Hwmk 6

Math 528 Summer Session 1

Due 5/28 (Friday at 11:59 pm)

1 CarnEvil Ride

Suppose you are designing a bungee carnival ride where the passenger and craft is pulled up and down by a large bungee cable suspended from a piston. To pick out the optimal engine designed to move the ride, you can study the differential equation

$$mh'' + \gamma h' + kh = F_0 \cos(\omega t)$$

where $h(t)$ is your height as a function of time, m is the mass of the craft, γ is the dampening force, k is the spring constant of the bungee material, and F_0 is the maximum force output of the engine. The weight of the craft is around 640 pounds. The spring constant of your bungee cable is 20 lbs/ft. The dampening system reports a value of 200 pounds of force for an object moving $10 ft/s$. The last thing you will need is the following formula that provides the equation for the amplitude (C) of a forced, steady periodic oscillation:

$$C = \frac{F_0}{\sqrt{(k - m\omega^2)^2 + (\gamma\omega)^2}}$$

(a) [1 point] To begin this problem, use the following relationships and the data provided above, to solve for constants m and γ. Weight (W) is the measurement of force exerted by gravity (g) on your mass (m) thus: $W = mg$ (Note gravity is $32\frac{ft}{s}$). The force exerted by your repelling device (here specified in pounds), is directly proportional to your velocity (v) thus: $W = \gamma v$.

(b) [1 point] Using the calculated values, what frequency ω should the piston move to be at the same frequency of the system/ the homogeneous solution (this will keep the ride oscillations more predictable)?

(c) [1 point] Using the calculated values (including ω), what should the maximum power output of the engine (F_0) be to reach a amplitude of 50 ft (height 100ft)?

(d) [5 points] With the calculated F_0 above, solve the differential equation for your awesome ride using variation of parameters (find the particular solution). But wait... Oh no! The dampening system has malfunctioned making $\gamma = 0$ (you will also need to recalculate the homogeneous here)! This happened at the peak height at 100ft ($h(0) = 100, h'(0) = 0$). This problem will be difficult, feel free to use wolfram alpha or some other online calculator to calculate middle steps.

(e) [2 points] At what speed will the riders meet their inevitable collision with the ground? This will be at the first intersection with $h = 0$