(s+4)}} + \frac{B}{(s-1)}$$

$$\Rightarrow 4s+17 = A(s-1) + B(s+4)$$

$s=1$

$4+17 = B \cdot 5$

$\frac{21}{5} = B$

$s=-4$

$1 = -5A$

$\boxed{A = -\frac{1}{5}}$

A B

$$\frac{6}{(s-2)(s+4)(s-1)} = \frac{C}{(s-2)} + \boxed{\frac{A}{(s+4)}} + \frac{B}{(s-1)}$$

$$6 = C(s+4)(s-1) + A(s-2)(s-1)$$
$$+ B(s-2)(s+4)$$

$s=2$

$$6 = 6C \Rightarrow \boxed{C = 1}$$

$$\boxed{B = -\frac{6}{5} \qquad A = \frac{1}{5}}$$

$$\tilde{Y}(s) = \cancel{0 \cdot \frac{1}{(s+4)}} + 3 \cdot \frac{1}{(s-1)} + \frac{1}{(s-2)}$$

$$\tilde{y}(\tilde{t}) = 3\mathcal{L}^{-1}\left(\frac{1}{s-1}\right) + \mathcal{L}^{-1}\left(\frac{1}{s-2}\right)$$

$$\tilde{y}(\tilde{t}) = 3e^{\tilde{t}} + e^{2\tilde{t}}$$

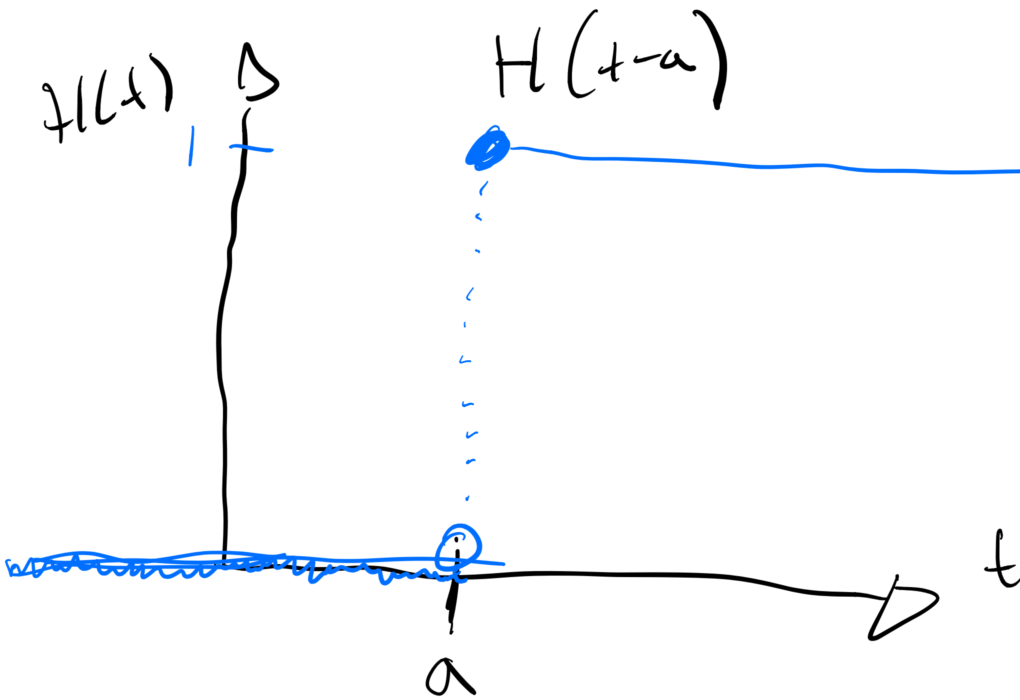
$$t = \tilde{t} + 1.5$$

$$\boxed{y(t) = 3e^{t-1.5} + e^{2t+3}}$$

$$f(y'', y', y) = \underbrace{[x]}_{\text{driver}}$$

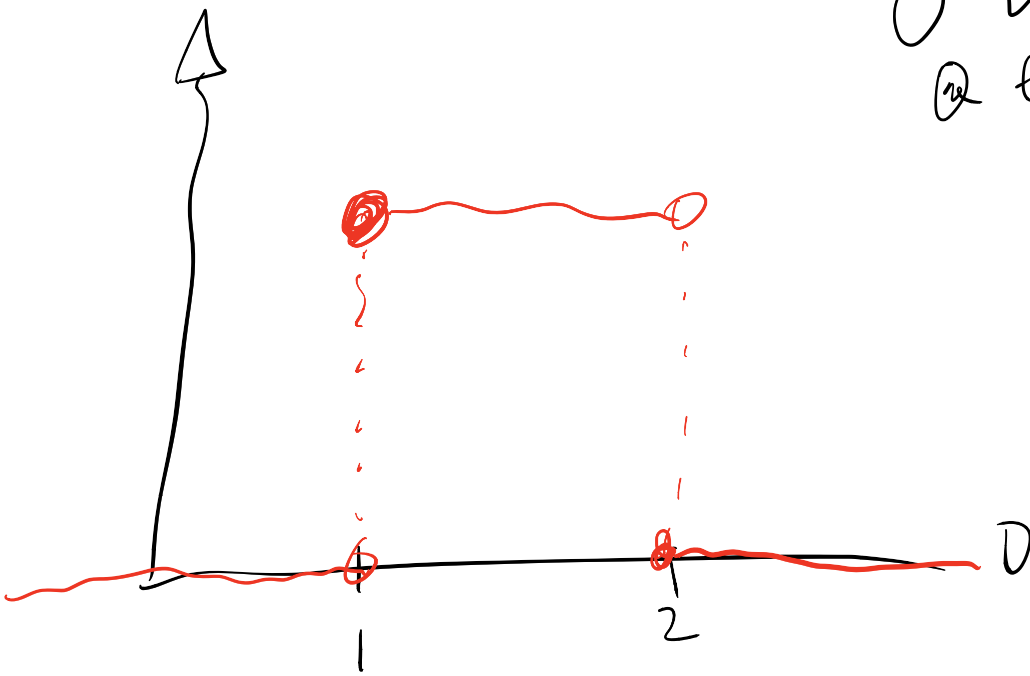
