$r(r-1)+b_0 r+c_0=0$

$r_1\,,\,r_2$

$\boxed{r_1-r_2>0}$

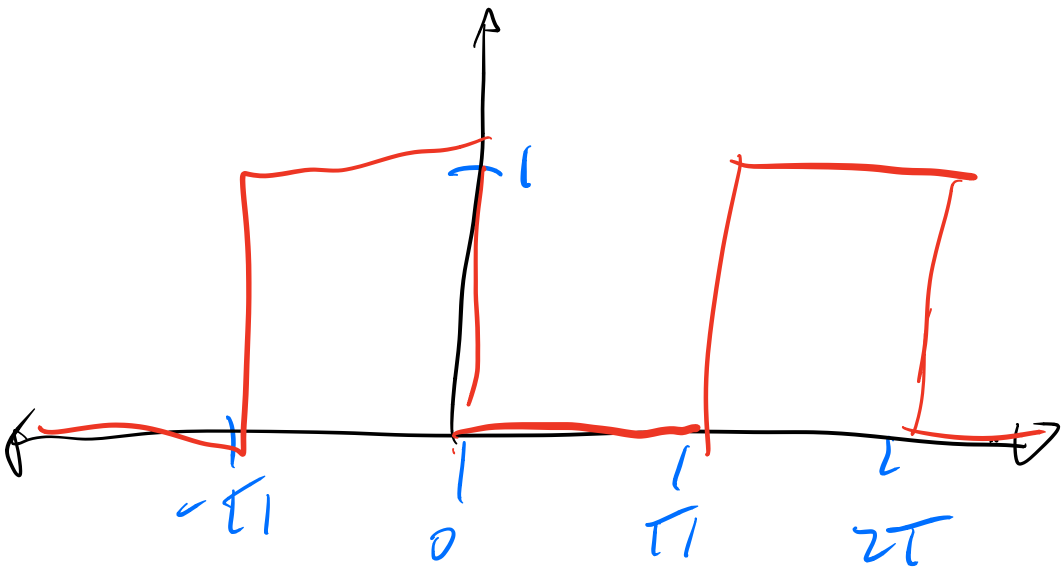
(I said $r_2-r_1>0$)

$$\boxed{y''+\beta y'+qy=r(t)}$$

β is the dampening

$$w/\ r(t)=\begin{cases}1 & 0<t<\pi\\ 0 & \pi<t<2\pi\end{cases}$$

periodic w/ period 2π



$$r(t) = \frac{1}{2} + \sum_{m=1}^{\infty} \frac{2}{(2m-1)\pi} \sin((2m-1)x)$$

$$= \frac{1}{2} + \sum_{m \text{ is odd}}^{\infty} \boxed{\frac{2}{m\pi} \sin(m k)}$$

$$y(t) = a_0 + \sum_{m=1}^{\infty} a_m \cos\left(\frac{2\pi}{p} x\right) + b_m \sin\left(\frac{2\pi}{p} x\right)$$

$$= a_0 + \sum_{m=1}^{\infty} a_m \cos(nx) + b_m \sin(nx)$$

$$y'' + \beta y' + q y = 0$$

$$y_H = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}$$

$$\lambda_{1,2} = \frac{-\beta \pm \sqrt{\beta^2 - 36}}{2}$$

first thing after solving homogeneous, is solve for A_0.

$$O(1): \; 9A_0 = \frac{1}{2} \Rightarrow A_0 = \frac{1}{18}$$

$$y = A_m \cos(mx) + B_m \sin(mx)$$

$$y' = -A_m(m)\sin(mx) + B_m(m)\cos(mx)$$

$$y'' = -A_m(m)^2\cos(mx) - B_m(m^2)\sin(mx)$$

$\theta(\cos(mx))$:

$$-A_m m^2 + \beta B_m m + 9A_m = 0$$

$\theta(\sin(mx))$:

$$\boxed{-B_m m^2 - \beta A_m m + qB_m \; \begin{array}{ll} = 0 & \text{if } m \text{ is even} \\ = \dfrac{2}{m\pi} & \text{if } m \text{ is odd} \end{array}}$$

for m is even case

$$A_m^E = \frac{1}{q}\left(A_m^2 - \beta B_m\right)$$

$$B_m^E = \frac{1}{q}\left(B_m^2 + \beta A_m\right)$$

for m is odd

$$A_m^o = \frac{-2\beta}{\ldots}$$

$$\pi((n^2-9)+\beta^2 n^2)$$

$$B_m^O = \frac{-2(n^2-9)}{n\pi((n^2-9)+\beta^2 n^2)}$$

$$y = y_h + \frac{1}{18} + \sum_{m \text{ is even}}^{\infty}\left(A_m^E \cos(m\pi x) + B_m^E \sin(m\pi x)\right)$$

$$+ \sum_{m \text{ is odd}}^{\infty}\left(A_m^O \cos(m\pi x) + B_m^O \sin(m\pi x)\right)$$

