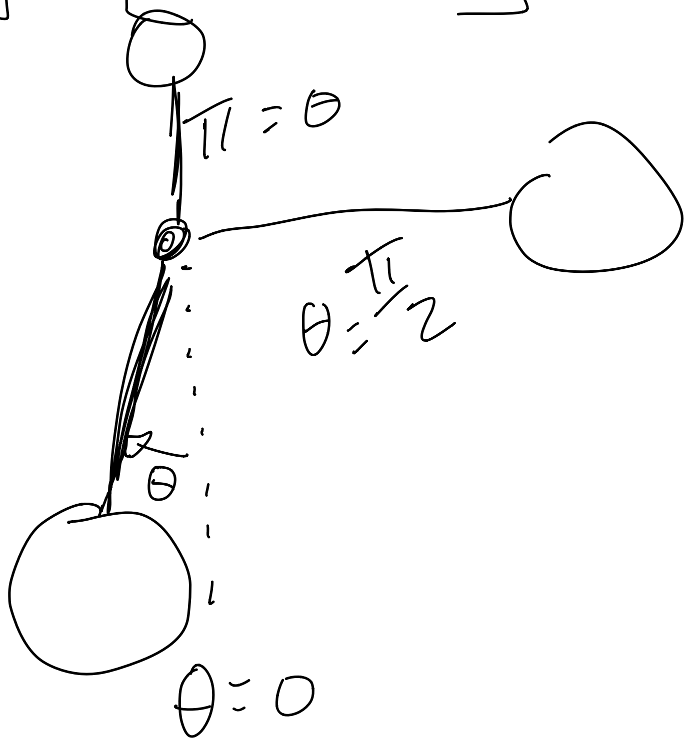
$$mL\theta'' + mg\sin(\theta) = 0$$

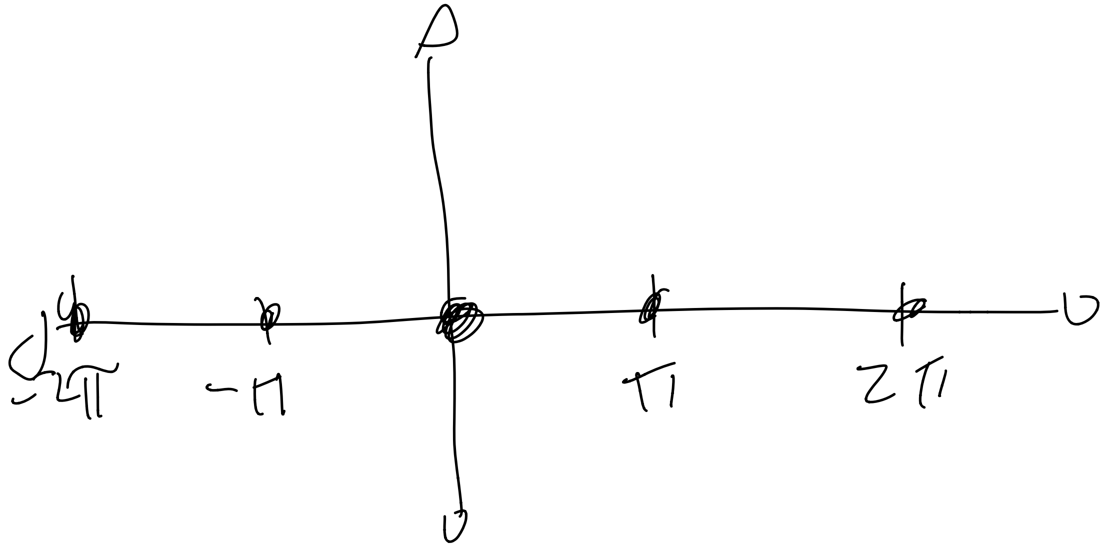
$$\downarrow$$

$$\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}' = \begin{bmatrix} \theta_2 \\ -k\sin(\theta_1) \end{bmatrix}$$



$$\theta_1' = 0 \qquad \theta_2' = 0 \implies \theta_1 = 0 \qquad \theta_2 = \pm n\pi$$

