Hwmk 15

Math 528 Summer Session 1

Due 6/15 (Tuesday at 11:59 pm)

1 “Wait a Minute... That’s a Bessel Function!”

The motion for the free undamped motion of a mass on an aging spring is given by:

$$m\frac{d^2y(t)}{dt^2} + ke^{-\alpha t}y(t) = 0$$

where $\alpha > 0$. The change of variables:

$$s = \frac{2}{\alpha}\sqrt{\frac{k}{m}}\,e^{-\alpha t/2}$$

can be used to transform the differential equation into a Bessel’s equation. You will be using this transformation to write down the general solution to the original differential equation in terms of Bessel functions.

(a) $\boxed{\text{2 points}}$ Rewrite the differential operator $\frac{d}{dt}$ in terms of s.

(b) $\boxed{\text{2 points}}$ Substitute s and $\frac{d}{ds}$ to transform the differential equation into the Bessel differential equation.

(c) $\boxed{\text{1 point}}$ What is ν

(d) $\boxed{\text{1 point}}$ What is the general solution?

2 Chains on the Brain Again

The radial displacement, $r(x)$ of a rotating chain in equilibrium is:

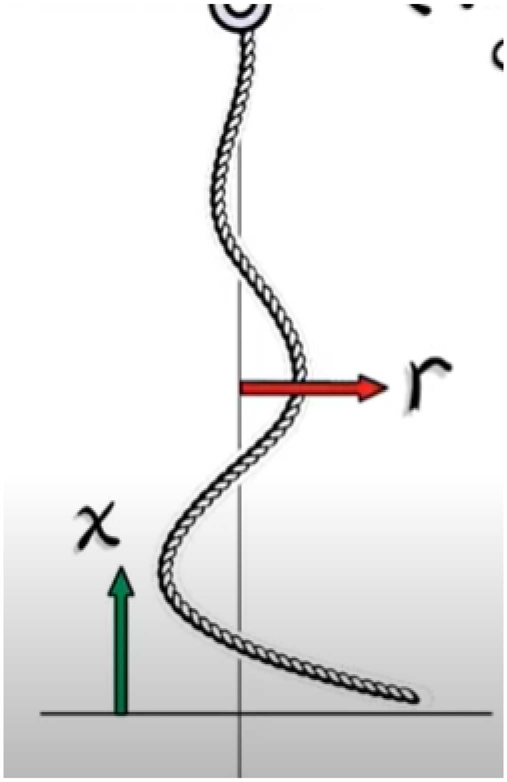
$$r(x) \sim J_0(2w\sqrt{(x/g)})$$

where w is the angular frequency, g is gravity and x is the distance from the bottom (check image below).

(a) $\boxed{\text{2 points}}$ Using the series definition of the Bessel function, calculate $J_0(2w\sqrt{(x/g)})$ up to $\mathcal{O}(x^2)$ (this is accurate for small w)

(b) $\boxed{\text{1 point}}$ Solve for the root(s).

(c) $\boxed{\text{1 point}}$ By what factor does my root(s) change if I double the rate I rotate the chain?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