11.5-11.6 General Fourier Series

$$[p(x)y']' + [q(x) + \lambda r(x)]y = 0$$

eig. value

eig. function

weighting function

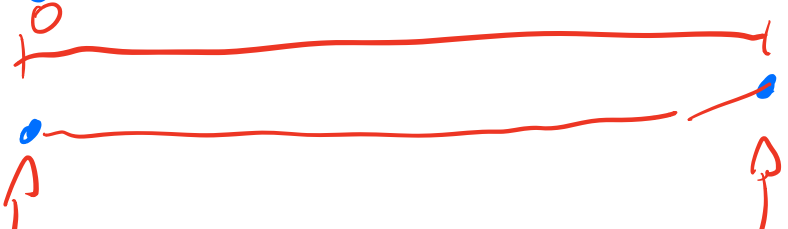
$$\boxed{A\vec{x} = \lambda\vec{x}}$$

Normally we had I.C.

$y(0) = A$, $y'(0) = B$

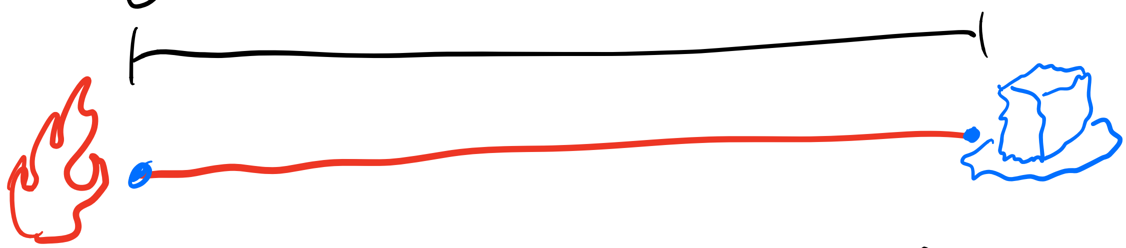
But now we have B.C.

0 L



$y(0) = 0$ $y(L) = 0$

0 L



$H(0) = 200$ $H(L) = -5$

(a) $k_1 y + k_2 y' = 0 \quad x = a$

(b) $l_1 y + l_2 y' = 0 \quad x = b$

where your S.L. problem is on the domain $[a, b]$

Trivial soln $y = 0$ is not relevant here

$$y'' + \lambda y = 0, \quad y(0) = 0$$
$$y(\pi) = 0$$

$$\ldots]' + [q(x) + \lambda r(x)]\, y = 0$$

$[p(x)y']' + [q$

eig. value

weighting function

eig. func

$p(x) = 1$

$q(x) = 0$

$r(x) = 1$

$$y'' + \lambda y = 0$$

$$x^2 + \lambda = 0$$

$$x = \sqrt{\pm\lambda}$$

$$\lambda < 0, \quad \lambda = 0, \quad \lambda > 0$$

$\boxed{\lambda < 0 \quad \lambda = -\alpha^2}$ $\alpha \in \mathbb{R}/0$

$x^2 - \alpha^2 = 0$

$x = \pm \alpha$

$y = c_1 e^{\alpha t} + c_2 e^{-\alpha t}$

$y(0) = 0$

$y(\pi) = 0$

$0 = c_1 + c_2 \Rightarrow \underline{c_1 = -c_2}$

$0 = c_1 \left(e^{\alpha\pi} - e^{-\alpha\pi} \right)$

$$0 = e^{\alpha\pi} - e^{-\alpha\pi}$$

$\alpha = 0$
$\Rightarrow \lambda = 0$

no non trivial soln.

$\lambda = 0$

$$y'' = 0$$

$$y(x) = c_1 x + c_2$$

$y(0) = 0$

$$\Rightarrow 0 = c_2$$

$y(\pi) = 0$

$$0 = c_1 \pi$$

$$\Rightarrow c_1 = 0$$

$\Rightarrow c_1 =$

Only trivial soln.

$\lambda > 0 \quad \lambda = \alpha^2$

$y'' + \alpha^2 y = 0$

$r^2 + \alpha^2 = 0$

$r = \pm i\alpha$

$y = c_1 \cos(\alpha x) + c_2 \sin(\alpha x)$

$y(0) = 0$

$\Rightarrow c_1 = 0$

$y(\pi) = 0$

$$0 = c_2 \sin(\alpha\pi)$$

$$0 = \sin(\alpha\pi)$$

$$\Rightarrow \alpha = (\dots, -2, -1, 0, 1, 2, \dots)$$

$$\Rightarrow \alpha^2 = \lambda \Rightarrow \lambda = n^2$$
$$n \in \mathbb{N}$$

eigenvalue is $\lambda = n^2$ $n \in \mathbb{N}$

eigen function $y = c_2 \sin(nx)$

or

eigen function: $\operatorname{span}\{\underline{\sin(nx)}\}$

$$\int^b (\dots)\, y_n(x)\, dx = 0$$

$$\int_a r(x)\, y_m(x)\, y_n(x)\, dx =$$

$$\int_0^{\pi} 1 \sin(mx)\sin(nx)\,dx = 0 \quad \text{if } m \neq n$$

$$= \gamma \quad \text{if } m = n$$

$$(1-x^2)y'' - 2xy' + n(n+1)y = 0$$

legendre eqn.