$$(\theta_1, \theta_2) = (0, \pm n\pi)$$



$$g = \theta_2$$

$$h = -k\sin(\theta_1)$$

$$\begin{bmatrix} 0 & 1 \\ \boxed{-k\cos(\theta_1)} & 0 \end{bmatrix}$$

Crit. points: $(0, \pm n\pi)$

if n is even

$$\Rightarrow -k\cos(2m\pi)$$

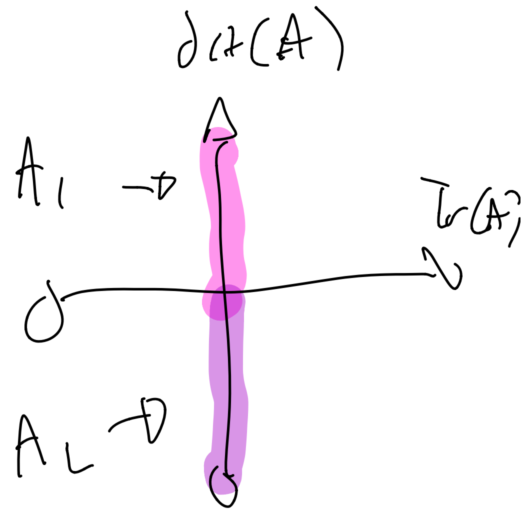
$$= -k \Rightarrow \begin{bmatrix} 0 & 1 \\ -k & 0 \end{bmatrix} = A_1$$

