Hmwk 16

1a)

$$f(x) = |x| \text{ for } -\pi < x < \pi$$

$$p = 2\pi \Rightarrow L = \pi$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |x| \, dx = \frac{\pi^2}{2\pi} = \frac{\pi}{2}$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} |x| \cos\left(\frac{\cancel{\pi}}{\cancel{\pi}} nx\right) dx$$

$$= \frac{1}{\pi} \left(\int_{-\pi}^{0} -x \cos(nx) \, dx + \int_{0}^{\pi} x \cos(nx) \, dx \right)$$

$$= \frac{2}{\pi n^2}\left(\underbrace{\cos(n\pi) - 1} + n\pi \sin(n\pi)\right)$$

n is even $= 0$

n is odd $= -2$

$b_n = 0$ since $f(x)$ is even

$$= \frac{-4}{\pi n^2} \text{ for } n \text{ odd}$$

$$\boxed{|x| = \frac{\pi}{2} + \sum_{n \text{ odd}}^{\infty} \frac{-4}{\pi n^2}\cos(nx)}$$

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16) $f(x) = 1 - \frac{x^2}{4}$

$$a_0 = \frac{1}{4}\int^{2} 1 - \frac{x^2}{\,} \, dx = \frac{1}{4}\left(x - \frac{x^3}{12}\right)\Big|^{2}$$

$$\int_{-2} \quad 4 \quad \Big|_{-2}$$

(mathematical)

$$= 2/3$$

$$a_n = \frac{1}{2}\int_{-2}^{2}\left(1 - \frac{x^2}{4}\right)\cos\left(\frac{n\pi x}{2}\right)dx$$

(simp even)

$$= \int_{0}^{2}\left(1 - \frac{x^2}{4}\right)\cos\left(\frac{n\pi x}{2}\right)dx$$

(mathematical)

$$= \left(4\,\xcancel{\sin(\pi n)}^{\,0} - 4\pi n\,\underbrace{\cos(\pi n)}_{(-1)^n}\right)\frac{1}{(\pi n)^3}$$

$$= \frac{4(-1)^{n+1}}{\pi^2 n^2}$$

$b_n = 0$ since $1 - \frac{x^2}{4}$ is odd!

$$\Rightarrow \boxed{f(x) = \frac{2}{3} + \sum_{n=1} \frac{4(-1)^{n+1}}{\pi^2 n^2} \cos\left(\frac{n\pi x}{2}\right)}$$

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2. $LI'' + RI' + \frac{1}{C} I = E'(t)$

$$E(t) = \begin{cases} 100(t - t^2) & -\pi < t < 0 \\ 100(t + t^2) & 0 < t < \pi \end{cases}$$

$I(0) = 0$

$I'(0) = ?$

We know:

Not necessary since you never use the I.C.s

$$LI' + RI + \frac{1}{C} Q = E(t)$$

$Q(0)=0 \Rightarrow LI'(0)+R\cancel{I(0)} = E(0)$

0

$$I'(0) = \frac{E(0)}{L} = 0$$

$\Rightarrow I'(0)=0$

2a) Homogeneous part

$$LI'' + RI' + \frac{1}{C}I = 0$$

$$Lx^2 + Rx + \frac{1}{C} = 0$$

$$\Rightarrow x = \frac{-R \pm \sqrt{R^2 - \frac{4L}{C}}}{2L}$$

(Plugging in $R=10$, $L=1$, $C=.1$)

$(5+\sqrt{15})x$ $\quad$ $(5-\sqrt{15})x$

$$\Rightarrow \boxed{y_H(x) = c_1 e \qquad + c_2 e}$$

2b) $p = 2\pi \Rightarrow L = \pi$

$$E'(t) = \underbrace{\begin{cases} 100(1-2t) & -\pi < t < 0 \\ 100(1+2t) & 0 < t < \pi \end{cases}}_{\text{Even!}}$$

$$\Rightarrow b_n = 0$$

$$a_0 = \frac{100}{\pi} \int_0^{\pi} 1 + 2t \, dt = \frac{100}{\pi} \cdot (\pi + \pi^2)$$

even

$$a_n = \frac{200}{\pi}\int_0^{\pi} (1+2t)\cos(nx)\,dx$$

$$= \frac{200}{\pi}\,\frac{(2\pi+1)\cancel{\sin(\pi n)}\,n + 2\cos(\pi n) - 2}{n^2}$$

0 even

n even = 0

odd $= \frac{-4}{n^2}\cdot\frac{200}{\pi}$

$= (-1)^n 400$

$$\boxed{f(t) = 100(1+\pi^2) + \frac{400}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n - 1}{n^2}\cdot\cos(nx)}$$

2c.

$$\sum_{}^{\infty} a_n \cos(nx)$$

$$y(x) = a_0 + \sum_{n=1} a_n \cos(nx)$$

$$+ \sum_{n=1}^{\infty} b_n \sin(nx)$$

Looking at a_0 first

$y(x) = a_0$

$$\Rightarrow \frac{a_0}{10} = 100(1+\pi^2)$$

$$\boxed{a_0 = 1000(1+\pi^2)}$$

Plugging in the rest

$\theta(\sin(nx))$:

$-n^2 b_n - 10 n a_n + 10 b_n = 0$

$\theta(\cos(nx))$:

$$-n^2 a_n + 10 n b_n + 10 a_n = \frac{400}{\pi}\left(\frac{(-1)^n - 1}{n^2}\right)$$

Splitting into two cases is easier for n is even or odd, but it using mathematica!

$$\Rightarrow \boxed{a_n = \frac{800(n^2 - 10)}{n^2(100 + 80n^2 + n^4)\pi} \qquad b_n = \frac{-8000}{(60n^3 + n^5 + 100n)\pi}}$$

(80n

Do not solve for C_1 and C_2! although I included I.C.s I didn't request it. much to hard!