$$\left[(1-x^2)y'\right]' + \lambda y = 0$$

$$\lambda = n(n+1)$$

$$(1-x^2)y'' - 2x y' + \lambda y$$

has soln. legendre polynomials

$P_n(x)$

$$\int_{-1}^{1} \underset{P_0}{1} \cdot \underset{P_1}{x} \, dx = 0$$

$$\frac{x^2}{2}\Big|_{-1}^{1} = \frac{1}{2} - \frac{1}{2} = 0$$

$$\boxed{\int_{-1}^{1} P_n(x)\, P_m(x)\, dx = 0 \text{ if } m \neq n}$$

General Fourier series

which allows you to represent a function as any soln. to a orthogonal basis found via S.L..

$$f(x) = \sum_{m=0}^{\infty} a_m y_m(x)$$

Ex: $$f(x) = \sum_{m=0}^{\infty} a_m P_m(x)$$

a_m?

fixed n

$$\int^1 P_n(x) f(x)\, dx = \sum^{\infty} a_m \int^1 P_n(x) P_m(x) dx$$

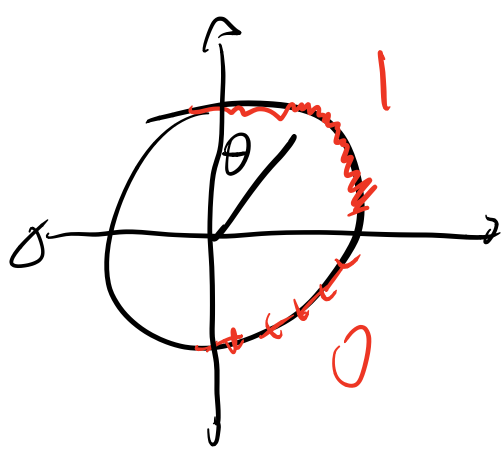
$\int_{-1}$ ... $m=0$ ...

$m=n$

$$\frac{2}{2m+1}$$

$$\int_{-1}^{1} P_m(x) f(x)\, dx = \frac{2}{2m+1} a_m$$

$$\boxed{a_m = \frac{2m+1}{2} \int_{-1}^{1} P_m(x) f(x)\, dx}$$



$f(\theta) = \begin{cases} 1, & 0 < \theta < 90^\circ \end{cases}$

$$f(\theta) = \begin{cases} 1, \\ 0, & 90^\circ < \theta < 180^\circ \end{cases}$$

let $z = \cos\theta$

$$f(z) = \begin{cases} 1, & 1 \geq z > 0 \\ 0, & 0 \geq z > -1 \end{cases}$$

$P_0(z) = 1$, $P_1(z) = z$

$P_2(z) = \frac{1}{2}(3z^2 - 1)$,

$P_3(z) = \frac{1}{2}(5z^3 - 3z)$

$$f(z) = \sum_{m=0}^{\infty} c_m P_m(z)$$

$$a_m = \frac{2m+1}{2} \int_{-1}^{1} P_n(x) f(x)\, dx$$

$$c_0 = \frac{1}{2} \int_{-1}^{1} f(z) P_0(z)\, dz$$

$$= \frac{1}{2} \int_{0}^{1} P_0(z)\, dz = \frac{1}{2}$$

$$c_1 = \frac{3}{2} \int_{-1}^{1} f(z) P_1(z)\, dz$$

$$= \frac{3}{4}$$

$$C_2 = \frac{5}{2}\int_{-1}^{1} f(x)\, P_2(z)\, dz$$

$$= 0$$

$$C_3 = -\frac{7}{16}$$

$$f(z) = \frac{1}{2} P_0(z) + \frac{3}{4} P_1(z) - \frac{7}{16} P_3(z) + \ldots$$

$$\boxed{\begin{aligned} f(\theta) = {} & \frac{1}{2} P_0(\cos\theta) + \frac{3}{4} P_1(\cos\theta) \\ & - \frac{7}{16} P_3(\cos\theta) + \ldots \end{aligned}}$$

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$$\tilde{x}^2 y''(\tilde{x}) + \tilde{x}\, y'(\tilde{x}) + (\tilde{x}^2 - n^2)\, y(\tilde{x}) = 0$$

$$y = J_n$$

$$x = \tilde{x}/k$$

$$k^2 x^2 y''(kx) + kx\, y'(kx) + (k^2 x^2 - n^2)\, y(kx) = 0$$

$$\left[x\, y'(kx)\right]' + \left(\frac{-n^2}{x} + \lambda_x\right) y(kx) = 0$$

$$\cancel{x}\; y''(kx)\, k$$

$p(x) = x$

$q(x) = -\frac{n^2}{x}$ $r(x) = x$

$\lambda = -n^2$

$$\boxed{\int_0^R x J_n(x) J_m(x)\,dx = 0}$$

except when $m = n$