Physics of the system

You putting "something" into the system or doing something to the system



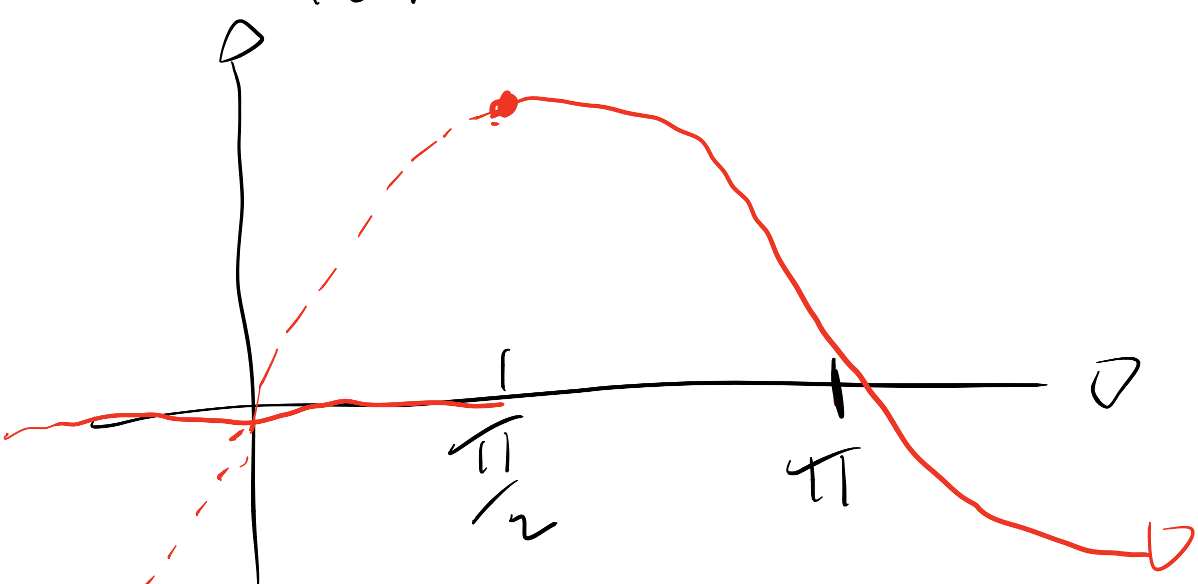
$$f(t) = H(t-1) - H(t-2)$$

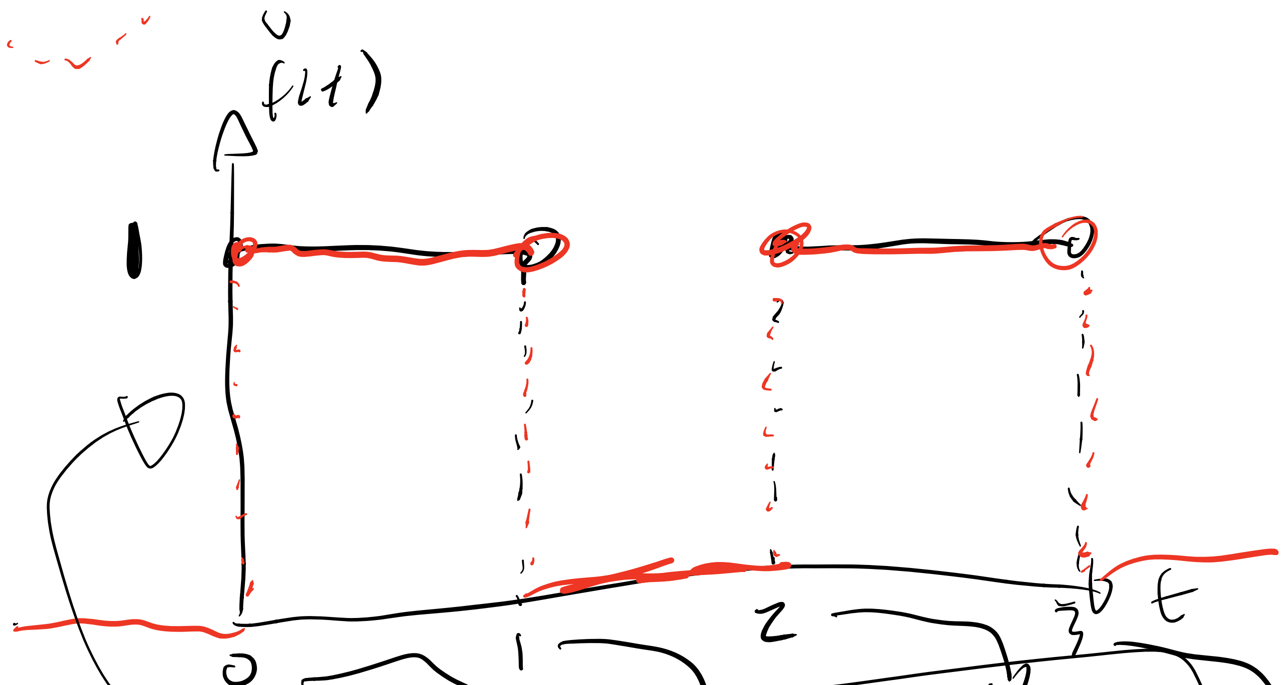
0 → 1
@ $t = 2$



$t = \frac{\pi}{2}$ I start oscillating my thing like a sinusoid

$$f(t) = H\left(t - \tfrac{\pi}{2}\right)\sin(t)$$





$$f(t) = H(t) - H(t-1) + H(t-2) - H(t-3)$$

$$\mathcal{L}\big(H(t-a)\,f(t-a)\big) = e^{-as}F(s)$$

$$\mathcal{L}\big(H(t-a)\,f(t)\big) = e^{-as}F(t+a)$$

$$\mathcal{L}\left(H(t-2)t\right)$$

$$= e^{-2s}\mathcal{L}\left(f(t+2)\right)$$

$$= e^{-2s}\mathcal{L}\left(t+2\right)$$

$$\boxed{= e^{-2s}\left(\frac{1}{s^2}+\frac{2}{s}\right)}$$

Problem 38

RLC

$R = 4\,\Omega$

$R =$

$L = 1H$

$C = 0.05F$

$E(t) = 34\, e^{-t}\, V$ for $t \in (0,4)$

$= 0$ for $(-\infty, 0] \cup [4, \infty)$

$$\boxed{L I'' + R I' + \frac{I}{C} = E'(t)}$$