if n is odd

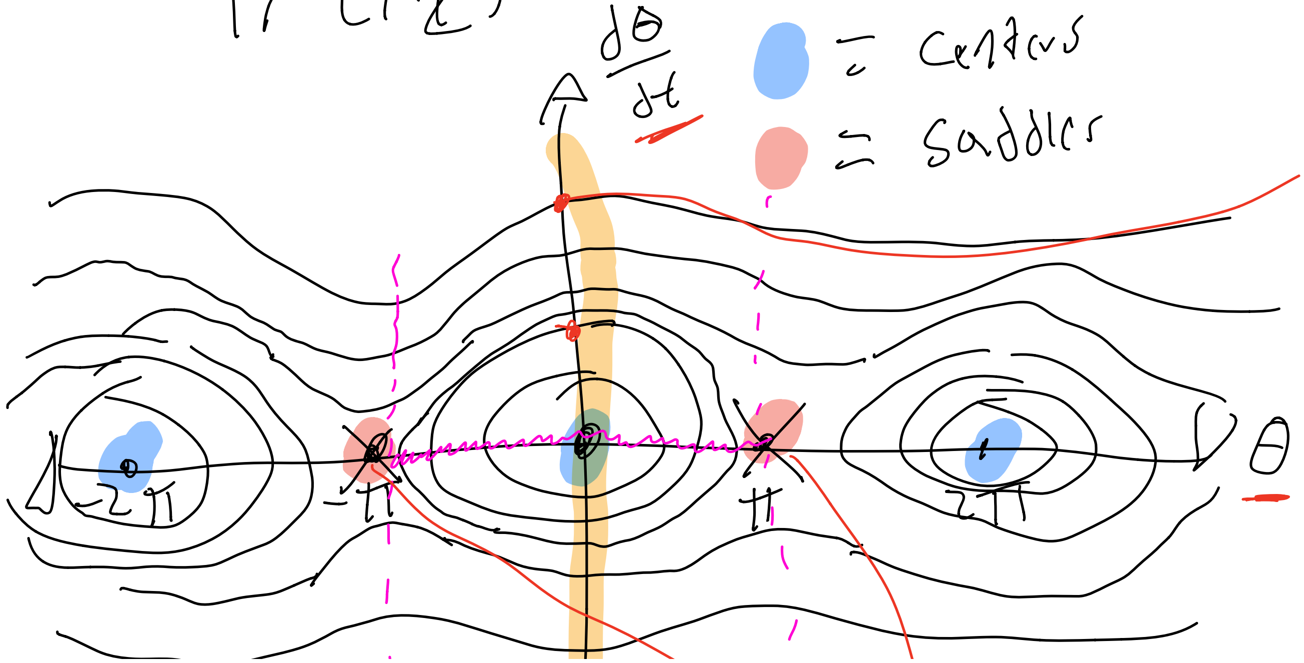
$$\Rightarrow -k\cos((2m+1)\pi)$$

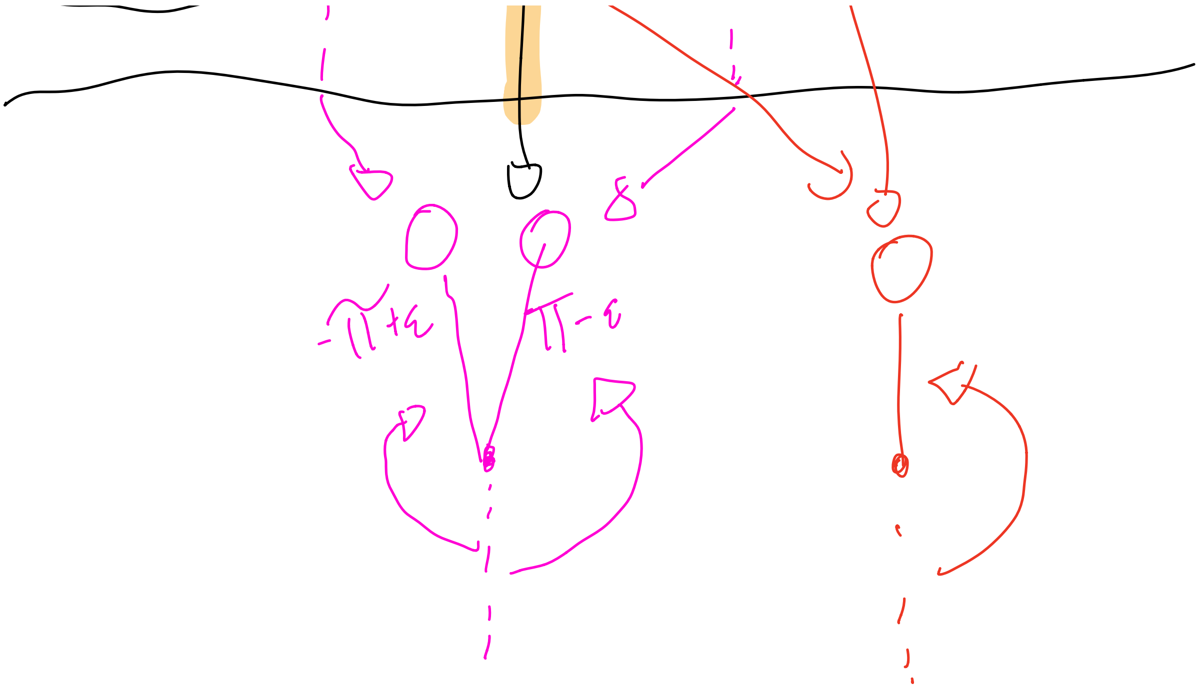
$$= k \Rightarrow \begin{bmatrix} 0 & 1 \\ k & 0 \end{bmatrix} = A_2$$

$$\det(A_1) = k$$
$$\mathrm{Tr}(A_1) = 0$$
$$\Rightarrow \text{center}$$



$$\det(A_2) = -k$$
$$\mathrm{Tr}(A_2) = 0$$
$$\Rightarrow \text{saddle}$$





S.1, S.2, (S.3?)

Laplace Transforms

- L.T. turn DEs to Algebraic eqns.

- Make nonhomogeneous

problems easier

- Make it easier to work with (on/off) drivers

$$\mathcal{L}(f(t)) = \int_0^\infty e^{-st} f(t)\, dt = F(s)$$

$$\mathcal{L}(1) = \int_0^\infty e^{-st}\, dt = -\frac{1}{s} e^{-st} \Big|_0^\infty$$

$$= 0 - -\frac{1}{s} = \boxed{\frac{1}{s}}$$

$$\mathcal{L}(t^n) = \int_0^\infty e^{-st} t^n\, dt$$

let $u = st$

$\Rightarrow du = s\,dt$

$\frac{du}{s} = dt$

$$= \int_0^\infty e^{-u} \left(\frac{u}{s}\right)^n \frac{du}{s}$$

$$= \frac{1}{s^{n+1}} \boxed{\int_0^\infty u^n e^{-u}\, du}$$

$$= \frac{1}{s^{n+1}} \Gamma(n+1)$$

$$= \frac{1}{s^{n+1}} n!$$

pg 248-251 Table of Laplace transforms

trans...

$$\mathcal{L}(t^2) \quad n=2$$

$$= \frac{2}{s^3}$$

$$\mathcal{L}^{-1}(\mathcal{L}(f(t))) = f(t)$$

$$\mathcal{L}^{-1}\left(\frac{2}{s^3}\right) = t^2$$

---

linear:

$$\mathcal{L}(af(x) + bg(x))$$

$$= a\mathcal{L}(f(x)) + b\mathcal{L}(g(x))$$

$= a\mathcal{L}($

$$\mathcal{L}\left(6t^2 + e^{5t}\sin(3t)\right) \quad 1$$

$$= 6\mathcal{L}(t^2) + \mathcal{L}\left(e^{5t}\sin(3t)\right)$$

$$= \frac{12}{s^3} +$$

$$\mathcal{L}\left(e^{at}f(t)\right) = F(s-a)$$

$$\boxed{\text{Ex} \quad \mathcal{L}\left(t^2 e^{5t}\right) = \frac{2}{(s-5)^3}}$$

$$3\,\mathcal{L}\left(\frac{1}{3}\sin(3t)\right)$$

$$\mathcal{L}\left(\frac{1}{\omega}\sin\omega t\right) = \frac{1}{s^2+\omega^2}$$

