Hmwk 10

$$LI'' + RI' + \frac{I}{C} = E'(t)$$

$$I(0) = 0$$

$$0 = Q(0) \Rightarrow LI'(0) + RI(0) + \cancel{Q(0)}^{0}$$

$$= LI'(0) + R\cancel{I(0)}_{0} = E(0) \quad (\sin(0) = 0)$$
$$0$$

$$\Rightarrow I'(0) = 0$$

1a) $255\cos(t) \quad 0 < t < 2\pi$

$$\Rightarrow \boxed{E'(t) = 255\cos(t)\left(H(t) - H(t-2\pi)\right)}$$

1b) $\mathcal{L}(LI'') = L\left(s^2\tilde{I} - sI(0) - I'(0)\right)$

$= Ls^2\tilde{I}$

$\mathcal{L}(RI') = R\left(s\tilde{I} - \cancel{I(0)}^{\,0}\right)$

$= Rs\tilde{I}$

$\mathcal{L}\left(255\cos(t)\left(H(t) - H(t-2\pi)\right)\right)$

$= 255\left(\mathcal{L}\left(\cos(t)H(t)\right) - \mathcal{L}\left(\cos(t)H(t-2\pi)\right)\right)$

(s-shifting)

$= 255\left(\left(\frac{s}{s^2+1}\right) - \left(e^{-2\pi s}\frac{s}{s^2+1}\right)\right)$

$$= 225 \frac{s}{s^2+1}\left(1 - e^{-2\pi s}\right)$$

$$L s^2 \tilde{I} + R s \tilde{I} + \frac{\tilde{I}}{C} = 225 \frac{s}{s^2+1}\left(1 - e^{-2\pi s}\right)$$

$$\boxed{\tilde{I} = 225 \frac{s}{(s^2+1)(L s^2 + R s + \frac{1}{C})}\left(1 - e^{-2\pi s}\right)}$$

$$\frac{s}{(s^2+1)(s^2+2s+10)}$$

$$= \frac{1}{85}\left(\frac{-9s-20}{(s^2+2s+10)} + \frac{9s+2}{(s^2+1)}\right)$$

$$= \frac{1}{85}\left(-9\left(\frac{s+1}{(s+1)^2+9}\right) - \frac{11}{3}\left(\frac{3}{(s+1)^2+9}\right) + 9\left(\frac{s}{s^2+1}\right) + 2\left(\frac{1}{s^2+1}\right)\right)$$

$$= \frac{1}{85}\left(-9\mathcal{L}^{-1}\left(\frac{s+1}{(s+1)^2+9}\right) - \frac{11}{3}\mathcal{L}^{-1}\left(\frac{3}{(s+1)^2+9}\right) + 9\mathcal{L}^{-1}\left(\frac{s}{s^2+1}\right) + 2\mathcal{L}^{-1}\left(\frac{1}{s^2+1}\right)\right)$$

(s-shifting)

$$= \left(\frac{-9}{85}\cos(3t) - \frac{11}{175}\sin(3t)\right)e^{-t} + 9\cos(t) + 2\sin(t)$$

$\frac{1}{85} \qquad 85$

$f(t)$

Notice that we have $-F(s)e^{-2\pi s}$ as the second piece to invert

$$\Rightarrow -\mathcal{L}^{-1}\left(F(s)e^{-2\pi s}\right)$$

$$= -f(t-2\pi)H(t-2\pi)$$

Thus:

$$y(t) = 225\left(f(t) - f(t-2\pi)H(t-2\pi)\right)$$

Notice that

$$f(t) = \left(\frac{-9}{85} \cos(3t) - \frac{11}{175} \sin(3t) \right) e$$

$$+ 9 \cos(t) + 2 \sin(t)$$

(since $\cos(x)$ and $\sin(x)$ are 2π periodic)

$$\Rightarrow f(t-2\pi)$$

$$= \frac{-9}{85} \cos(3t) - \frac{20}{255} \sin(3t)\, e^{-t+2\pi}$$

$$+ 9 \cos(t) + 2 \sin(t)$$

$$\Rightarrow f(t) - f(t-2\pi)$$

$$= \underbrace{\left[\left(\frac{-9}{85} \cos(3t) - \frac{11}{175} \sin(3t) \right) e^{-t} \left(1 - e^{2\pi} \right) \right]}_{g(t)}$$

Thus for

	$y(t)$
$t < 0$	0
$0 < t < 2\pi$	$f(t) \cdot 225$
$2\pi < t$	$g(t) \cdot 225$

2f I meant to specify to just plot the non transient solution (so not the homogeneous piece)

Plotted in Mathematica