Hmwk 18

1. a. $f(x) = e^{2ix}$ if $-1 < t < 1$

$\qquad\qquad 0$ elsewhere

$$\hat{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\, e^{i\omega x}\, dx$$

(integral for $f(x)=0$ is 0)

$$= \frac{1}{\sqrt{2\pi}} \int_{-1}^{1} e^{2ix} e^{i\omega x}\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-1}^{1} e^{(2+\omega)ix}\, dx$$

let $\alpha = (2+\omega)$

(usub works here)

$$= \frac{1}{\sqrt{2\pi}} \left(\frac{1}{\alpha i}\right) e^{\alpha ix} \Big|_{-1}^{1}$$

$$= \frac{-i}{\sqrt{2\pi}\,\alpha}\left(e^{\alpha i x} - e^{-\alpha i x}\right)$$

$$\left(\sin(\theta) = \frac{1}{2i}\left(e^{i\theta} - e^{-i\theta}\right)\right.$$

$$= \frac{2\sin(\alpha)}{\alpha\sqrt{2\pi}} = \boxed{\sqrt{\frac{2}{\pi}}\,\frac{\sin(2+\omega)}{(2+\omega)}}$$

1b. (similar to above)

$$\hat{f}(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-1}^{0} x e^{-x} e^{i\omega x}\,dx$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-1}^{0} x e^{(i\omega-1)x}\,dx$$

$$(i\omega - 1) = \alpha$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-1}^{0} x e^{\alpha x}\,dx$$

([illegible] by parts

(cal 2 trick w/ integr [illegible]
to remove x)

$$= \frac{1}{\sqrt{2\pi}} \left(\frac{e^{-\alpha}}{\alpha} - \frac{1}{\alpha} \int_{-1}^{0} e^{\alpha x} \, dx \right)$$

$$= \frac{1}{\sqrt{2\pi}} \left(\frac{e^{-\alpha}}{\alpha} - \frac{1}{\alpha^2} e^{\alpha x} \right) \Bigg|_{-1}^{0}$$

$$= \dots = \frac{e^{-\alpha} - 1}{\alpha^2} + \frac{e^{-\alpha}}{\alpha}$$

or

$$= \boxed{\frac{e^{-\alpha}\left(\alpha - e^{\alpha} + 1\right)}{\alpha^2} \quad w/ \alpha = (i\omega - 1)}$$

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$$

F.1.

$$\Rightarrow \cancel{\frac{1}{\sqrt{2\pi}}}\int_{-\infty}^{\infty} \frac{\partial^2 u}{\partial t^2}\, e^{i\omega x}\, dx = \cancel{\frac{1}{\sqrt{2\pi}}}\int_{-\infty}^{\infty} \frac{\partial^2 u}{\partial x^2}\, e^{i\omega x}\, dx$$

(Integral is in x) (Page 527 (10))

$$\Rightarrow \frac{\partial^2}{\partial t^2}\int_{-\infty}^{\infty} u e^{i\omega x}\, dx = -\omega^2 \int_{-\infty}^{\infty} u\, e^{i\omega x}\, dx$$

$$\Rightarrow \boxed{\frac{\partial^2 \hat{u}(\omega, t)}{\partial t^2} = -\omega^2\, \hat{u}(\omega, t) \quad \text{or} \quad \ddot{\hat{u}} = -\omega^2 \hat{u}}$$