$\omega = 3$

$$3\left(\frac{1}{s^2+9}\right)$$

$$\boxed{= \frac{3}{(s-5)^2+9}}$$

$$① = \frac{12}{s^3} + \frac{3}{(s-5)^2+9}$$

$$\mathcal{L}(t^2) = \frac{2}{s^3}$$

$$\mathcal{L}(e^{at}t^2) = \frac{2}{(s-a)^3}$$

6.2

If we are planning on solving DEs we will need to transform derivatives!

$$\mathcal{L}\left(\frac{\partial f}{\partial t}\right) = \mathcal{L}(f') = s\underbrace{\mathcal{L}(f_{(t)})} - f(0)$$

$$= s\,F_{(s)} - f(0)$$

$$\mathcal{L}(f'') = s^2\mathcal{L}(f) - s f(0) - f'(0)$$

$$\mathcal{L}(f'') - s\,\underbrace{\mathcal{L}(f)}_{F(s)} - sf(0) - f'(0)$$

$y(0),\ y'(0)$

$$\mathcal{L}(f^{(n)}) = s^{(n)}\underbrace{\mathcal{L}(f)}_{F(s)} - s^{(n-1)} f(0)$$

$$- s^{n-2} f'(0) - \dots - s^{0} f^{(n-1)}(0)$$

$$\mathcal{L}\left(\int_0^t f(u)\,du\right) = \frac{F(u)}{u}$$

$$\mathcal{L}(t^2)$$

$F(u)\quad \frac{1}{u}$

$$= 2\mathcal{L}\left(\int_0^t u\,du\right) = 2 \cdot \frac{1}{s^2} \cdot \frac{1}{s}$$

$$= \boxed{\frac{2}{s^3}}$$

$$y'' + ay' + by = r(t) \qquad y(0) = c_1,\ y'(0) = c_2$$

$$\mathcal{L}(\text{LHS}) = \mathcal{L}(\text{RHS})$$

$$\mathcal{L}(y'') = s^2 \overset{Y(s)}{\mathcal{L}(y)} - s\,\overset{c_1}{\cancel{y(0)}} - \overset{c_2}{\cancel{y'(0)}}$$

$$\mathcal{L}(y') = a\left(s\mathcal{L}(y) - \cancel{y(0)}\right)$$

$$a\mathcal{L}(y') = a[s\mathcal{L}(y) - \underbrace{y(0)}_{c_1}]$$

$$b\mathcal{L}(y) = b\underbrace{\mathcal{L}(y)}_{Y(s)}$$

$$\text{LHS}: Y\underbrace{(s^2+as+b)}\underbrace{-sc_1-c_2-ac_1}$$

$$\text{RHS}: \mathcal{L}(r(x)) = R(x)$$

$$Y(s) = \frac{R(x) + c_1(s+a) + c_2}{s^2+as+b}$$

$$\mathcal{L}^{-1}(\text{LHS}) = \mathcal{L}^{-1}(\text{RHS})$$

$$y(t) = \mathcal{L}^{-1}(\text{RHS})$$

- Laplace Transform DE using Table
- Solve for $Y(s) = F(s)$
- Invert $F(s)$ via table

$$y'' + 3y' - 4y = 6e^{2t-3}$$

$$y(1.5) = 4, \quad y'(1.5) = 5$$

$$t = \tilde{t} + 1.5$$

$$\tilde{y}'' + 3\tilde{y}' - 4\tilde{y} = 6e^{2\tilde{t}}$$

$$\tilde{y}(0) = 4, \quad \tilde{y}'(0) = 5$$

drop ~

$y'' + 3y' - 4y = 6e^{2t}$

$y(0) = 4 \quad , \quad y'(0) = 5$

$\mathcal{L}(y'') = s^2 Y - s4 - 5$

$3\mathcal{L}(y') = 3(sY - 4)$

$-4\mathcal{L}(y) = -4Y$

$\mathcal{L}(6e^{2t}) = 6\mathcal{L}(e^{2t})$

$= \dfrac{6}{s-2}$

$Y(s^2 + 3s - 4) - s4 - 17$

$Y(s^2+3s-4)$

$$= \frac{6}{s-2}$$

$$\boxed{Y(s) = \frac{s\cdot 4+17+\frac{6}{s-2}}{s^2+3s-4}}$$