Hwmk 16

Math 528 Summer Session 1

Due 6/16 (Wednesday at 11:59 pm)

1   "There's Your Sine"

Find the Fourier series of the following functions:

(a) $\boxed{\text{2 points}}$

$$f(x) = |x| \quad \text{for} \quad -\pi < x < \pi \quad \text{and} \quad f(x + 2\pi) = f(x)$$

(b) $\boxed{\text{2 points}}$

$$f(x) = 1 - \frac{x^2}{4} \quad \text{for} \quad -2 < x < 2 \quad \text{and} \quad f(x + 4) = f(x)$$

2   AC/DC

Suppose you have a RLC circuit component and you supply it an alternating current (AC). We can model the current in the component $I(t)$ via the differential equation:

$$LI'' + RI' + \frac{1}{C}I = E'(t)$$

where L is the inductance, R is the resistance, C is the capacitance, and E is the electromotive force. Suppose the resistance is 10 ohms, the inductance is 1 henry and the capacitance is 0.1 farads. You have an alternating electromotive force that is periodic with period 2π:

$$E(t) = \begin{cases} 100(t - t^2) & \text{if } -\pi < t < 0 \\ 100(t + t^2) & \text{if } 0 < t < \pi \end{cases}$$

Initial conditions are that $I(0) = 0$ and $Q(0) = 0$ (where $Q(t) = \int I(t)dt$):

(a) $\boxed{\text{1 point}}$ Solve the homogeneous problem (the transient element)

(b) $\boxed{\text{2 points}}$ Transform $E'(t)$ (the driver)

(c) $\boxed{\text{3 points}}$ Using a Fourier Series anzats, solve for the coefficients of your solution.