$$L I' + R I + \frac{1}{C} \underbrace{\int I\, dt}_{Q(t)} = E(t)$$

$I(0) = 0$

$Q(0) = 0$

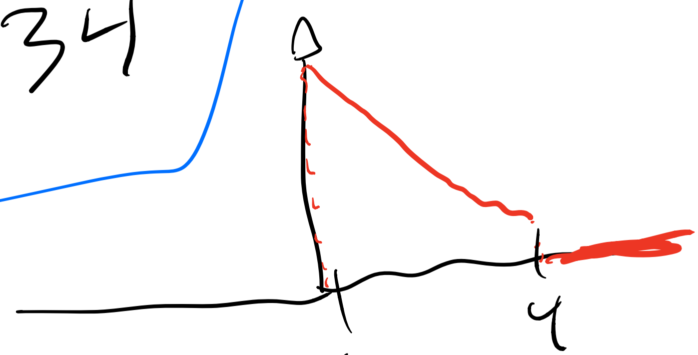
$\hookrightarrow I'(0) = ?$

$t = 0$

$$L I'(0) + R \cancel{I(0)}^{\,0} = E(0)$$

$$I'(0) = \frac{E(0)}{L} = \frac{34}{1}$$

$$\boxed{I'(0) = 34}$$



$$E(t) = 34e^{-t}H(t) - 34e^{-t}H(t-4)$$

$$\boxed{E'(t) = -34e^{-t}H(t) + 34e^{-t}H(t-4)}$$

$$\mathcal{L}(E'(t))$$

$$= -34\mathcal{L}\left(e^{-t}H(t)\right)$$

$$+34\mathcal{L}\left(e^{-t}H(t-4)\right)$$

$$= -34\mathcal{L}(e^{-t}) + 34e^{-4s}\mathcal{L}(e^{-(t+4)})$$

$f(t) = e^{-t}, \quad a = 0$

$\boxed{\begin{aligned} f(t) &= e^{-t} \\ a &= 4 \end{aligned}}$

$$\mathcal{L}(H(t-a)f(t-a))$$

$$= e^{-as}F(s)$$

$$\mathcal{L}\left(H(t-a)f(t)\right) = e^{-as} F(t+a)$$

$$= -34\frac{1}{s+1} + 34\frac{e^{-4s+4}}{s+1}$$

$$\mathcal{L}\left(e^{-t-4}\right) = e^{4}\mathcal{L}\left(e^{-t}\right)$$